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GRAIN SIZE: THE FABRIC OF (BRITTLE) FRACTURE OF POLYCRYSTALS

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MICROSTRUCTURE AND FRACTURE

More than two hundred and fifty years ago, Réaumur [1] reported that the quality of a steel material was established by the fineness of its microstructure. In recording this observation, Réaumur noted that the distribution of grains within the material was able to be revealed by fracturing a piece of it and examining the surfaces of separation. There is an important connection between these statements. No doubt, the correlation of the quality of a material with the smallness of the scale of its microstructure seemed reasonably correct, as far as concerned the ambient mechanical properties of steel materials, at least, because the fracture strength of steel is itself found to be greater as the average size of the grains is smaller. This is particularly true for the (brittle) cleavage fracture strength of iron and steel materials as reported nearly twenty-five years ago in the careful experiments of Petch [2]. A recent assessment of results for the brittle fracture strength dependence on average grain diameter for a number of iron and steel materials has been made by Madhava [3]. The data are shown in Figure 1 according to the Petch analysis which predicts that

$$\sigma_f = \sigma_{of} + k_f d^{-1/2} \quad . \tag{1}$$

The references for these data are given in Table 1 including the reference to the original results of Petch.

The total data in Figure 1 do show that the fracture strength increases as the grain size is refined even though there are considerable differences between the results obtained in the various studies. The lowest fracture stresses in Figure 1 are reported for the intercrystalline fracture of three iron materials. Nevertheless, these low values of fracture stress are shown to be larger than the stress values calculated from the Sack equation [4] describing the Griffith fracture condition for a circular crack of size equal to the average grain diameter (see Table 1). The reason for all of the measured fracture stresses in Figure 1 being larger than is predicted by the Griffith fracture condition is normally taken to be the requirement for crystalline materials that plastic flow must necessarily accompany the most brittle fracture process. This plastic flow, which may occur to a vanishingly small degree for extremely brittle failures, affects the fracture process in two ways. For a crack-free material, an amount of deformation must occur initially to produce an internal concentration of stress of sufficient intensity to cause a micro-cleavage crack to form; hence, the presence of σ_{of} in equation (1) for the movement of dislocations and the influence of different values of σ_{of} on the measurements in Figure 1. The growth of the cleavage crack, certainly past one grain diameter, may

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require an additional amount of plastic flow at the crack tip; hence, the internal concentration of stress measured in equation (1) by k_f may be greater even than is specified by the Griffith condition, i.e. $k_f \geq [\pi E \gamma_s / (1 - \nu^2)]^{1/2}$ from equation (1) and Table 1. The largest fracture stresses in Figure 1 are reported for the condition whereby fracture was preceded by plastic yielding. These results are consistent with the presumed influence of plastic flow on raising the fracture strength of materials.

The striking appearance of brittle cleavage fracture surfaces which is caused by the varying orientations of individual micro-cleavage cracks within the polycrystal grains and by the differing areas of the grains covered by these micro-cracks is revealed by the two reasonably similar scanning electron micrographs of Figure 2a,b. These fracture examples are taken from separate steel materials which have failed under quite different circumstances. Figure 2a shows the fracture surface obtained for crack-free (plain carbon) 1010 steel which was tested by Madhava [3] in tension at 4.2°K. Figure 2b is from the research study of Stonesifer [5] involving A533 B steel material tested at 77°K in the form of a pre-cracked compact specimen designed for fracture mechanics evaluation of the toughness of the material in plane strain deformation. In this latter case, the fracture stress is characterized by the equation, after Irwin [6] and Orowan [7]:

$$\sigma_f = K_{Ic}(\pi a_e)^{-1/2} \quad . \tag{2}$$

Stonesifer found that the prior austenite grain size, which in Figure 2b is only slightly larger than the ferrite grain size for this A533 B specimen, had a significant infleunce on determining the nature of the fracture surface morphology and, correspondingly, on the fracture toughness of the material.

