EFFECTS OF MEAN STRESS AND FREQUENCY ON FATIGUE CRACK GROWTH IN HIPS

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ABSTRACT

The two-stage line zone model is used to describe fatigue crack growth in a high impact polystyrene (HIPS) polymer. It is shown that some modifications are necessary in order to describe the effects of varying mean stress conditions on fatigue crack propagation rates. Such modifications have removed some difficulties inherent in the original model and they give constant craze stress values independent of mean stress. The moderate frequency sensitivity of crack growth rates recorded appears to be reflections of changes in the Young's modulus (E), or equivalently, the critical stress intensity factor (K_c) through the loss factor (tan Δ) for the β transition of the matrix polystyrene polymer.

KEYWORDS

Fatigue; polymer; mean stress; frequency sensitivity; R ratio; craze stress.

INTRODUCTION

Fatigue crack growth data for polymers are usually analysed according to the Paris power-law equation (Hertzberg et al, 1970). Although reasonably successful for the description of fatigue in a wide range of polymers the approach does not offer any link with established fracture parameters, such as crack opening displacement and craze stress. However, an attempt has been made recently to describe fatigue in polymers in terms of these parameters (Williams, 1977; Mai & Williams, 1979) so that the craze stress, σ_c, in the crack tip Dugdale line plastic zone is reduced to a new value, $\alpha\sigma_c$, upon unloading and reloading. This results in a two-stage line zone (Williams, 1977) and using the critical crack tip opening displacement fracture criterion reloading in each cycle gives the growth of the zone and thus a fatigue crack growth equation of the form:

$$\frac{da}{dN} = \frac{\pi}{8} \frac{1}{(1-\alpha)^2 \sigma_c^2} \left(K^2 - \alpha K_c^2 \right) \qquad (1)$$

where K is the maximum cyclic stress intensity factor and K_c is the fracture toughness of the polymer. This equation has been shown to give a good description of fatigue crack growth rates in PMMA, Nylon 66, PC and GPS (Williams, 1977;

Mai & Williams, 1979). In addition, it can accommodate both environmental and temperature effects on fatigue (Mai & Williams, 1979). However, the two-stage line zone model is not completely satisfactory. For example, the craze stress, σ_c, predicted from equation (1) is too high and it increases too rapidly with mean stress or R (K_{min}/K_{max}) ratio.

The purpose of the present paper is to extend the two-stage line zone fatigue model, to investigate the effects of R ratio and frequency on fatigue crack growth in a two-phase polymer, HIPS. Modifications necessary for equation (1) to give consistent craze stress values independent of R ratios are discussed.

EXPERIMENTAL WORK

The polymer used was a high impact grade polystyrene supplied by Nylex Plastics Pty. Ltd. (Australia) in the form of 4.7 mm thick extruded sheets. The rubber content was 7.5% and tensile tests at a strain rate of 0.005 (s^{-1}) gave E = 2.2 GPa, σ_y = 21 MPa and ε_f = 17%. All the fatigue experiments were conducted at ambient conditions (22°C and 50% R.H.) on single-edge-notched specimens with dimensions 4.7 x 70 x 210 mm^3 using a Shimadzu closed-loop servo-pulser testing machine with sinusoidal load waveforms. Precracks were introduced in the specimens by sharp dead fly-cutters, R ratios of 0 to 0.5 and frequency levels of 0.01 to 25 Hz were investigated in this work. Crack lengths (a) were measured periodically with a travelling microscope to an accuracy of 0.01 mm and plotted against the number of elapsed cycles (N). The crack growth rates, da/dN, were then determined from the a-N plots using a computer programme written for this purpose and K was calculated from the relevant stress intensity factor equation given by Brown and Srawley.

EXPERIMENTAL RESULTS

Figure 1 shows the R ratio effects on fatigue crack growth rates which are represented by the conventional log da/dN - log ΔK plot according to the Paris equation, i.e.

$$\frac{da}{dN} = A(\Delta K)^m \qquad (2)$$

Clearly, da/dN is insensitive to changes in R ratio and this behaviour is similar to that obtained for the matrix polymer, GPS (Mai & Williams, 1979) but different to other polymers like PMMA and epoxy resins which show remarkable R ratio effects (Arad et al, 1972; Sutton, 1974). A plausible explanation (Yap, 1980) is that HIPS does not display crack closure for R > 0 so that both the effective and applied ΔK values are equal. Crack closure is likely at R = 0 but this effect is not apparent in Fig. 1.

