EVALUATION OF PROCESS ZONE BY USING Jext INTEGRAL UNDER LARGE SCALE YIELDING

— STRUCTURE OF J INTEGRAL —

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ABSTRACT

The Jext integral is considered as the release rate of the potential energy per unit advance of the crack tip and the distributed dislocations. A compact type specimen is analysed by F.E.M using finite deformation theory. On the basis of these results, the relation between the process zone and evaluated values of the Jext integral along several contours is studied. At the same time the values of the J and J*ext integrals are compared with them. These results indicate a relationship between the process zone and the values of the Jext integral. This relation is discussed as it depends on the load and the size of the process zone and the following equations are obtained:

$$\text{Jext.pz} = C\,P\,r = \frac{2C}{\sigma_Y} J\,P \qquad (\,C \text{ about } 30\ \text{m}^{-1}\,)$$

where Jext.pz: Jext integral along a path very close to process zone
P: load
r: representative distance of contour from the crack tip along the x axis
σ_Y: yield stress

KEYWORDS

Jext integral; distortion tensor; Peach-Koehler force; process zone; J integral

§1 INTRODUCTION

It is undeniable that J_{IC} is one of the best fracture criteria of elastic-plastic fracture mechanics which has appeared up to the present. There have been various discussions on the structure of the J integral, but it is not yet completely settled. It is well known that at the tip of a crack there is an intensely deformed nonlinear zone called the process zone, and in this zone there occurs the development of holes, tearing and other fracture process which resist the current plasticity treatment.

Rice and Johnson (1970) have conducted analysis in this region using slip line theory and more recently large strain, finite element analysis, (recent work of Rice and McMeeking,1975). This problem is also treated by McMeeking (1977a,1977c) and Atluri (1977) and the J integral is discussed as a parameter characterizing the near tip field.

On the other hand, Kageyama and Miyamoto (1978) introduced the concept of Jext which dose not lose physical meaning in the elastic-plastic state. This concept is considered to be the release rate of the potential energy per unit advance of the crack tip and the distributed dislocations. Jext is equivalent to Q_ℓ which was introduced by Bilby (1973).

In this paper, a compact type specimen is analysed by the finite element method using finite deformation theory. The evaluation of the process zone is made by calculating Jext in the near tip field. These results are compared with those of infinitesimal deformation and slip line theory relating to the connexion between Jext, J, J*ext and P.

§2 Jext AND ITS PHYSICAL MEANING

The Jext integral is based on the energy consideration of a body with eigen-distortions. Consider the case when an eigen-distortion β^*_{ji} is distributed in a body and an applied force acts on the boundary of the body. In the two-dimensional case the Jext integral is defined by

$$\mathrm{Jext} = \int_\Gamma (W^e dx_2 - T_i\beta_{1i} ds) \qquad (2.1)$$

where Γ is a closed path, W^e is the elastic strain energy density, T_i is the x_i component of the traction vector on ds, and β_{1i} is the elastic distortion tensor. By using the dislocation density tensor Eq. (2.1) is transformed to

$$\mathrm{Jext} = \int_\Omega (\sigma_{i2}\alpha_{3i} - \sigma_{i3}\alpha_{2i}) dx_1 dx_2 \qquad (2.2)$$

where Ω is the domain enclosed by Γ, σ_{ij} is the stress tensor and α_{ij} is the dislocation density tensor and is given as follows:

$$\alpha_{ij} = \varepsilon_{ik\ell}\partial_k\beta_{\ell j} = -\varepsilon_{ik\ell}\partial_k\beta^*_{\ell j} \qquad (2.3)$$

where ∂_k means $\partial/\partial x_k$ and $\varepsilon_{ik\ell}$ is the permutation tensor. Equation (2.2) represents the x_1 component of the Peach-Koehler force on continuously distributed dislocations in the region Ω. In case Γ encloses a small region near the crack tip, the crack may be replaced with the equivalent distribution of dislocations. Jext can be interpreted as a potential energy release rate when the crack tip and dislocations move by $\delta\xi_1$ in the x_1 direction:

$$\mathrm{Jext} = -\lim_{\delta\xi_1\to 0} \frac{\delta\Pi}{\delta\xi_1}\bigg|_{\Omega,\alpha_{ij},\mathrm{crack}} \qquad (2.4)$$