The similar appearances of the fracture surfaces which are shown in Figure 2 for these two engineering materials are roughly indicative of the same degree of brittleness in them even though this brittleness was ahcieved by means of different external testing conditions. The fracture stress for the 1010 specimen at 4.2°K was measured to be 930 MPa and this is greater than the 164 MPa which should apply at 77°K for this A533 B material subject to the condition of it containing a surface crack nearly 13 mm deep. The (ε = .002) yield stress of the A533 B material at 77°K in a crack-free condition was found to be 1020 MPa and this compares with 840 MPa for the yielding of the 1010 specimen in compression at 4.2°K. The brittle fracture stress measurement for the 1010 steel material is plotted in Figure 1 according to its ferrite grain diameter of 0.03 mm. If the fracture stress determined for the A533 B material according to equation (2) is plotted in Figure 1 by substituting the appropriate $a_e^{-1/2}$ value for $d^{-1/2}$, say $a_e^{-1/2}$ = 0.28 $mm^{-1/2}$, then, the value of K_{Ic} is required to be clearly greater than any k_f value measured for the fracture stress-grain size experiments. The value of K_{Ic} = 33 MPa$\cdot$m$^{1/2}$ for this steel as compared with the value of k_f = 3.3 MPa$\cdot$m$^{1/2}$ which is determined for the grain size dependence of the fracture stress of 1010 steel in Figure 1. This typically large value of K_{Ic} is attributed to the controlling influence of plastic deformation on determining the unstable growth of a macroscopic crack in even the most brittle circumstance of any fracture mechanics experiment.

Another view of the relationship between the microstructure of a material and the nature of cleavage cracking within it is shown in Figure 3, this time from a study by Prasad [8] involving polycrystalline zinc material which has been deformed in compression at 4.2°K. The cleavage cracks are

revealed as white bands in this micrograph which has been produced with polarized light from a surface film put on to the specimen in a post-deformation anodization treatment. The technique has been found to be useful for revealing the relationship between the pattern of grains and the orientation or extent of cleavage cracking within hexagonal close packed metals. Extensive deformation twinning is observed within the microstructure of this zinc material, and, as for the cleavage cracks, the twin bands are related to the crystallographic orientations of the grains.

Figure 3 is important for one reason because the formation of these cleavage cracks in an externally applied compressive stress field gives emphasis to the local tensile character of the internal concentrations of stress within the slip bands which have produced the cracks. More significantly, perhaps, the figure is useful for describing the importance of information which, sometimes, either is totally absent or, at least, is not able to be observed easily in studies of cracking processes within the microstructures of materials. The deformation by slip which has produced the cracking and twinning events of Figure 3 is not able to be observed with this metallographic technique. The absence of critical information about the role of slip deformation - the primary agent - in effecting cleavage is true, also, for most studies of fracture surfaces such as are shown in Figures 2a, b. A close examination of Figure 2a is required to detect evidence of the extensive deformation by twinning which has occurred in this steel material. Thus, the connection between the microstructure of materials and their fracture properties, which has been described in historical times by Réaumur, has not been carried forward today as far as scientists and engineers desire because of this problem of less than complete information being obtainable with current observational techniques. Modern theoretical models for understanding fracture processes are normally described on the level of atomic dimensions via the dislocation events which are operative.

THE BRITTLENESS OF CRACK-FREE POLYCRYSTALS

The dislocation theory for the ductile-brittle transition behavior of steel and related materials, such as the refractory metals, has been developed by Cottrell [9] and by Petch [10]. Their analyses represent the starting point for our current understanding of the brittle fracture process within essentially crack-free polycrystalline materials. Both analyses are based on the result computed by Stroh [11] for the stability of a crack formed from an idealized (slip band) dislocation pile-up, as follows:

$$\sigma[n'b] = c'\gamma_s \quad , \tag{3}$$

where σ is the applied stress, n' is the number of dislocations, and c' is a numerical constant. Cottrell evaluated $n'b$ and σ in terms of several equations:

$$n'b = (\sigma_y - \sigma_{oy})d/2\mu \tag{4a}$$

$$\sigma = c''\sigma_y \tag{4b}$$

$$\sigma_y = \sigma_{oy} + k_y d^{-1/2} \quad , \tag{4c}$$

where σ_y is the yield stress of the material and σ_{oy} is the grain size independent friction stress component of the yield stress. Beyond equations (4a)-(4c), Cottrell proposed that the onset of brittle fracture was controlled by the break-out of cleavage past one grain diameter so that γ_s

should be replaced by a surface energy term including the plastic work associated with the unstable growth of the cleavage crack, say, γ_p. The combination of terms in equations (3) and (4) gives the implicit specification of a ductile-brittle transition in the well-known Cottrell form:

$$k_y[\sigma_{oy}d^{1/2} + k_y] = C\mu\gamma_p \quad . \tag{5}$$

Equation (5) is particularly useful for indicating the effect on the potential brittleness of a material of those factors determining the yield stress according to the Hall [12] - Petch [2] grain size dependence in equation (4c). Brittleness is promoted: (a) by a large k_y value, measuring the slip band stress concentration for propagating plastic yielding across grain boundaries; (b) by a large σ_{oy} value, for the average friction stress to move the dislocations within slip bands; and (c) by a large grain size, the length over which an appreciable internal concentration of stress may be generated. At low temperature, σ_{oy} is increased appreciably due to the reduced availability of thermal fluctuations for assisting the movement of the large number of dislocations involved in the general plastic yielding of a material, hence, its brittleness is enhanced.