The effects of frequency (w) on fatigue crack growth rates are given in Fig. 2 which shows that da/dN decreases moderately with increasing frequency. The experimental data fit equation (2) well and the values of A and m are given in Table 1. Since the slopes of the curves are essentially constant (m ≈ 3.3) the experimental results can be superposed to give a single master curve represented by:

$$\frac{da}{dN} = 0.70\ w^{-0.12}(\Delta K)^{3.3} \qquad (3)$$

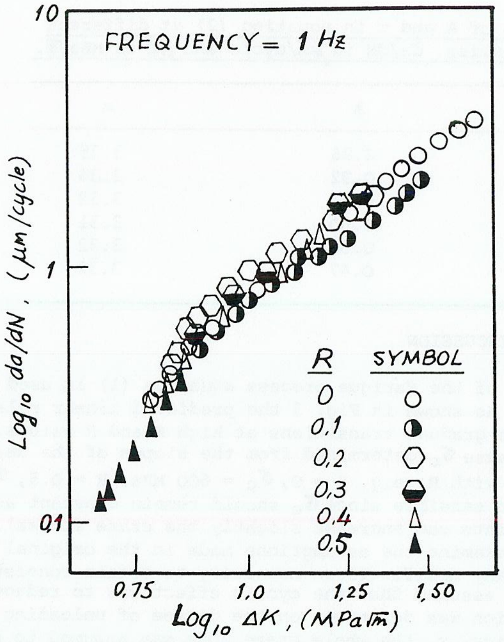

Fig. 1. R ratio effects on fatigue crack growth.

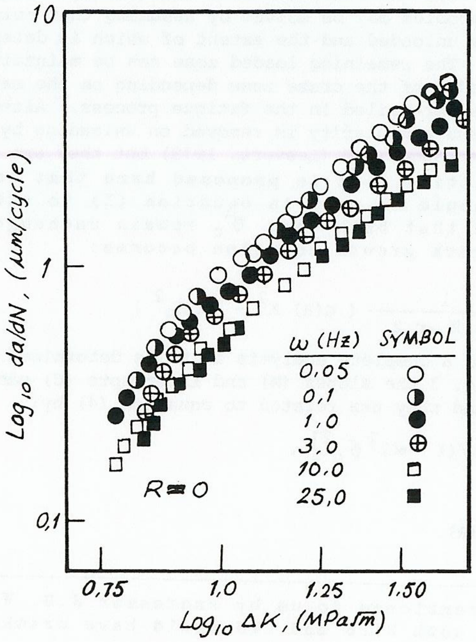

Fig. 2. Frequency effects on fatigue crack growth.

TABLE 1 Values of A and m in equation (2) at different frequencies. da/dN in μm/cycle and ΔK in MPa$\sqrt{m}$.

Frequency (Hz)	A	m
0.05	0.96	3.35
0.10	0.92	3.34
1.00	0.71	3.32
3.00	0.60	3.31
10.00	0.52	3.32
25.00	0.47	3.32

ANALYSIS AND DISCUSSION

To elucidate the mechanics of the fatigue process equation (1) is used to analyse the experimental results. As shown in Fig. 3 the predicted linear relationships are obtained, but there are gradual transitions at high K and R values. A major difficulty now emerges because σ_c determined from the slopes of the da/dN versus K^2 plots increases rapidly with R. e.g. R = 0, σ_c = 600 MPa; R = 0.5, σ_c = 2.5 GPa. These results are not sensible since σ_c should remain constant as a material property (although constraints can increase slightly the craze stress). It is therefore necessary to re-examine the assumptions made in the original model (Williams, 1977) and make any modifications necessary to obtain consistent σ_c values. It was previously assumed that the cyclic effect was to reduce the craze stress and that the reduction was dependent on the degree of unloading so that $\alpha = 1$ when R = 1. In particular, the whole craze zone was assumed to be partially unloaded. This seems unrealistic from simple considerations of the mechanical properties of the craze and thus the existence of a stress singularity on unloading (Leevers, 1979). This problem may be solved by assuming that only part of the craze zone is completely unloaded and the extent of which is determined by the mean stress conditions. The remaining loaded zone can be maintained either at the leading or trailing edge of the craze zone depending on the material used and the deformation mechanism prevailed in the fatigue process.[1] Alternative mechanisms in which the stress singularity is removed on unloading by actual blunting or voiding have been suggested (Leevers, 1979) but they are considered as less likely possibilities. It is proposed here that in general an effective K value should be used in equation (1) to account for the R ratio effects and that both α and σ_c remain unchanged. Thus, the modified fatigue crack growth equation becomes:

$$\frac{da}{dN} = \frac{\pi}{8} \frac{1}{(1-\alpha)^2 \sigma_c^2} \left(G(R)\, K^2 - \alpha K_c^2 \right) \qquad (4)$$

where G(R) in the absence of a complete analysis is to be determined experimentally and G(0) = 1.0. From Fig. 3 the slopes (M) and intercepts (C) can be obtained easily as a function of R and they are related to equation (4) by:

$$M(R) = \pi/8\left[G(R)/(1-\alpha)^2 \sigma_c^2\right], \qquad (5)$$

and

$$C(R) = \alpha K_c^2/G(R). \qquad (6)$$

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[1] This idea was first mentioned to us by Professor J.G. Williams. It would appear that both HIPS and GPS would have crack tip loaded zones.

