USE OF THE CALCULATION OF INTEGRAL J_1

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INTRODUCTION

The method of finite elements applied to the analysis of stresses and strains is of major interest in fracture mechanics. In particular, calculation of the integral J_1 or similar integrals by these methods leads to useful applications. Two types of application are presented here, one for the fast, relatively low cost determination of K_I in linear elastic fracture mechanics, and the second for the examination of certain conditions of validity of initiation criteria based of J for an elastic plastic material. These applications are illustrated for a number of calculation results obtained by means of the CEASEMT system developed at Saclay [1-2].

DETERMINATION OF K_I IN LEFM

A number of different methods are available to determine K_I by means of a calculation program by finite elements [3-4]. Most of these involve determination of certain values (displacement, stress, etc...) as a functio of distance from the crack front (polar radius). Hence these are actually derivation methods. Consequently, they require a fine mesh and are therefore costly. This explains why they are rarely employed for industrial calculations.

Methods possessing an integration character are far more preferable. Furthermore, the method of finite elements gives better results on overall values such as energy, than on the detailed distriubtion of strains or stresses. The integral J_1 hence appears more suitable for the determination of K_I since it is a curvilinear integral resulting from an integration in the entire plane region enclosed by the integration path.

Calculations performed with the CEASEMT system showed that this method is, technically, fairly accurate and inexpensive. The PASTEL module enables calculation of J_I simultaneously on several contours.

A simple example is provided by the results obtained for a square plate exhibiting a crack whose length 2a is 1/4 of the side, subjected to a uniform tensile stress S on the sides parallel to the crack. The table one gives the reduced value F of $K_I = FS\sqrt{\pi a}$, the deviation in relation to known values [5] and the total cost of the calculation expressed in equivalent seconds of an IBM 360/91 computer for the different meshes employed. For reasons of symmetry, the calculation only covers a quarter of the plate. Some of these meshes are shown in Figure 1.

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It can be seen that satisfactory accuracy (2%) is achieved for low calculation cost. Moreover, the mesh is obtained automatically and requires very little labor.

This method is obviously limited to the area of validity of J_1 (homogeneity of the material, flatness of the crack). However, it is suitable for dealing with certain interesting cases, such as pseudo-cracks without sharp fronts.

As an example, a pseudo-crack with a thickness of 0.1 mm located at the center of a beam bent at three points was subjected to calculations (Figure 2). By definition, the reduced stress intensity factor is equal to:

$$F_J = \frac{1}{S}\sqrt{\frac{EJ}{(1-\nu^2)\,\pi a}} \tag{1}$$

Where S is the reference stress (in this case equal to the maximum bending stress of the uncracked part) and J is calculated on four contours surrounding the entire crack front.

a(mm)	0.2	0.5	1	2
F_J	1.046	1.016	0.998	0.959

The standard deviation on the four contours is always less than 0.010. As for the calculation cost, it is less than 50 equivalent seconds for each case.

This method may be extended to three-dimensional cases, by calculation of the vectorial integral J on different surfaces enclosing the elements of the crack tip. Another development under way at Cadarache deals with thin shells, using both the integral J and the integral L of Knowles and Sternberg [6].

VALIDITY CONDITIONS OF THE CRITERION J_C

The integral J is employed as a crack propagation initiation criterion [7]. Two arguments may be employed to justify this view. The first relates to the energy available during crack propagation. The second is based on the property of independence of the contour, making J a characteristic of the crack front (like K_I in LEMF). For both arguments, it is necessary for J to be path-independent.

It has been shown that J is independent of the contour for non-linear elastic materials. It was suggested to extend this property to plastic materials by using the "strain energy" W, namely, the density of work received [8]. However, the validity of this extension is debatable.

A necessary condition for J to be independent of the contour may be written [9]. A defect vector W_k^* is defined, with a volume density of:

$$W_k^* = W_{,k} - \sigma_{ij}\varepsilon_{ij,k} \tag{2}$$

Where:

$$W = \int_0^{ij} \sigma_{ij}\,d\varepsilon_{ij}$$

($W_{,k}$ is the derivative with respect to x_k).

The surface and linear densities are similarly defined. This vector is null when the spatial variation in "strain energy" dW is equal to that which would result from the spatial variation in "strain", in other words:

$$dw = \sigma_{ij}\,d\varepsilon_{ij}$$

Applied to the surface of the body are the surface defect vectors $\bar{W}^{**}$ which, when no loads are applied, are equal to $W\bar{n}$ ($\bar{n}$ normal to the surface). It can easily be shown that the overall defect vectors applied have a null resultant (Figure 3).

It is easy to show that integrals J_1 and L_1 are the resultants of the defect vectors located in the volume V bounded by the integration surface:

$$\vec{J} = \int_V \vec{W}^* dv \tag{3}$$

$$\vec{L} = \int_V (\vec{OM} \wedge \vec{W}^*)\, dv \tag{4}$$

The conditions of independence of J and L are hence reduced to $\bar{W}^*_k = 0$.