Petch obtained in his analysis [10] an explicit value of the ductile-brittle transition temperature which is measured for the fracturing energy absorbed in Charpy impact tests. This was accomplished by expressing the temperature dependence of σ_{oy} in one way as

$$\sigma_{oy} = Be^{-\beta T} \tag{6}$$

where B and β are experimental constants [13]; and, also, by assuming for this ductility transition temperature that the value of k_y was increased to k_f*, the slope of the true ductile fracture stress dependence on the reciprocal square root of the grain size [14]. By substitution for σ_{oy} and k_y in equation (5), for the transition condition at $T=T_D$,

$$T_D = \beta^{-1}[\ell nB - \ell n\ (C\mu\gamma_p k_f^{*-1} - k_f^*) - \ell n\ d^{-1/2}] \quad . \tag{7}$$

Heslop and Petch [15] showed that the predicted grain size dependence of T_D was in agreement with experimental results measured for mild steel material and, also, they showed that an increase in T_D could be accounted for if the yield stress of a material were raised by the addition of a temperature independent contribution, say, $\Delta\sigma_{oy}$*, as might occur due to solid solution strengthening, precipitation hardening, or even neutron irradiation of the material.

The Cottrell and Petch analyses for the ductile-brittle transition behavior of materials have been related by Armstrong [16] to the experimental condition of this transition being specified since the beginning of this century by the yield stress of the material becoming equal to the tensile brittle fracture stress, i.e.

$$\sigma_y = \sigma_f \quad . \tag{8}$$

Because both stresses show a Hall-Petch stress-grain size dependence, these dependences can be substituted directly into equation (8) and the parameters rearranged in the form of equation (5) developed by Cottrell as

$$k_y[\ \sigma_{oy}d^{1/2} + k_y] = k_y\ [\sigma_{of}d^{1/2} + k_f] \quad . \tag{9}$$

Thus, on this basis, the value of $C\mu\gamma_p$ in equation (5) is obtained as

$$C\mu\gamma_p = k_y[\sigma_{of}d^{1/2} + k_f] \quad . \tag{10}$$

This experimental condition which applies at the ductile-brittle transition requires, then, that the value of γ_p is not an independent parameter; it is larger itself either as the value of k_y is increased, as the grain size is increased, or as the fracture stress parameters, σ_{of} and k_f, are increased.

The dislocation theory leads to the expected inequality $k_f > k_y$ for separate model calculations of hypothetical brittle cracking versus plastic yielding processes. Therefore, the balance reflected in equations (8) and (9) is achieved for a given grain size by increasing the value of σ_{oy} over that of σ_{of} to account, when multiplied by $d^{1/2}$, for the difference between k_f and k_y. The condition that $\sigma_{of}<\sigma_{oy}$ means that the increase in γ_p with increase in grain size is smaller than applies for the collection of terms on the left side of equation (5) - and for the idealized case of $\sigma_{of} = 0$, which should apply for a pre-existing crack, the value of γ_p is independent of the grain size.

An advantage of describing the ductile-brittle transition in terms of equal values of the yield stress and the tensile fracture stress, as given by equation (8), is that these stresses are reasonably well-defined experimentally, despite the apparent scatter in Figure 1, and the theoretical considerations for understanding the transition behavior are mainly those involved in understanding the controlling factors for these individual stresses. The yield and tensile fracture stresses are related to those stresses which are operative in the Charpy impact testing procedure by accounting for the effect of the strain rate on the yield stress through reducing the parameter β in equation (6) and by accounting for the effect of the notch on raising the yield stress through a multiplying factor α in equation (8). In this manner the Charpy impact transition temperature, T_D, is evaluated in an analogous way to that given by Petch as

$$T_D = \beta^{-1}[\ell n(\alpha B) - \ell n\ \{(k_f - \alpha k_y) + (\sigma_{of} - \alpha\Delta\sigma_{oy}^*)d^{1/2}\} - \ell n d^{-1/2}] \quad . \tag{11}$$

Equation (11) has been applied by Armstrong [17] to evaluating the T_D dependence on ferrite grain size for mild steel. Figure 4 shows a comparison of this calculated result with separate experimental measurements which have been reported for the ferrite grain size dependence of a number of iron and steel materials [18-21] and for the prior austenite grain size dependence of T_D measured for A533 B steel [5] and for an iron-nickel alloy material [22]. The agreement of results for the ferrite grain size dependence appears to be satisfactory. The indication for the different average values of the T_D results for the A533 B steel and the iron-nickel alloy - but otherwise an austenite grain size dependence which is similar to the ferrite result - is mainly attributed by Armstrong and Stonesifer [23] to a large value of $\Delta\sigma_{oy}$* for A533 B steel and to an effective small (martensitic) ferrite grain size for the iron-nickel alloy.

