AN INTEGRAL EQUATIONS METHOD FOR RESOLUTION, IN OPENING MODE, OF THE PROBLEM OF PLANE CRACKS AT FREE SURFACE

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ABSTRACT

A mathematical representation of the opening of a crack of arbitrary shape, loaded normally to its plane is achieved by means of a scalar integral equation involving elastic potentials ; this approach allows in particular an estimation of the influence (compared to the case of similar geometry in a three-dimensional infinite body) of a plane free surface normal to the crack plane. A numerical description of the problem has also been considered ; it has enabled us to control the efficiency of the method in the particular case of semi-elliptical surface cracks. The accuracy of the process allows the results to fit inside the range of the most credible results computed by different authors (Raju Newman, 1979 ; Rooke Cartwright, 1976 ; Smith Sorensen, 1976) and this with a much shorter computing time (4s CPU on CDC 7600). This is due to the fact that the method is particularly well adapted to the crack problem. In particular, the separation of modes represents a big improvement on the classic integral method.

KEYWORDS

Edge surface crack ; integral equations ; semi-elliptical crack ; numerical method; discretization.

INTRODUCTION

The problem of linear homogeneous isotropic elasticity in regular shaped bodies is studied with satisfactory results using current techniques such as finite element methods or vectorial integral equations. See (Cruse 1969 ; Lachat and Watson 1976) for the latter. Less classical shapes -like solids showing edges, angular points or cracks- can, at the present time, be approximated only by some numerical subtilities affecting the shape functions describing the elements in the immediate neighbourhood of the singularity. Unfortunately, these improvements are sometimes only a compromise and it seems a priori more satisfactory to use directly the theoretical results (Rigolot, 1978) as far as integral equations, applied to angular geometries, are concerned.

Similarly, we are inclined to remove the restrictions on the use of Somigliana formula and to use a very direct analytical method for plane cracks. Thus, our concern is entirely directed to the solid displacement field in the neighbourhood of the discontinuity. The boundaries chosen for the solid are either at infinity or ideally described as a plane surface free from any stress. The cracked solid problem is thus free from any interference with undesirable boundary conditions and the attention is focused on the geometry of the crack itself. It is therefore not surprising that the theory, thus developed, leads to an integral equation only involving expressions linked to the cracked area. Another point of interest is the way the modes are decoupled in the equations ; this separation will be shown clearly for the problem of a crack in an infinite body and we will use it, to advantage, to show the influence of the free surface on the opening displacement of the crack. The singularity in the neighbourhood of the crack intersecting a plane free surface was studied in theoretical works (Benthem, 1977,1980 ; Bazant and Estenssoro, 1979). We will not deal with the asymptotic problem considered by these authors but only the global aspect of the opening crack behaviour will be considered. A different formulation of the same problem is given by Bueckner (1977).

1. THEORETICAL POSITION OF THE PROBLEM OF PLANE CRACK IN THREE-DIMENSIONAL INFINITE MEDIUM.

We follow in this first paragraph recent results obtained by Bui (1975, 1977). The fundamental problem considered constitutes the basic mathematical tool which is later used to treat the more complicated problem with interference of a free surface. It is also a reference case for which in particular the stress intensity factor associated with a point on the contour is independent of Young's modulus and Poisson's ratio. The fact of having precise solutions for elliptical cracks adds interest to this kind of representation since it allows comparisons to be made. We will try to find the displacement field resulting form arbitrary loading of a structure containing a plane crack of arbitrary shape. We propose to represent this displacement by means of three integral equations governing the discontinuities in the displacement when crossing over the crack.

1.1 Fundamental tensors in the linear isotropic elastic body.

Let x_i $(i = 1, 2, 3)$ be the cartesian coordinates of the point x. The basis vectors are e_i. The displacement field u(x) satisfies the elastostatic equation :

(1) $$Lu \equiv \mu\Delta u + (\lambda + \mu)\,\nabla\nabla u = -\,f \qquad \text{in a domain } \Omega$$

where λ,μ are the Lamé constants and f is the body force. Kupradze (1963) had introduced a generalized stress vector operator P^n defined as follows :

(2) $$P^n u = (\alpha + \mu)\,\frac{\partial u}{\partial n} + \beta n.\nabla u + \alpha n \wedge \nabla \wedge u$$

where α,β are any real numbers satisfying the relation $\alpha + \beta = \lambda + \mu$ and n the unit vector normal to the area where the stress vector is applied. He established the equivalent of the Betti's reciprocal relation

(3) $$\int_\Omega (u.Lv - v.Lu)dx = \int_{\partial\Omega} (uP^n v - vP^n u)ds$$

We will notice that the physical stress vector operator T^n is included in the class P^n (for $\alpha = \mu$ and $\beta = \lambda$).