This interpretation of the Jext integral is applicable independently of the mechanism of plastic deformation. Therefore, the Jext concept is available for use not only in the incremental theory of plasticity but also in the physical theory of plasticity, and we can apply it, for example, to the unloading process and to stable crack propagation.

If the contour Γ passes through the elastic region, Jext is equal to J and is independent of the choice of contour. But if the contour passes through the plastic region, the Jext integral depends on the choice of contour. For the study of elasto-plastic fracture mechanics, the region Ω should be related to the crack tip fracture process zone and Jext is expected to represent the physical state of the process zone.

Next consider a case when eigen-strain is distributed in a body D and a constant applied traction acts on the surface $|D|$. If the distribution of eigen-strain changes by $\delta\varepsilon^*_{ij}$ in the domain Ω, the change of the potential energy is given by

$$\delta\Pi = -\int_\Omega \sigma_{ij}\delta\varepsilon^*_{ij} dv \qquad (dv = dx_1dx_2dx_3) \qquad (2.5)$$

The J*ext integral is defined as the change of potential energy per unit parallel movement of the eigen-strain distribution in Ω. In the two-dimensional case, there is a simple relation between J, Jext and J*ext:

$$\mathrm{J^*ext} = -\lim_{\delta\xi_1\to 0}\frac{\delta\Pi}{\delta\xi_1} = -\int_\Omega \sigma_{ij}\partial_1\varepsilon^*_{ij} dv$$

$$= \mathrm{Jext} - \int_\Gamma T_i\beta^*_{1i} ds \qquad (2.6)$$

$$\mathrm{Jext} = \int_\Gamma (W^e dx_2 - T_i\beta_{1i} ds) \qquad (2.1)$$

$$\therefore \mathrm{J^*ext} = \int_\Gamma [W^e dx_2 - T_i(\beta_{1i}+\beta^*_{1i}) ds]$$

$$= \int_\Gamma [W^e dx_2 - T_i\partial_1 u_i ds] \qquad (2.7)$$

$$\mathrm{J} = \int_\Gamma [W dx_2 - T_i\partial_1 u_i ds] \qquad (2.8)$$

$$= \int_\Gamma [(W^e+W^p) dx_2 - T_i(\beta_{1i}+\beta^*_{1i}) ds] \qquad (2.9)$$

or

$$\mathrm{J^*ext} - \mathrm{Jext} = -\int_\Gamma T_i\beta^*_{1i} ds \qquad (2.6)$$

$$\mathrm{J} - \mathrm{J^*ext} = \int_\Gamma W^p dx_2 \qquad (2.9)$$

$$\mathrm{J} - \mathrm{Jext} = \int_\Gamma [W^p dx_2 - T_i\beta^*_{1i} ds] \qquad (2.10)$$

§3 THE BEHAVIOR OF Jext IN THE C-T SPECIMEN

A compact type specimen, shown in Fig. 3.1, is analysed by the finite element method using finite deformation theory for a 2-dimensional plane strain state. The material properties are shown in Table 3.1.

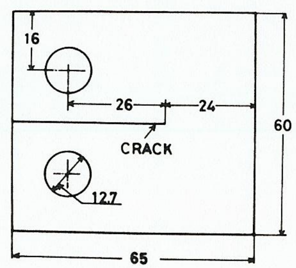


Fig. 3.1 Compact type specimen

TABLE 3.1 Material Properties

Young's modulus	E = 206 GPa
Poisson's ratio	ν = 0.3
Yield stress	σ_Y = 549 MPa
Hardening ratio	H = 981 MPa

Fig. 3.2 shows the finite element mesh of the half part of the C-T specimen. As recent results (McMeeking,1977b) show that the finite element calculations modelled the blunting of an initially-sharp crack, even though the tip actually had a finite root-radius in the undeformed configuration, so considering the blunting of the crack tip, the crack is assumed to be a slit whose width is 0.125 mm before deformation. To evaluate the J, Jext and J*ext values, the path integral method is employed, and the thick lines show the sixteen contours for calculation of J.