The stress-grain size analysis for the ductile-brittle transition behavior of iron and steel materials, as described for equation (8), has been carried over by Armstrong [24] to describe the onset of brittleness in hexagonal close-packed (hcp) metals. The situation is shown schematically in Figure 5. The brittle fracture stress of these materials, too, is found experimentally not to change in any significant way with the temperature or the strain rate. However, for decreasing temperature or increasing strain rate, the yield stress of an hcp metal such as zinc, say, as measured in compression, is increased due to increases in both the σ_{oy} and k_y parameters of equation (4c). Thus, the temperature enters into equation (8) in two ways also. The k_y temperature dependence is important generally and the dislocation theory for it has been described by Armstrong as

$$k_y = c''' \, m[m^* \mu b \tau_c]^{1/2} \quad , \tag{12}$$

for which m is the Taylor orientation factor for the distribution of slip systems required to maintain continuity of the material during straining, m^* is a less-restrictive Sachs orientation factor, and τ_c is the concentrated stress required for propagation of deformation across the grain boundary during general yielding of the material.

Prasad, Madhava and Armstrong [25] have shown for the polycrystalline zinc material revealed in Figure 3 that the temperature dependence of k_y followed the prediction from previous metallographic observations of τ_c being controlled by slip on the $\{11\bar{2}3\}$ $\langle 11\bar{2}2 \rangle$ systems. The friction stress σ_{oy} for this material followed the temperature dependence corresponding to slip on $\{0001\}$ $\langle 11\bar{2}0 \rangle$ systems, as expected. The extensive twinning in Figure 3 does not enter, therefore, into either the σ_{oy} or k_y values which are measured over a range of temperatures in this case. For zinc, as was previously indicated for beryllium and magnesium materials, the temperature dependence of k_y was found to be very much larger than that for σ_{oy} and so the σ_{oy} temperature might be neglected in obtaining an approximate relationship for T_D from equation (8). The study verified the result that a relatively small grain sizes the value to T_D should follow the predicted dependence:

$$T_D = \beta_c^{-1} \, \ell n[c^* m^2 m^* \mu b B_c k_f^{-2}] + [2(\sigma_{oy} + \Delta\sigma_h - \sigma_{of})\beta_c^{-1} \, k_f^{-1}]d^{1/2} \tag{13}$$

where β_c and B_c apply for τ_c according to equation (6) and $\Delta\sigma_h$ is a work hardening contribution added to σ_y so as to specify the value of T_D for the flow stress at some small value of strain becoming equal to the tensile fracture stress. In a separate study of other polycrystalline zinc materials, Pszonka [26] has shown that the grain size dependence of equation (13) applies very well for the brittle fracture transition observed in tension.

THE BRITTLE FRACTURE OF PRE-CRACKED MATERIALS

The K_{Ic} characterization of brittleness brought on by the presence of a macroscopic crack within a material, as prescribed by equation (2), naturally excludes any direct consideration of grain size on fracture toughness. Nevertheless the K_{Ic} parameter is able to be related to the preceding description of the onset of brittleness in crack-free materials via the Dugdale [27] or Bilby-Cottrell-Swinden [28] models of continuum crack growth with an associated plastic zone at the crack tip. Keer and Mura [29] have compared the theoretical bases for calculations of this type when described in terms of continuous distributions of dislocations or in terms of discrete dislocations. Yokobori [30] has considered the energetics of fracture involving such models. Most recently, Bilby [31] has reviewed the connection between cracks and dislocations for this fracture description in terms of the equations

$$(a_s - a)/a = \sec(\pi\sigma_f / 2\sigma_y) - 1 \tag{14a}$$

$$\Phi_a/a = (4\sigma_y/\pi M)\ell n(a_s/a) \tag{14b}$$

where $a_s \equiv (a+s)$; the critical displacement parameter $\Phi_a = (2\gamma_p/\sigma_y)$; and, the modulus factor $M = \mu/2(1-\nu)$ or $\mu(1+\nu)/2$ for plane strain versus plane stress deformation. Armstrong [32] has pointed out that equation (14a) is able to be usefully expanded by series approximation so as to change it into a form that is directly comparable with equation (2). The result has been obtained by Stonesifer [5] that

$$\sigma_f = (8^{1/2}\sigma_y/\pi) \, [s/a_a]^{1/2} \quad . \tag{15}$$

A comparison of equations (2) and (15) shows for $a_s \simeq a_e \simeq a$, a linear dependence of σ_f on $a^{-1/2}$ requires a constant plastic zone size to characterize the fracture toughness. The comparison gives the result

$$K_{Ic} = (8s/\pi)^{1/2}\sigma_y \quad . \tag{16}$$

The same result is obtained from equation (14b) by using the condition that $K_{Ic}^2 = 4M\gamma_p$.