Let the Green's tensor of the infinite elastic body be $V_i^k(x,y)$ (also identified as Kelvin Somigliana tensor) ; the fundamental field $V^k(x,y)$ satisfies from its definition :

(4) $$LV^k(x,y) = -\,\delta(x - y)e^k$$

where δ is the Dirac measure concentrated at the point y as point of $\mathbb{R}^3$; the components of tensor V_i^k are obtained by differentiation of (4) with respect to x. Kupradze shows that in these conditions, Somigliana's relationship can still be

applied with P^n operator, which yields :

(5) $$d(x)u_k(x) = \int_{\partial\Omega} \{V^k(x,y).P^{n(y)}u(y) - u(y).P^{n(y)}V^k(x,y)\}\, dS_y$$

where :

$d = 1$ if $x \in \Omega$
$d = 1/2$ if $x \in \partial\Omega$ regular point
$d = 0$ if $x \in \Omega'$ exterior to Ω .

A study of elastic potentials on a non regular surface (Rigolot 1978) gives a generalization of (5) where d(x) is replaced by $C_i^j(x)u_j(x)$ and C_i^j is an expression in relation with the solid angle Ω and inertia product p_i^j :

$$C_i^j(x) = -\,\frac{1}{8\pi(1-\nu)}\,\{(1 - 2\nu)\,\delta_i^j\Omega + 3p_i^j\}$$

In equation (5), the term $P^{n(y)}V^k(x,y)$ is singular for points x on $\partial\Omega$, the singularity is due to the normal derivative of $1/\rho$, so that for $d = 1/2$, the associated integral must be understood in the sense of a principal value.

We restrain now the choice of P^n in forcing non normal derivatives to disappear ; the corresponding choice P_o^n is obtained in an unique way with a particular couple (α_o,β_o) ; we thus obtain the Kupradze-Bashelishvili tensor, defined by :

(6) $$B_j^k(x,y,n(y)) = 2\,P^{n(y)}\,V^j(x,y)\,e^k$$

which will enable us to describe the jump in the displacement field across the crack :

(7) $$B^k(x^{\pm},y,n(y)) = \pm\,\delta(x - y)e^k$$

δ is here the Dirac measure in relation with a plane P with n(y) unit normal vector.

1.2 Describing the displacement discontinuities as a sum of elastic potentials.

We express (5) in an equivalent way in considering the displacement field of the infinite body as a sum of potentials connected with preceding tensors.

So we define a single layer potential in relation with density ψ_k :

(8) $$C_i(x) = 2\int_{\partial\Omega} V_i^k(x,y)\,\psi_k(y)\,dS_y$$

and a double layer potential associated with density ϕ_k :

(9) $$D_i(x) = \int_{\partial\Omega} B_i^k(x,y,\,n(y)\,)\,\phi_k(y)\,dS_y$$

Description of the displacement field is presented as :

(10) $$u_i(x) = C_i(x) + D_i(x)$$

We will not discuss the properties of these potentials but content ourselves with the expression of the double layer potential discontinuity across the crack surface :

(11) $$D_i(x^+) - D_i(x^-) = 2\phi_i(x)$$

with the further statement that the general determination on the lower face of $\partial\Omega$:

(12) $$D_i(x^-) = -\,\phi_i(x^-) + \int^{*}_{\partial\Omega} B_i^k(x,y,n(y))\phi_k(y)dS_y$$

is considerably simplified when $\partial\Omega$ is a plane ; with this assumption, the principal value vanishes and the density $\phi_i(x)$ is simply equal to the displacement on the upper face while $-\phi_i(x)$ is the displacement of the lower one. The double layer potential seems likely in a very perceptible way as describing exactly the jump of displacement field whereas the single layer potential supplies the boundary conditions. It can be proved -in deriving the tensions by differentiation of the potentials- that these conditions are satisfied when following group of relations are met (Bui 1975, 1977) :