Fig. 3.3 shows the computed load P versus load- line displacement curve. In this analysis the general yield load per unit width is 3.11 MN/m. And also, the general yield load by slip line analysis (Shiratori and Miyoshi,1979) is obtained by the following equation:

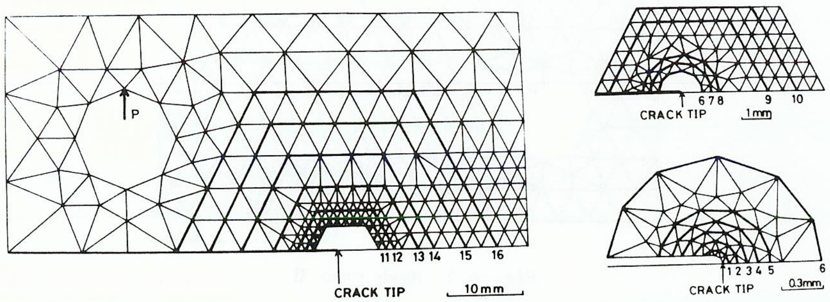


Fig. 3.2 Finite element mesh(1) of C-T specimen

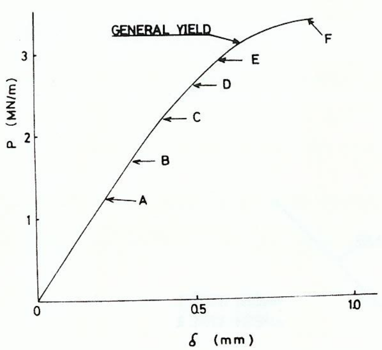


Fig. 3.3 Load P versus load line displacement δ

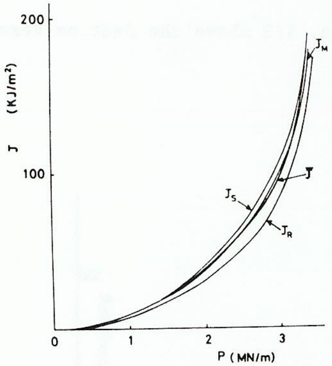


Fig. 3.4 J versus P

$$T = 2kb \{ C_2(R/b)-1 \} \quad (3.1)$$

where yield stress in shear $k = \sigma_Y/\sqrt{3} = 317$ MPa (3.2)

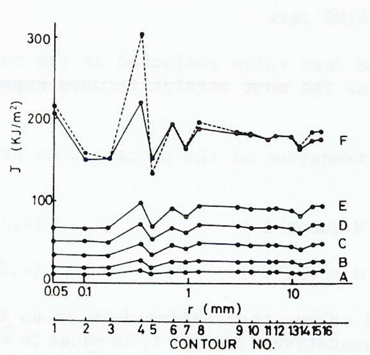
ligament width $b = 24$ mm (3.3)

and $C_2 = 2.57$ (3.4)

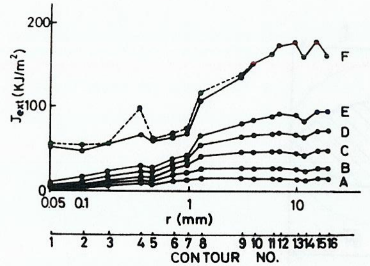
$$R/b = 1.052[-(\frac{W}{b}-1)+\{(\frac{W}{b}-1)^2+0.740(\frac{W}{b}-\frac{1}{2})\}^{\frac{1}{2}}] = 0.471 \quad (3.5)$$

Substituting Eq. (3.2)~(3.5) in Eq. (3.1), we obtain T=3.20 MN/m. This result agrees well with that of finite element analysis.