A direct utility of equation (15) is to demonstrate the effect of the plastic zone size s on modifying the linear $a^{-1/2}$ dependence of σ_f as (σ_f/σ_y) increases, even to the extent of approaching 1.0. This consideration has been demonstrated in one way for results to be expected for PMMA (polymethylmethacrylate) material by Armstrong [32]. The same consideration is shown in Figure 6 for the dependence of σ_f on $a^{-1/2}$ which is computed from very accurate measurements of K_{Ic} at different a values for tempered 4340 steel as determined by Jones and Brown [33]. Also shown in Figure 6 is the σ dependence on $a^{-1/2}$ which was obtained for PMMA material in the study of Williams and Ewing [34]. The greater toughness of steel material relative to PMMA is established in the figure by the increased slope of the steel result which corresponds to a larger value of the plastic zone size s.

The influence of the polycrystal grain size on the fracture toughness of a material as expressed in K_{Ic} should follow a Hall-Petch dependence because of the yield stress factor in equation (16). This dependence has been verified by Stonesifer and Armstrong [35] for the ferrite grain size dependence of a number of steel results [36,37] and for prior austenite grain size dependence of A533 B steel material. Figure 7 indicates this Hall-Petch dependence may apply for the fracture toughness of several aluminum alloys in sheet form as reported by Hahn and Rosenfield [38] from a study of Thompson, Zinkham and Price [39]. A Hall-Petch dependence has been established for the yield strength of recrystallized 7075 aluminum alloy by Waldman, Sulinski and Markus [40]. In this latter study, the grain diameter was measured perpendicular to the rolling direction in a longitudinal direction. The k_y value for the material of 0.16 MPa•m$^{1/2}$ compares favorably with the range of values $0.12 < k_y < 0.19$ MPa•m$^{1/2}$ reported in an analysis of previous studies by Armstrong [41]. Thompson and Zinkham [42] have indicated for the results in Figure 7, plus others, that the microstructures for these alloys ranged from being coarse equiaxed grains to being highly elongated and almost completely unrecrystallized grains - though the grain size through the thickness of the sheet was able to be correlated with the fracture toughness of the alloys. A quantitative evaluation of the fracture toughness dependence on grain size in Figure 7 does indicate that the plastic zone size should have to increase as the average grain size is reduced for these materials. A similar result is indicated for the high temperature fracture toughness measurements for

A533 B steel material which have been obtained by Stonesifer [5]. One consideration for this result is that the post-yield fracture mechanics description proposed by Heald, Spink and Worthington [43] might apply in these cases so that the value of σ_f should be employed in place of σ_y in equation (16) and, correspondingly, the grain size dependence of K_{Ic} should be greater, as measured, for example, by k_f or k_f*.

A special application of equations (15) and (16) may be to describing the fracture toughness dependence on grain size for polycrystalline ceramic materials. This occurs because these materials fracture brittlely due to the presence of cleavage cracks within the microstructure even at stress levels near to those required for general plastic yielding. Armstrong [41] has suggested that the curvature due to the dual effect of the plastic zone size in equation (15) may be responsible for the bimodal fracture stress dependence on grain size frequently reported for these materials. Bradt, Dulberg and Tressler [44] have interpreted the effect of surface finishing on the strength - grain size dependence of MgO material to be explicable in terms of an equation similar to equation (15). The measurement of an increased fracture energy as the grain size is reduced has been reported for Al_2O_3 by Simpson [45]. An increased fracture energy has been measured by Ahlquist [46] as the yield strength is increased for alkali halide materials.

Finally, it should be noted that despite the differences which have been described for the models of fracture on the macroscale versus the microscale there is one more connection to be made between the various equations describing the operative mechanics of the cracking processes. This is best illustrated, perhaps, by combining equation (16) with $K_{Ic}{}^2 = 4M\gamma_p$ to give

$$\sigma_y^2 s = (\pi/2)M\gamma_p \; . \tag{17}$$

This result for the fracturing energy involved in the failure of a brittle material is to be compared with equation (5) from Cottrell for the fracture transition as rewritten in the form