$$(13) \qquad \begin{cases} \psi_\alpha(x) = \dfrac{2\ \mu^2}{\lambda+3\mu}\dfrac{\partial\phi_3}{\partial x_\alpha} & (\alpha = 1,\ 2) \\ \psi_3(x) = -\dfrac{2\ \mu^2}{\lambda+3\mu}\left(\dfrac{\partial\phi_1}{\partial x_1} + \dfrac{\partial\phi_2}{\partial x_2}\right) \end{cases}$$

$$(14) \qquad \frac{\lambda\mu}{2\pi(\lambda+2\mu)} \overset{*}{\int_S} \left(\frac{\partial\phi_1}{\partial y_1} + \frac{\partial\phi_2}{\partial y_2}\right) \frac{\partial}{\partial x_1}\left(\frac{1}{\rho(x,y)}\right) dS_y + \frac{\mu}{2\pi}\overset{*}{\int_S} \frac{\partial\phi_1}{\partial y_\alpha}\frac{\partial}{\partial x_\alpha}\left(\frac{1}{\rho}\right) dS_y = T_1(x)$$

$$(15) \qquad \frac{\lambda\mu}{2\pi(\lambda+2\mu)} \overset{*}{\int_S} \left(\frac{\partial\phi_1}{\partial y_1} + \frac{\partial\phi_2}{\partial y_2}\right) \frac{\partial}{\partial x_2}\left(\frac{1}{\rho(x,y)}\right) dS_y + \frac{\mu}{2\pi}\overset{*}{\int_S} \frac{\partial\phi_2}{\partial y_\alpha}\frac{\partial}{\partial x_\alpha}\left(\frac{1}{\rho}\right) dS_y = T_2(x)$$

$$(16) \qquad \frac{\mu(\lambda+\mu)}{\pi(\lambda+2\mu)}\overset{*}{\int_S} \frac{\partial}{\partial x_\alpha}\left(\frac{1}{\rho(x,y)}\right)\frac{\partial\phi_3}{\partial y_\alpha}\, dS_y = T_3(x)$$

Relations (13) express the necessary connection between densities ; (14), (15), (16) are the requested fundamental equations. It should be noted that mode I (16) is entirely uncoupled from modes II and III, represented in equations (14) and (15); integration domain is restricted to the cracked surface S as densities are elsewhere zero.

2. THEORETICAL ASPECT OF THE PROBLEM OF THE EDGE-PLANE SURFACE CRACK: OPENING MODE

The plane of the crack being normal to the free surface, we extend the theory developed in the preceding paragraph in order to simulate the surface crack problem ; the adjustement is obtained by means of a superposition of two problems of fundamental type corresponding to two orthogonal plane cracks. The first one refers to the outline S of the studied crack F and its mirror image $\bar{F}$ against the plane of the free surface (fig. 1). The second one refers to a surface Σ large enough to be interpreted as the free infinite plane surface. The superposed problem in relation with the "crossed-cracks configuration" will represent in a convenient way the

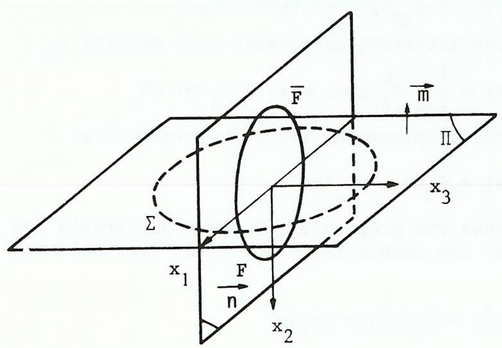


Fig. 1. Crossed-cracks configuration.
Physical crack F, mirror image $\bar{F}$.

problem of the edge crack, providing that boundary conditions are met on $F \cup \bar{F}$ (imposed normal pressure) and on Σ (free from any traction).