Since G(0) = 1 so that G(R) = M(R)/M(0). The critical craze zone size (r_c) can be determined from equations (5) and (6). Table 2 gives the values of G, M, C and MC as a function of R. α is determined from equation (6) using K_c = 1.91 MPa$\sqrt{m}$ in the low stress conditions (Yap, 1980). Figure 4 shows a plot of G(R) versus R in which the experimental data are bounded by the curves: $G(R) = (1-R)^2$ and $G(R) = (1-R)^3$. The data for GPS (Mai & Williams, 1979) are also plotted in this figure and they fall between the same two curves thus indicating the same deformation mechanism for these two materials in the fatigue fracture process. As required and indeed shown in Table 2 the modifications to equation (1) give values of α, σ_c and r_c essentially independent of R ratio. The large σ_c values have been reduced by cyclic fatigue to $\alpha\sigma_c$ values which are comparable in magnitude to static craze stresses of HIPS. An explanation for this large reduction in σ_c has been given recently in terms of molecular fracture mechanisms (Williams, 1980). Analysis of the experimental data above the transitions shown in Fig. 3

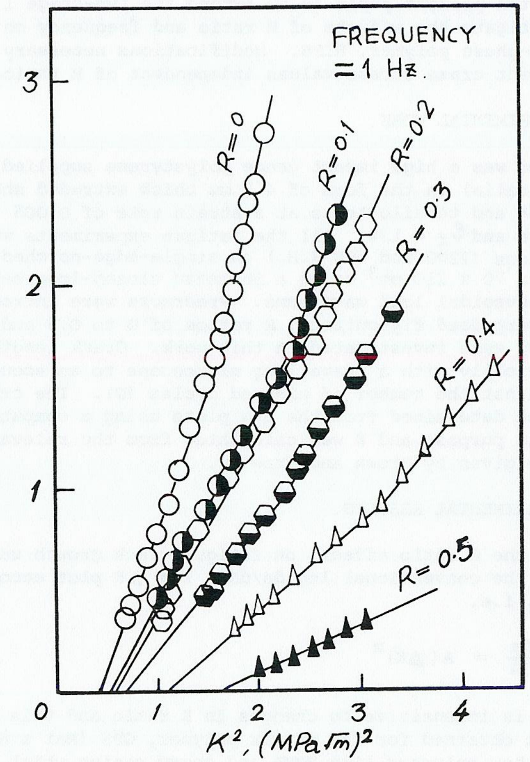

Fig. 3. Fatigue crack growth data plotted in accordance with equation (1).

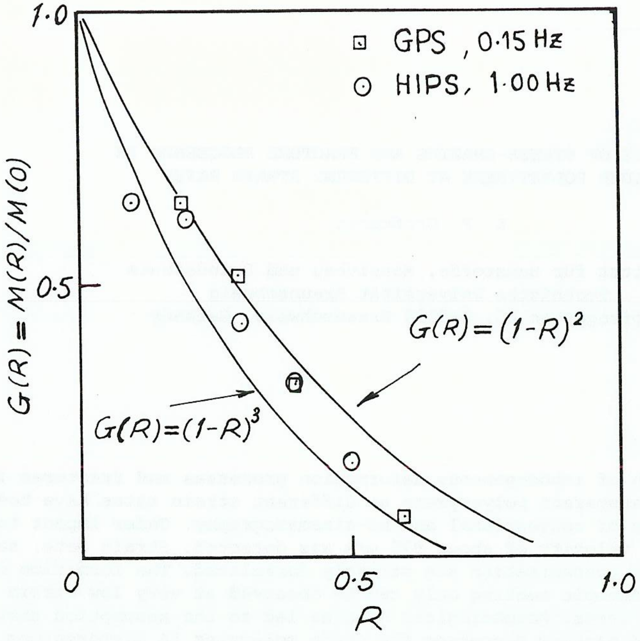

Fig. 4. Variation of G with R.

according to equation (4) gives $\alpha = 0.10$, $\alpha\sigma_c$ = 45 MPa and r_c = 9 μm. These results suggest that approximately the same fatigue craze stresses are obtained for both high and low K_c levels. This behaviour is thus different to that obtained for PVC and GPS (Williams, 1980; Mai & Williams, 1979).