$$\sigma_y^2 d\ [1 - (\sigma_{oy}/\sigma_y)] = C\mu\ \gamma_p \; . \tag{18}$$

Putting aside comparative questions about the strain rate and triaxiality of the stress state, the equations are nicely matched by considering for the onset of brittleness in a crack-free material that the slip length was entered into equation (3) as a measure of the dislocation displacement involved in forming a cleavage crack. The presence of σ_{oy} is due to the finite friction resistance which is attributed to the average shear strength of individual crystals. For the limiting case of brittleness to be obtained for a crack containing material, then, it might be imagined according to equation (17) that the value of s could be reduced to a dimension of the order of the polycrystal grain diameter as described for equation (18). The decrease in K_{Ic} for steel as the temperature decreases is well-known. The explanation for it is produced by the dramatic decrease in s which must occur to override the appreciable increase in σ_y accompanying the decrease in temperature. A lower limiting size for the micro-cleavage events signally the onset of brittle fracture at the crack tip of pre-cracked specimens has been estimated by Ritchie, Knott and Rice [47] as occurring at a distance of two average grain diameters ahead of the crack tip. Stonesifer and Cullen [48] have obtained striking confirmation of this estimate for the fracture toughness testing of A533 B steel at 77°K - hence the connection of equations (17) and (18) seems reasonable on the basis of microstructural observations, also. The implication that the most brittle of cleavage fractures are controlled by events occurring locally within the microstructure of the material, on the scale of several grains at most, must contribute something to the observed scatter of experimental results in Figure 1.

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Table 1 - Brittle Fracture Stress Investigations for Iron and Steel Materials

SYMBOL	Material and Remarks	Testing Temperature °K	Reference
○	Mild Steel, 0.1 w/o C.	4.2	N. M. Madhava, Ph.D. Thesis, Md., 1975.
⬭	Fe-3% Si, 0.037 w/o C.	77	D. Hull, Acta Met., 9, (1961), 191.
⌑	Manganese Steel, 0.15 w/o C, 1.8 w/o Mn.	60 to 140	F. de Kazinczy and W.A. Backofen, Trans. ASM, 53, (1961), 55.
▷	Mild Steel, 0.155w/o C.	77	N.J. Petch, Progress in Metal Physics, 5, 1954, p. 1; J. Iron and Steel Institute, 174, (1953), 25.
▽	Ingot Iron, 0.036 w/o C.	77	Ibid.
◁	Spectrographic Iron, 0.07 w/o C.	77	Ibid.
⊕	Iron, 0.03 w/o C.	77	J.R. Low, Progress in Material Science, 12, 1963, pl
⬭	Rimmed Steel, 0.07 w/o C.	78	J.R. Low, "Relation of Properties to Microstructure", ASM, Metals Park, Ohio, 1954, p. 163.
◇	Rimmed Steel Decarburized; Intercrystalline Fracture	78	Ibid.
0	Mild Steel, 0.2 w/o C.	77	F.S. Deronja and M. Gensamer, Trans. ASM, 51, (1959), 666.
⬭	Mild Steel, 0.2 w/o C.	4.2	Ibid.
⌂	Steel M, 1.3 w/o Mn, 0.16 w/o C.	77	G.T. Hahn, M. Cohen and B.L. Averbach, J. Iron and Steel Institute, 200, (1962), 634.
D	Steel E, 0.22 w/o C, 0.36 w/o Mn.	77	G.T. Hahn, M. Cohen, B.L. Averbach and W.S. Owen, "Fracture", 1959, (New York, John Wiley), p. 91; J. Iron and Steel Institute, 200, (1962), 634.
⌔	Steel F2, 0.039 w/o C.	77	G.T. Hahn, B.L. Averbach, W.S. Owen and M. Cohen, "Fracture", 1959, (New York, John Wiley), p. 91.
ᗡ	Steel X-52, 0.26 w/o C, 1.15 w/o Mn	77	A.R. Rosenfield and G.T. Hahn, Trans. ASM, 59, (1966), 962.
△	Armco Iron, 0.022 w/o C.	128 to 183	E.A. Almond, D.H. Timbres and J.D. Embury, "Fracture 1969", (London, Chapman and Hall), p. 253.
⊞	EN 2 Steel, 0.15 w/o C; Yielding Preceded Fracture.	77	D. Hull and I.L. Mogford, Phil. Mag., 3, (1958), 1213.
■	High Purity Quenched Iron, 0.0025 w/o C; Intercrystalline Fracture	77	C.E. Richards, C.N. Reid and R.E. Smallman, Trans. Japan Inst. Metals, 1968, Vol. 9 Supplement, p. 961
▼	Decarburised NPL Iron, 0.018 w/o C; Intercrystalline Fracture	77	E.A. Almond, D.H. Timbres and J.D. Embury, Phil. Mag., 23, (1971), 971.
▲	Decarburised Armco Iron; Intercrystalline Fracture.	218	Ibid.
	$\sigma = 351.5 + 3.30 d^{-1/2}$ MPa ; d in m.		A.R. Rosenfield and G.T. Hahn, Trans. ASM, 59, (1966), 962.
	$\sigma = \left[\pi E \gamma/(1-\nu^2)\right]^{1/2} d^{-1/2}$		R.A. Sack, Proc. Phy. Soc. (London) 58, (1946), 729.