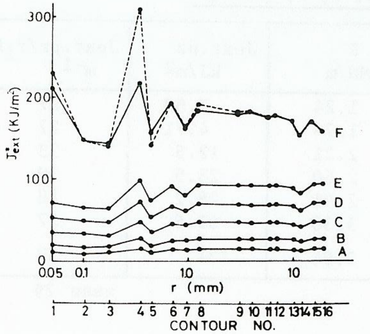
Fig. 3.4 shows the J-P curve. In this, $\bar{J}$ is the average of sixteen contours, and J_R, J_M and J_S are evaluated by the P-δ curve in Fig. 3.3, where J_R, J_M and J_S are evaluated by Rice's (1973), Merkle's (1974) and Shiratori's (1979) equation respectively. In this case J_M agrees best with $\bar{J}$. Fig. 3.5 shows the values of J, Jext and J*ext on each contour at the load stages A~F, in Fig. 3.3, where r shows the representative distance of the contour measured from the crack tip on the x axis. The J value shows path independence, and the Jext value path dependence. The results for infinitesimal deformation are shown by the dotted line. Fig. 3.6 shows the difference between the J, Jext and J*ext values.



(a) J



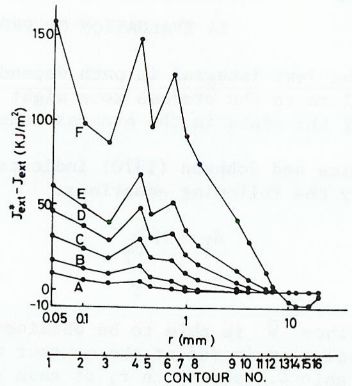
(b) Jext



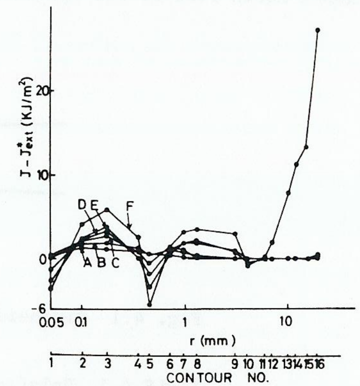
(c) J^*_{ext}

------infinitesimal deformation

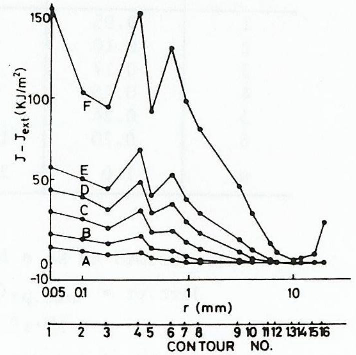
Fig. 3.5 J, Jext and J*ext distribution on each contour



(a) J*ext-Jext $(=-\int_\Gamma T_i \beta^*_{1i} ds)$



(b) J-J*ext $(=\int_\Gamma W^P dx_2)$



(c) J-Jext $(=\int_\Gamma [W^P dx_2 - T_i \beta^*_{1i} ds])$

Fig.3.6 Difference between J, Jext and J^*_{ext}

§4 EVALUATION OF PROCESS ZONE USING Jext

The Jext integral is path dependent, and the Jext value evaluated at the contour close to the process zone might be thought as the most straightforward expression of the state in the process zone.

Rice and Johnson (1970) indicate that the dimensions of the process zone are estimated by the following equations:

$$\delta_T = M \frac{J}{\sigma_Y} \qquad \text{(M about 1)} \qquad (4.1)$$

$$\overline{W} = \alpha\, \delta_T \qquad \text{(} \alpha \text{ about 2)} \qquad (4.2)$$

Since $\overline{W}$ is able to be obtained from the J value, then define Jext.pz as the Jext value evaluated at the contour whose representative distance r_i is equal to $\overline{W}$ (Fig. 4.1). Table 4.1 shows the r_i of each contour. $\overline{J}$ value and load P at $r_i=\overline{W}$, and Jext.pz. Further, in Table 4.1 the result of the different mesh divisions shown in Fig. 4.2 is also indicated as contour No. a. This mesh is used as a standard mesh for the Round Robin Test of C-T specimen by the JSME PSC 21 subcommittee.