TABLE 2 - Parametric values of equation (4)

R	M(R) μm/(MPa$\sqrt{m}$)^2	C(R)	G(R)	M.C. (μm)	α	σ_c (GPa)	$\alpha\sigma_c$ (MPa)	r_c (μm)
0	1.40	0.40	1.00	0.56	0.100	0.56	56	4.5
0.1	0.91	0.51	0.65	0.46	0.091	0.58	53	4.2
0.2	0.89	0.55	0.62	0.48	0.094	0.57	54	4.4
0.3	0.60	0.67	0.43	0.41	0.081	0.57	46	4.4
0.4	0.45	1.07	0.32	0.48	0.095	0.57	54	4.4
0.5	0.24	1.60	0.17	0.40	0.082	0.58	48	4.2

The frequency effects on fatigue in polymers are well-known. It has been hypothesised that frequency sensitivity on fatigue crack growth is associated with the β-peak related segmental motions (Skibo et al, 1977). However, changes in da/dN with frequency may also be caused by similar changes in K_c due to the viscoelasticity of the secondary (usually β) transitions (Williams, 1977). Since $K_c^2 = E^2 e_c \delta_c$ and both e_c, δ_c remain essentially constant K_c changes due to frequency are therefore reflections of the time dependence of E. Thus, $K_c \propto w^{\tan\Delta}$, where $\tan\Delta$ is the loss factor appropriate to the particular transition concerned. When cast in the form of the Paris equation the frequency sensitivity of da/dN is determined by the product: $-m\tan\Delta$. Figure 5 shows a plot of log A versus log w and the slope of which gives $m \tan\Delta = 0.12$ so that $\tan\Delta = 0.036$. This $\tan\Delta$ value is in reasonable agreement with that reported previously for a similar HIPS polymer (Wagner & Robeson, 1970). Since the glass transition of the rubber phase will not occur until at very low temperatures it is proposed that the moderate frequency sensitivity of da/dN may be due to the β relaxation of the matrix polymer GPS.

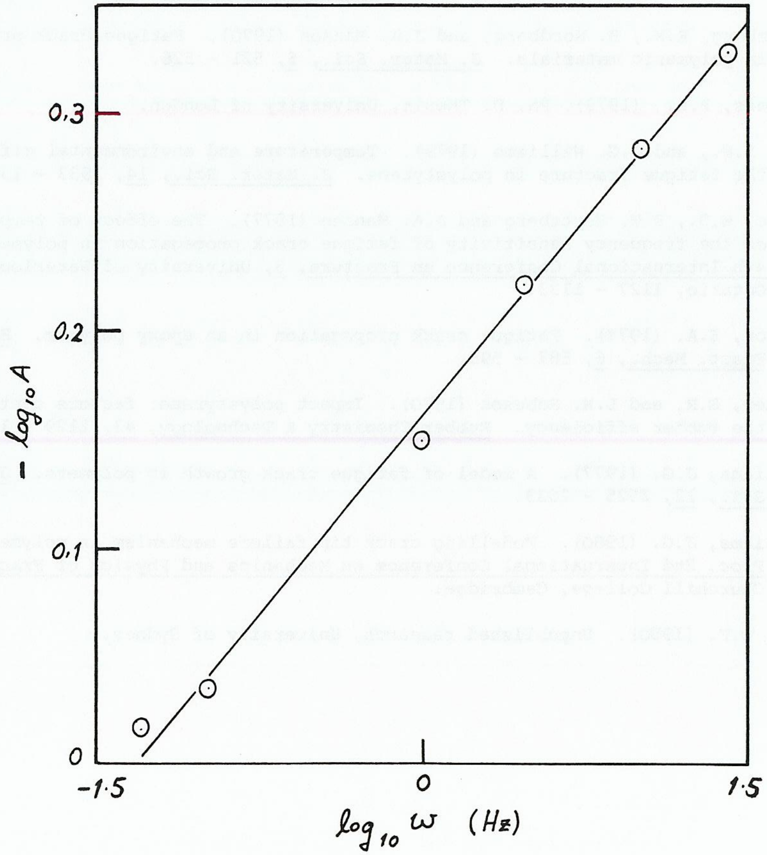

Fig. 5. A plot of log A versus log w.

CONCLUSIONS

The Williams two-stage line zone model has been modified to account for R ratio effects on fatigue crack growth in HIPS. Such modifications, albeit somewhat empirical at the present stage, have given constant craze stress (σ_c) values independent of R ratios. The moderate frequency sensitivity of da/dN appears to be a result of viscoelasticity effects of K_c through the loss factor (tanΔ) for the β- transition of the matrix polystyrene polymer.

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