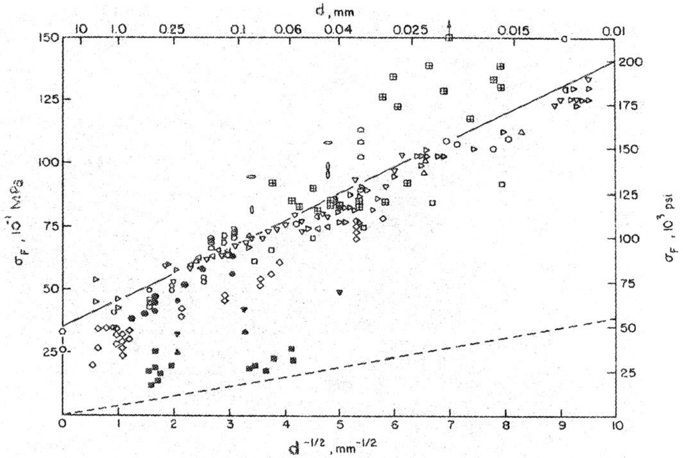
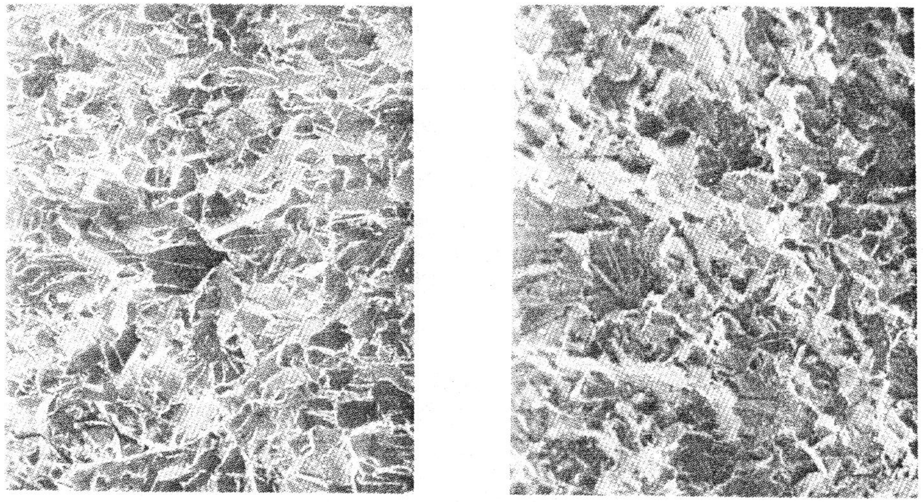


Figure 1 - Hall-Petch dependence for the brittle fracture stress of iron and steel materials, after Madhava (1975).



(a) (b)

Figure 2 - Scanning electron micrographs of polycrystal cleavage surfaces for two steel materials:
(a) tensile fracture at 4.2°K, after Madhava (1975); and
(b) fracture mechanics test at 77°K, after Stonesifer (1975).

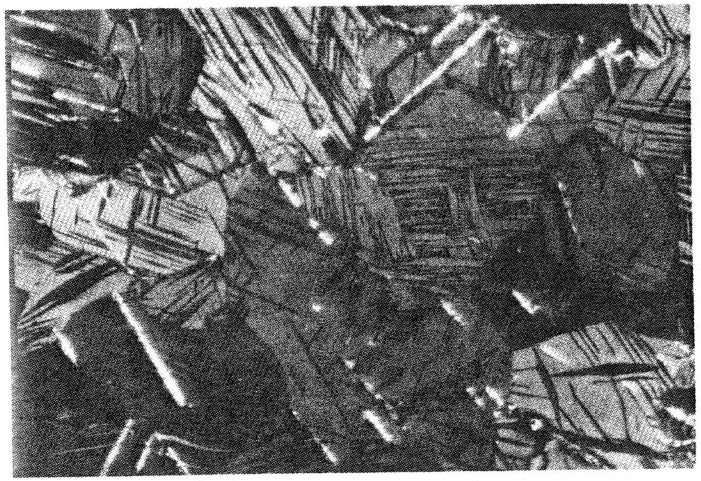


Figure 3 - Cleavage cracks within polycrystalline zinc compressed at 4.2°K, after Prasad (1974).