Problem 1 describing the crack $S = F \cup \bar{F}$ is characterized by densities ϕ_3 disposed on S. Using the notation of the preceding paragraph, the displacement field solution of this problem is expressed in general form as (10). We denote by σ_{33} the component observed in the plane P of these densities : its expression is as follows :

$$(17) \qquad e_3.T^n(C + D) = \frac{\mu(\lambda+\mu)}{\pi(\lambda+2\mu)}\overset{*}{\int_S}\frac{\partial}{\partial x_\alpha}\left(\frac{1}{\rho(x,y)}\frac{\partial\phi_3}{\partial y_\alpha}\, dS_y\right)$$

Problem 2 related to Σ is defined by means of densities β_2 disposed on Σ. The field displacement resulting from these densities is written :

$$(18) \qquad u_i(x) = \Gamma_i(x) + \Delta_i(x)$$

Γ and Δ denote respectively single and double layer potentials similar to C and D but related to β_2. By analogy with equation (17), we can write :

$$(19) \qquad e_2.T^m(\Delta + \Gamma) = \frac{\mu(\lambda+\mu)}{\pi(\lambda+2\mu)}\overset{*}{\int_\Sigma}\frac{\partial}{\partial x_\alpha}\left(\frac{1}{\rho(x,y)}\right)\frac{\partial\beta_2}{\partial y_\alpha}\, dS_y$$

The superposition described at the beginning of this paragraph consists in considering the field obtained by the sum of the 4 potentials originated by both kinds of densities. By symmetry, we can derive the required boundary conditions for the superposed problem, lack of shear stress in planes P and Π and existence of discontinuities $2\phi_3$ and $2\beta_2$ on S and Σ respectively.

Equations for the superposed problem.

☆ Considering any point x of the crack S, we impose that external tractions $T_3^d(x)$ be in equilibrium with, for one part, singular tractions (17) generated by densities ϕ_3 on S, for the other part with tensions originated by the densities β_2 on Σ and denoted by $e_3.T^n(\Delta + \Gamma)$.

The superposition of these influences is described by the following equation :

$$(20) \qquad \begin{cases} \forall\ x \in S \qquad x \notin S \cap \Sigma \\ e_3.T^n(\Delta + \Gamma) + \dfrac{\mu(\lambda+\mu)}{\pi(\lambda+2\mu)}\displaystyle\overset{*}{\int_S}\frac{\partial}{\partial x_\alpha}\left(\frac{1}{\rho(x,y)}\right)\frac{\partial\phi_3}{\partial y_\alpha}\, dS_y = T_3^d(x) \end{cases}$$

☆ Considering any point of the surface of discontinuity Σ, we prescribe in an analogeous way the zero-equilibrium between singular tractions resulting from β_2 densities acting in their plane and regular tractions originated by ϕ_3 densities on S.

$$(21) \qquad \begin{cases} \forall\ x \in \Sigma \qquad x \notin \Sigma \cap S \\ e_2.T^m(C + D) + \dfrac{\mu(\lambda+\mu)}{\pi(\lambda+2\mu)}\displaystyle\overset{*}{\int_\Sigma}\frac{\partial}{\partial x_\alpha}\left(\frac{1}{\rho(x,y)}\right)\frac{\partial\beta_2}{\partial y_\alpha}\, dS_y) = 0 \end{cases}$$

These equations, where the coupling between the surfaces S and Σ is clearly illustrated, solve our problem (Bui, Putot, 1979 ; Putot, 1980).

Remarks

Compact notations $e_3.T^n(\Delta + \Gamma)$ and $e_2.T^m(C + D)$ are used for the lengthy procedures followed in deriving, under the integral sign with T^n operators, the elastic

potentials.
The contribution $\sigma_{22}(C)$ derived from single layer potential can be expressed as follows :

$$\sigma_{22}(C) = e_2.T^{m(x)}(C)$$
$$= 2\int_S (-\lambda\nabla.V^1 - 2\mu V^1_{2,2})\psi_1(y)dS_y + 2\int_S (-\lambda\nabla V^2 - 2\mu V^2_{2,2})\psi_2(y)dS_y$$

It only remains to clarify $V^k_{i,j}$ with reference to the coordinate system. The contribution $\sigma_{22}(D)$ from double layer potential is calculated in a similar way :

$$\sigma_{22}(D) = e_2.T^{m(x)}(D)$$
$$= \int_S (-\lambda\nabla.B^3 - 2\mu B^3_{2,2})\ \phi_3(y)\ dS_y .$$

Here it is necessary to develop $B^3_{i,j}$. Analogeous quantities in connection with Γ and Δ are developed in a similar way.