In the plane case of a plane crack parallel to axis Ox (tip perpendicular to Oxy), the condition for J_1 (component of J along x) to be independent of the contour is hence $W^*_1 = 0$, or:

$$\frac{\partial W}{\partial x} = \sigma_{ij}\frac{\partial \varepsilon_{ij}}{\partial x} \tag{5}$$

This condition does not require the use of the constitutive equation of the material.

A sufficient but not necessary condition may be suggested for materials with potential mechanical energy W. It is sufficient for W not to depend explicitly on the point in question, but only on the state of strain ε_{ij} (the material must be homogeneous).

This condition may be applied to non-linear elastic materials. It may also be applied to materials exhibiting deformation type plasticity. In effect, if unloading were not to occur, a relationship would exist in finite terms between strains and stresses, introducing a mechanical potential.

Unfortunately, the plastic behavior of materials is rather of the incremental type, and no mechanical energy potential exists. Consequently, it is not certain that J is independent of the contour, and that a criterion based on J is entirely valid [10].

NUMERICAL STUDIES OF THE INDEPENDENCE OF J

Calculation results obtained with the method of finite elements, employing an incremental plasticity model, can serve to evaluate the independence of J. Consequently, in the special cases calculated, it is possible to appreciate the validity of a criterion based on the value of J. Such calculations have been performed [11] in a number of cases. It does not appear that highly significant variations of J with the contour occur. Moreover, it is difficult to establish whether the variations observed are real or due to numerical appearances. However, some authors [12] believe that J is not path-dependent when the path crosses plastic zones.

A number of calculations of this type were performed with the CEASEMT system. The plasticity model employed was that of Von Mises normal flow and law (Prandtl-Reuss equations) [13].

The above plate (2 x 2 mm square with crack 2a = 0.5 mm) was analyzed (plane strain) (E = 206,800 MPa - $\nu = 0.3$ - $\sigma_y = 310$ Pa). The load S consisted of a tensile force applied progressively to the sides parallel to the crack. The quarter plate mesh is represented in Figure 4. Figure 5 shows the variation of F_J with S for a material without strain hardening, and a material of which the tangent modulus is 1/10 of the Young's modulus.

For each of the 15 increasing values of stress S, over 20 contours, J was calculated, together with the portion due to elastic energy J_e, that due to plastic energy J_p and that due to the forces on the integration contour J_F. Table 2 gives, as an example, the results obtained for S = 260 MPa. It may be noted that J depends only slightly on the contour (unit N/m) despite a very broad plastic zone.

It should be observed that in all the cases dealt with, the loading was radial, in other words, all the forces applied increased proportionally. This procedure makes it possible to consider the behavior of the test sample like that of a material with deformation type plasticity. It is also interesting to analyse other types of loading and to compare them with the foregoing results. In effect, significant changes in strain may occur at certain points, and possibly local recovery in the elastic strain region. Analyses of this type are under way at Saclay on the plate previously investigated. Instead of increasing the applied stress S uniformly, it is progressively established at a selected value S_o, starting, for example, with the corners and moving towards the center of the side (Figure 6). Initial results obtained appear to differ from those obtained for radial loading up to S_o.

Consequently, calculations of J by the method of finite elements can serve to clarify the independence of the latter for material behavior which is not of deformation type plasticity, but incremental plasticity and visco-plasticity. They also make it possible to evaluate the effect of the loading procedure which, in actual structures, is not always of the radial type, but may be more complex.

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Table I

Mesh	nodes	elements	F	deviation	cost
A	25	32	0.855	- 17%	
B	81	128	0.962	- 8%	53
C	137	232	0.974	- 5%	61
D	289	512	1.009	- 2%	71
E	1089	2048	1.034	+ 0.3%	354

Table II

PATH	J_E	J_P	J_F	J	F_J
E1	26,33	235,57	249,42	511,32	1,479
E2	64,16	254,82	164,28	483,27	1,438
E3	72,76	308,19	108,31	489,27	1,447
E4	79,89	344,34	67,84	492,08	1,451
E5	87,70	300,01	102,69	490,40	1,448
E6	94,06	323,14	74,53	491,74	1,450
E7	100,09	336,24	56,41	492,76	1,452
E8	105,82	341,10	46,45	493,38	1,453
E9	111,39	341,10	41,26	493,77	1,453
E10	140,33	304,81	49,72	494,87	1,455
E11	171,86	271,49	53,25	496,61	1,458
E12	187,65	239,72	70,52	497,90	1,459
E13	203,93	179,33	116,81	500,08	1,463
E14	229,70	121,94	149,89	501,55	1,465
E15	236,29	69,80	196,16	502,26	1,466
E16	233,83	27,37	242,47	502,68	1,466
E17	206,94	0	296,11	503,06	1,467
E18	155,23	0	348,01	503,24	1,467
E19	127,17	0	375,99	503,16	1,467
E20	120,51	0	382,58	503,10	1,467