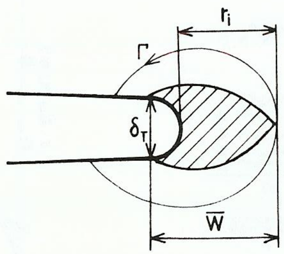


Fig. 4.1 Dimensions of process zone and contour

TABLE 4.1 Relation between r, $\overline{J}$, P, Jext.pz

Contour No. i	r_i (= $\overline{W}$) mm	$\overline{J}$ kJ/m^2	P MN/m	Jext.pz kJ/m^2	Jext.pz/r_iP = C m^{-1}
1	0.05	13.7	1.24	1.68	27
2	0.10	27.4	1.74	4.71	27
3	0.17	46.6	2.21	12.5	33
4	0.25	68.6	2.60	23.5	30
5	0.34	93.3	2.91	28.2	28
6	0.70	192.	3.38	63.6	27
a	1.0	275.	3.94	131.	33
					mean 29

Jext.pz value seemed to be a function of P and r:

$$\text{Jext.pz} = J_{ext.pz}(\text{ P, r })$$
$$= C \cdot P^m \cdot r^n \qquad \text{(C : constant)} \qquad (4.3)$$

The result of Table 4.1 indicates that C ≅ 30, m≅n≅1. Substituting these values in Eq. (4.3), we obtain

$$J_{ext.pz} = C \cdot P \cdot r \qquad (4.4)$$

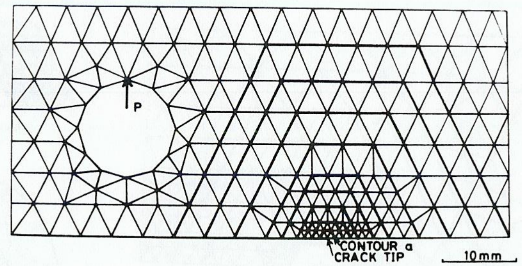


Fig. 4.2 Mesh type II

Further, substituting $r=\overline{W}=2J/\sigma_Y$ from Eq. (4.1), (4.2) in Eq. (4.4), next equation is obtained

$$J_{ext.pz} = \frac{2C}{\sigma_Y} J \cdot P \qquad (4.5)$$

Fig. 4.3 shows the Jext.pz versus P·J.

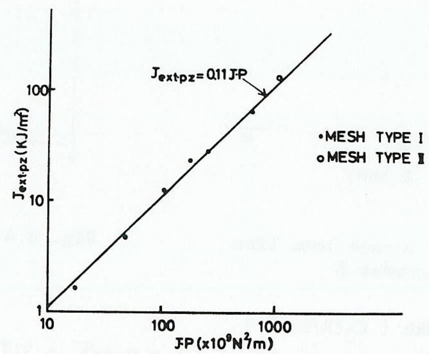


Fig. 4.3 Jext.pz versus P·J

§5 SUMMARY

A compact type specimen is analysed by the finite element method using finite deformation theory. The evaluation of the process zone is made by calculating Jext in the near tip field. The results are summarized as follows:

(1) Jext.pz =C·P·r (C ≅ 30 m^{-1})
where
Jext.pz : Jext value evaluated at a contour whose representative distance r equal to $\overline{W}$
P : load at $\overline{W}$ = r
r : representative distance of contour (=$\overline{W}$)

(2) by using the relation

$$r_i = \overline{W} = 2J/\sigma_Y$$

Jext.pz can be expressed as follows:

$$\text{Jext.pz} = \frac{2C}{\sigma_Y} J \cdot P$$

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