3. NUMERICAL PART

3.1 Generalities

We only used the integral equations (20) and (21) with mere collocation without for the present time the help of some more competitive numerical techniques (Nedelec, 1977). Equation (16) solving the crack problem in a three-dimensional infinite material is easily shown to express itself as a linear system :

(24) $\qquad A\Phi = T$

where Φ is a vector of unknown components Φ_3 at the different modes of the mesh ; A is a full matrix, non symmetrical in this case. The systems expressing equations (20) and (21) of the surface crack problem can be written as follows :

(25) $\qquad \begin{cases} A\Phi + B\beta = T \\ C\Phi + D\beta = 0 \end{cases}$

where A, B, C, D are matrices linked to applications of sets shown in the following diagram ;
Φ and β are vectors, their dimension being the number of nodes associated to S for the former, Σ for the latter (n_S and n_Σ).
Notation (n_S , n_Σ) indicates the number of lines and columns of the associated rectangular matrix :

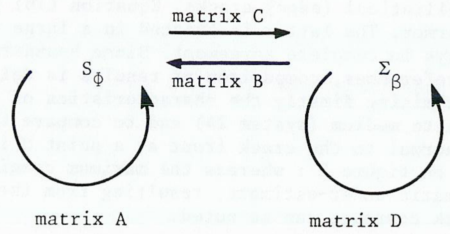
$S \xrightarrow{\text{matrix A}} S \qquad\qquad \Phi \longrightarrow \underset{(n_S,n_S)}{A}\ \underset{(n_S,1)}{\Phi}$

$\Sigma \xrightarrow{\text{matrix B}} S \qquad\qquad \beta \longrightarrow \underset{(n_S,n_\Sigma)}{B}\ \underset{(n_\Sigma,1)}{\beta}$

$S \xrightarrow{\text{matrix C}} \Sigma \qquad\qquad \Phi \longrightarrow \underset{(n_\Sigma,n_S)}{C}\ \underset{(n_S,1)}{\Phi}$

$\Sigma \xrightarrow{\text{matrix D}} \Sigma \qquad\qquad \beta \longrightarrow \underset{(n_\Sigma,n_\Sigma)}{D}\ \underset{(n_\Sigma,1)}{\beta}$

Exchanges can be summed up on the following diagram :



Square matrices A and D describing the singular part can be inverted and equation (25) written in the equivalent form :

(26) $\qquad (A - BD^{-1}C) = T$

showing clearly the perturbation $BD^{-1}C$ of the system (24) induced by the presence of the free surface.

It must be noticed that BUECKNER's formulation (1977) leads to a crack opening equation equivalent to (26).

3.2 Some remarks on "singular computation"

Computing matrices A and D require an integration in the sense of Cauchy's principal value ; we have chosen to adjust, by the method of least squares, the best fitting surface (generated by a straight line) containing the values ϕ and β which are assumed to be known at the different nodes of the polygone Q in the immediate neighbourhood of the pole. This procedure is a real improvement when compared to the numerical technique used in Bui (1977).

This method allows us to separate the partial stiffnesses due to the term I of the sum in (27). Putot's work (1980) will be referred to for more details.

(27) $\qquad \displaystyle\int_S \frac{\partial}{\partial x_\alpha}\left(\frac{1}{\rho(x,y)}\right)\frac{\partial\Phi}{\partial y_\alpha}\, dS_y = I + J + K$

(28) $\qquad \displaystyle I = \int^*_Q \frac{\Phi(y) - \Phi(x)}{\rho^3(x,y)}\, dS_y$

(29) $\qquad \displaystyle J = \int_{P-Q} \frac{\Phi(y)}{\rho^3(x,y)}\, dS_y = \int_{S-Q} \frac{\Phi(y)}{\rho^3(x,y)}\, dS_y$

(30) $\qquad \displaystyle K = -\ \phi(x)\int_{P-Q} \frac{dS_y}{\rho^3(x,y)}$

J requires a classical type of calculation (same kind as with the regular matrices B and C) with an integration by Gaussian points successively in all the elements without the pole x ; we chose a parabolic isoparametric interpolation. K is directly linked to the diagonal stiffness associated to node x ; its integration is easy.