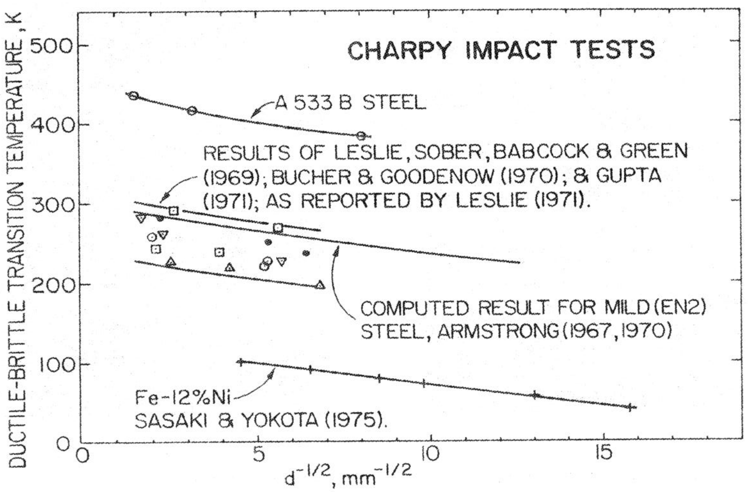


Figure 4 - Ductile-brittle transition temperature dependence on grain size for Charpy impact testing of steel materials, after Stonesifer and Armstrong (1977).

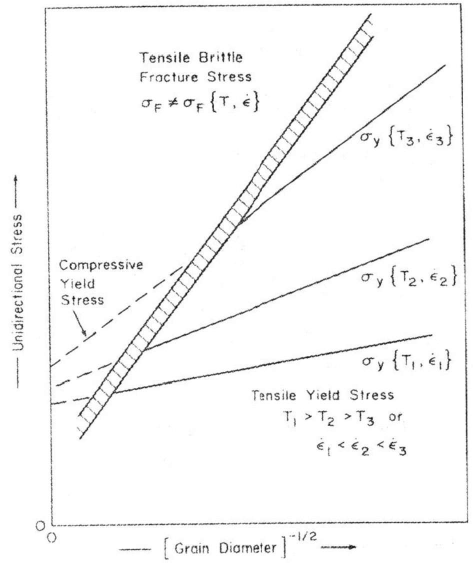


Figure 5 - The tensile ductile-brittle transition behavior of hcp materials according to a Hall-Petch analysis, after Armstrong (1968).

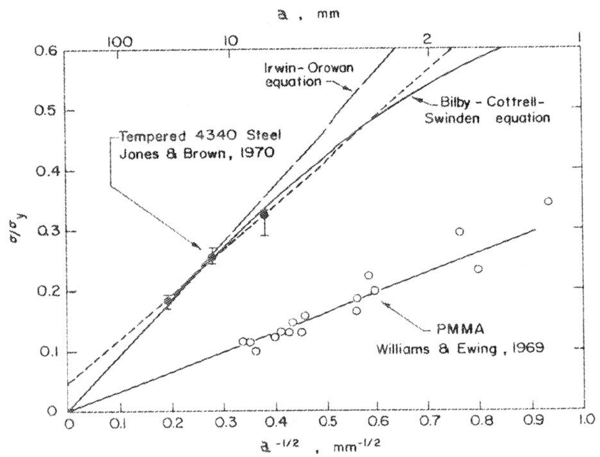


Figure 6 - The fracture stress dependence on crack size for steel, after Jones and Brown (1970), and for PMMA, after Williams and Ewing (1969).

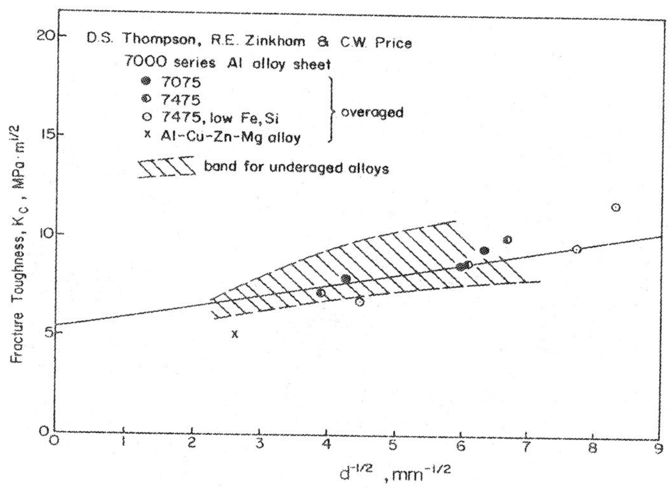


Figure 7 - The fracture toughness dependence on polycrystal grain size for variously treated aluminum materials; results of Thompson, Zinkham and Price (1974).