DEFINING THREE-DIMENSIONAL DAMAGE EFFECTS IN POLYMERS

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ABSTRACT

Polymer damage caused by microcracking and microvoiding can be readily defined for fibers or one-dimensional effects. However, in two- or three-dimensions, the problem becomes more complex because of isotropy group changes caused by strain induced non-isotropic crack arrays.

KEYWORDS

Damage; microcracks; microvoids; metatropic effects; change in isotropy group; three-dimensional damage.

INTRODUCTION

In the last century, Kelvin (1980) noted that the present state of a materials mechanical response was dependent on its entire past history of straining. Fitzgerald (1980) discussed the one-dimensional case wherein structurally induced damage must be measured from an equilibrium state to another neighboring equilibirum state. It was, therein, pointed out that assuming an elastic or viscoelastic range exists between the above two states of deformation, one can uniquely define a degree of damage.

Essentially, in the one-dimensional case, the definition of damage is a scalar quantity. Whether this scalar is taken as a stress ratio, a modulus ratio, or where applicable, an equilibrium free energy ratio, is immaterial since an equivalence exists between the various definitions.

This above simplicity is lost in the multi-dimensional case.

THREE-DIMENSIONAL DAMAGE

Second Order Damage Tensor

In the one-dimensional case, one might define damage, D, by the difference ratio between the stress, σ_0, in the undamaged fiber and the stress, σ_D , in the damaged $D = (\sigma_0 - \sigma_D)/\sigma_0$.

It is assumed that σ_0 and σ_D are measured at the same equilibrium elongations.

Where σ_D represents a damaged state, in the usual sense, the values of D will vary from 0, undamaged, to a value of 1, totally damaged.

An analogous expression in three-dimensions is in cartesian coordinates.

$$D_{ij} = [(\sigma_0)_{ik} - (\sigma_D)_{ik}](\sigma_0)^{-1}_{kj} = \delta_{ij} - (\sigma_D)_{ik}\,(\sigma_0)^{-1}_{kj} \qquad (1)$$

where δ_{ij} is the Kroneker delta.

The damage is now a second order tensor and is uniquely valued, if, and only if, the stress σ_D commutes with σ_0. Otherwise, the second term in Eq. (1) produces different results depending on whether the σ_D term is right or left multiplied.

Fourth Order Damage Tensor

In the one dimensional case cited above, the scalar damage defined can also be written in terms of secant moduli $\epsilon_0 = \sigma_0/\epsilon$ and $\epsilon_D = \sigma_D/\epsilon$ where ϵ is the infinitesimal "test" elongation strain. Then the damage becomes $D = (\epsilon_0 - \epsilon_D)/\epsilon_0$ and is numerically identical to the previous scalar definition.

For the three-dimensional case, the equilavent moduli or equilibrium elasticities are denoted by capital $\overset{0}{C}$ for undamaged and $\overset{D}{C}$ for damaged material. Thus, the analogous expression for damage expressed as the moduli or elasticities difference ratio is

$$D^{ij}_{k\ell} = \delta^{ij}_{k\ell} - (\overset{D}{C})^{ij}_{mm}\,(\overset{0}{C})^{mn\;-1}_{k\ell} \quad \text{with}$$
$$\overset{0}{\sigma}_{ij} = \overset{0}{C}{}^{ij}_{k\ell}\,\epsilon_{k\ell} \quad \text{and} \quad \overset{D}{\sigma}_{ij} = \overset{D}{C}{}^{ij}_{k\ell}\,\epsilon_{k\ell} \qquad (2)$$

The damage is now a fourth order tensor and is uniquely valued, if, and only if, the elasticity or modulus tensor $\overset{D}{C}$ commutes with $\overset{0}{C}$.

Further, neither the D_{ij} of Eq. (1) nor the $D^{ij}_{k\ell}$ of Eq. (2) are symmetric unless the second terms in their expressions commute.

The physical implication of the above statement is that, for example, the matrix representation of the product $\sigma_D\sigma_0^{-1}$ in Eq. (1) could have unity values along the diagonal and one off diagonal non-zero value. This result would imply no damage in any of three orthogonal directions, finite shear damage in a plane normal to one of the above directions and no shear damage in the other two normal planes.

Thus the matrix representation of D_{ij} could contain up to 9 distinct scalar values and that of $D^{ij}_{k\ell}$ up to 36 scalar values.

Eight Order Damage Tensor

Warburg (1890) conducted experiments on copper wires and demonstrated that a wire, previously twisted into its inelastic range and released, would, when hung with a weight, show twisting as well as elongation.

Materials which show both a change in their response fuction(al)s as well as a change in their isotropy groups after prior straining are said (Fitzgerald, 1975) to exhibit metatropic behavior.

I shall, henceforth, use damage and metatropic somewhat interchangeably. If the word damage connotes bad versus good to the reader, then substitute metatropic wherever the word damage occurs.

It logically follows, from the Kelvin and Warburg ideas, that one should compare the virgin and damaged elasticities of a material to determine both changes in the specific moduli values and changes in the isotropy group of the material.

That is, an initially isotropic material, after being strained uniaxially into an inelastic range would on subsequent straining probably exhibit transversely isotropic behavior with reduced moduli in the direction of prior straining. Laboratory straining history could produce the undamaged elasticity $\overset{0}{C}_{ijk\ell}$ defined preiously and later the damaged elasticity $\overset{D}{C}_{ijk\ell}$ could be determined.

To a first order, then, one could define a metatropic function, $\mathcal{M}$, relating the two elasticities through

$$\overset{D}{C}_{ijk\ell} = \mathcal{M}^{ijk}_{mnop}\;\overset{0}{C}_{mnop}$$
$$\text{or}\quad \mathcal{M}^{ijk\ell}_{mnop} = \overset{D}{C}_{ijk\ell}\;(\overset{0}{C}_{mnop})^{-1} \qquad (3)$$

$\mathcal{M}$ above is of necessity an eighth order tensor relating the two fourth order elasticities, just as the elasticities are fourth order relating the second order stresses and strains.

Again, $\mathcal{M}$ is not in general symmetric and in its greatest generality is expressible as a 21 x 21 matrix with 441 scalar entries.

For the isotropic to transversely isotropic case mentioned earlier, however, $\mathcal{M}$ can be expressed as a diagonal 9 x 9 matrix with only 5 non-unity entries.

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