4. RESULTS AND CONCLUSIONS

We have been interested to test the computer program for elliptical (infinite medium) and semi-elliptical (edge) cracks. Equation (16) gives us the analytical solution to the former. The latter is treated in a large number of references, which are not always in complete agreement. Since boundary conditions are not identical in all the references, comparison of results is delicate. The procedure adopted consists in examining firstly the characteristics of the opening of elliptical cracks in an infinite medium (system 24) and to compare this with analytical results. A section normal to the crack front at a point M is compared with these opening profiles, on figure 2 : whereas the maximum opening agrees within 1 %, elsewhere a systematic under-estimate, resulting from the excessive rigidity of the mesh near the crack contour, can be noted.

This has an effect on the determination of the local stress intensity factor deduced from results for nodes near the contour. In the worst case this can reach 10 %. We have made use of these observations and assumed that the same under-estimate must be present for edge cracks. Results have been obtained (for uniform tension) which agree well (figures 3a, 3b) with the points calculated in the references, which we consider to be most reliable. One advantage of the program is its very short time of execution (4 sec. C.P. on a C.D.C 7600). The mesh used is shown on fig. 4. It consists of 37 nodes for F, 61 nodes for Σ^+. We have also calculated crack openings resulting from polynomial type loads. The resulting diagrams are given in the figures 5.

We have also begun to examine the influence of Poisson's ratio on crack opening in the presence of a free surface. (This influence is null for an infinite medium). Our approach has been similar to that of Bazant, 1979, who, for a stabilised crack, has attempted to relate Poisson's ratio to the angle, between the tangent to the crack contour and the free surface, at their intersection.

In practice, we have deformed the network of the semi-elliptical crack, in order to subject it to a non-normal incidence. We have also sought to assure a constant stress intensity factor in the neighbourhood of the surface (which carries the hypothesis of continuity near the surface suggested by Bazant and Estenssoro in an asymptotic manner).

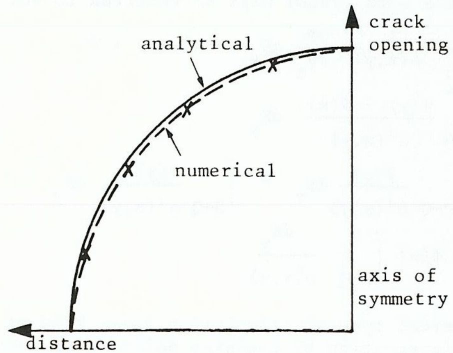


Fig. 2. Comparison between analytical and numerical solutions.

The relationship found between the angle of incidence and Poisson's ratio, is of the correct order of magnitude although the mesh is probably insufficiently dense to allow a true comparison. However this question does not, in our opinion, merit a very detailed study, since the assumption of continuity at the point of intersection with the free surface is not really justified. Thus there is some uncertainty about the asymptotic value of the angle of incidence. In other words, the values found using the above assumptions are approximately justified near the free surface, but not too near !

A fairly large field of studies of different geometrical configurations remains to be carried out ; research concerning iso-K_I cracks, studies of neighbouring surface cracks using the numerical technique of sub-structuration, studies of cracks of quite different shape to semi-ellipses. It should be noted that practical surface cracks encountered in industry are rarely semi-elliptical.

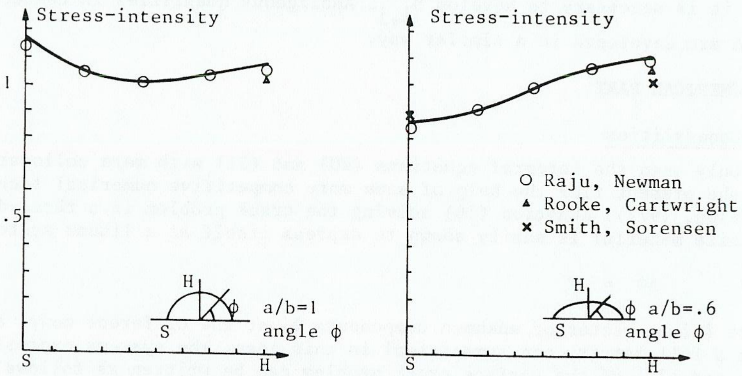


Fig. 3a,b. Numerical results for semi-elliptical cracks. Normalized stress-intensity factor $K_I \phi_0/(\sigma\sqrt{\pi a})$, with $\phi_0(a/b)$ complete elliptical integral of the second kind.

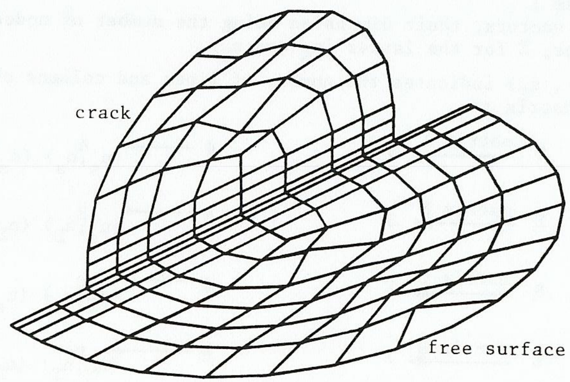


Fig. 4. Mesh for semi-elliptical crack and free surface.

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REFERENCES

Bazant, Z.P., and L.F. Estenssoro (1979). Int. J. Solids Struct., vol. 15.
Benthem, J.P. (1977). Int. J. Solids Struct., vol. 13.
Bueckner, H.F. (1977). Proc. Int. Conf. on Fracture Mechanics and Technology, Hong-Kong, March 21-25, 1977.
Bui, H.D. (1975). C.R. Acad. Sc., Paris, 5 Mai 1975.
Bui, H.D. (1977). J. Mech. Phys. Solids, vol. 25.
Bui, H.D., and C.J. Putot (1979). C.R. Acad. Sc., Paris, 29 Janvier 1979.
Cruse, T.A. (1969). Int. J. Solids Struct., 5, p. 1259-1274.
Green, A.E., and I.N. Sneddon (1950). Proc. Cambridge Phil. Soc., 46.
Kupradze V.D. (1963). Progress in Solids Mechanics, vol. III, edited by Sneddon and Hill. North Holland, Amsterdam.
Lachat J.C., and J.O. Watson (1975). A.S.M.E Conf. on Integral Equations Methods, New York, June 1975.
Nedelec J.C. (1977). Ecole d'été d'Analyse Numérique (IRIA), Juin 1977.
Putot C.J. (1980). Thèse, Université Paris VI.
Raju I.S., and J.C. Newman (1979). Eng. Fract. Mech., vol. 11.
Rigolot C. (1978). C.R. Acad. Sc., Paris, 29 Mai 1978.
Rooke D.P., and D.J. Cartwright (1976). Compendium of stress intensity factors. H.M.S.O. London.
Smith F.W., and D.R. Sorensen (1976). Int. J. of Fract., vol. 12 n° 1.

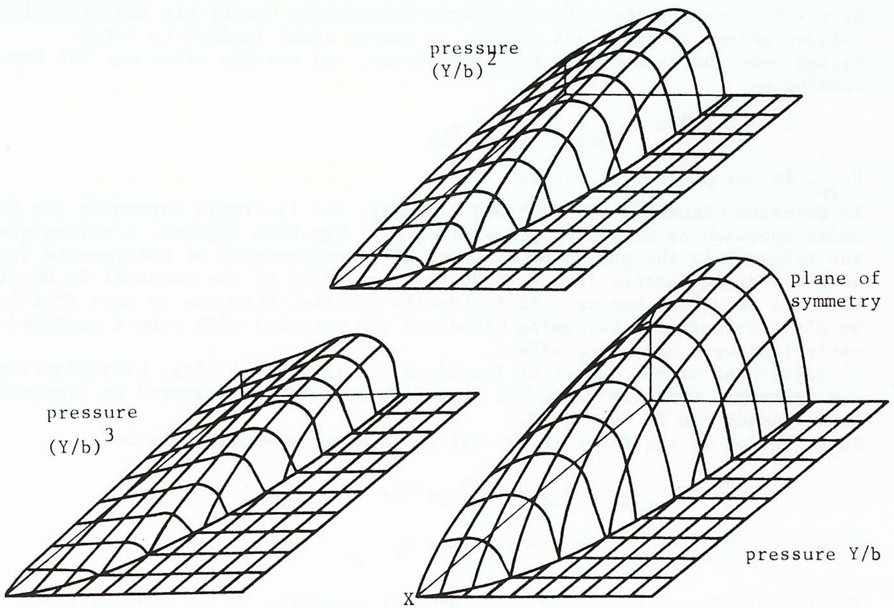

Fig. 5. Crack opening under different loading conditions.