J unit N/m

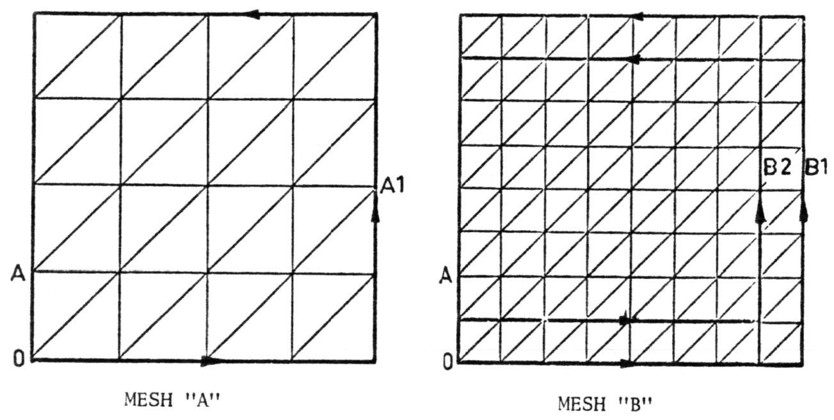

Figure 1 Meshs used in K_I computation

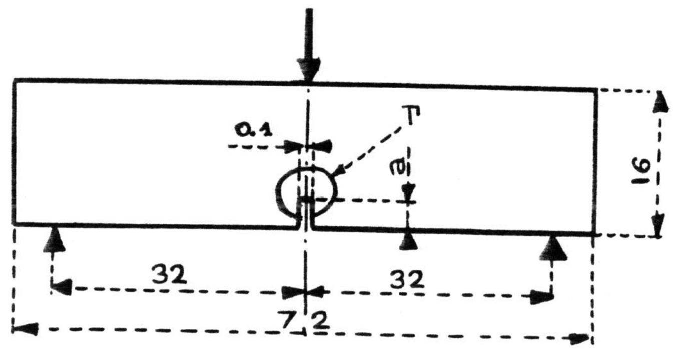

Figure 2 Open crack in a beam

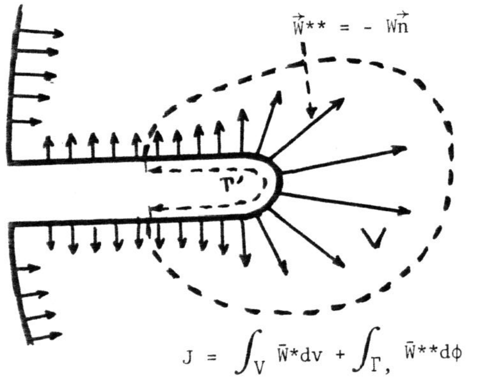

Figure 3 Defect vectors along a crack

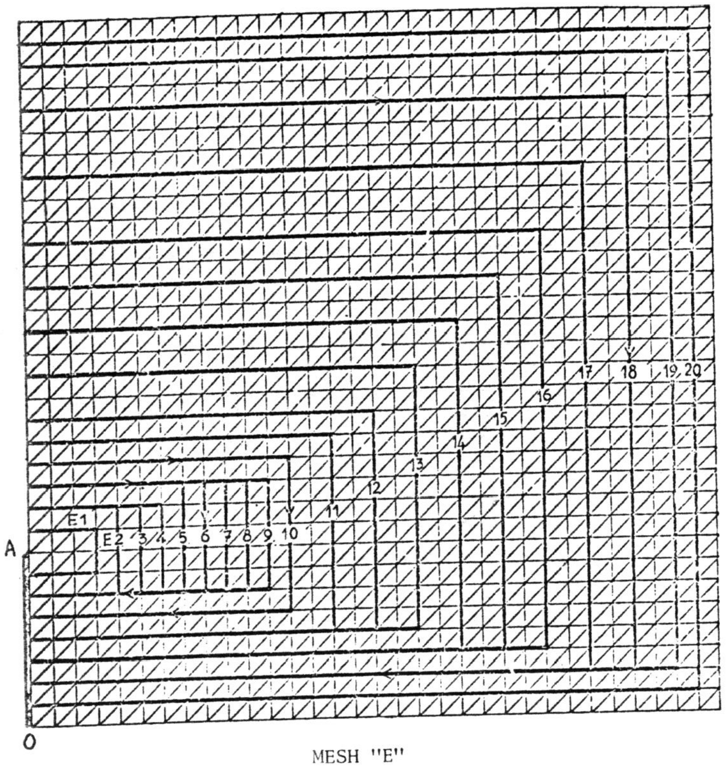

Figure 4 Mesh used for plastic computation

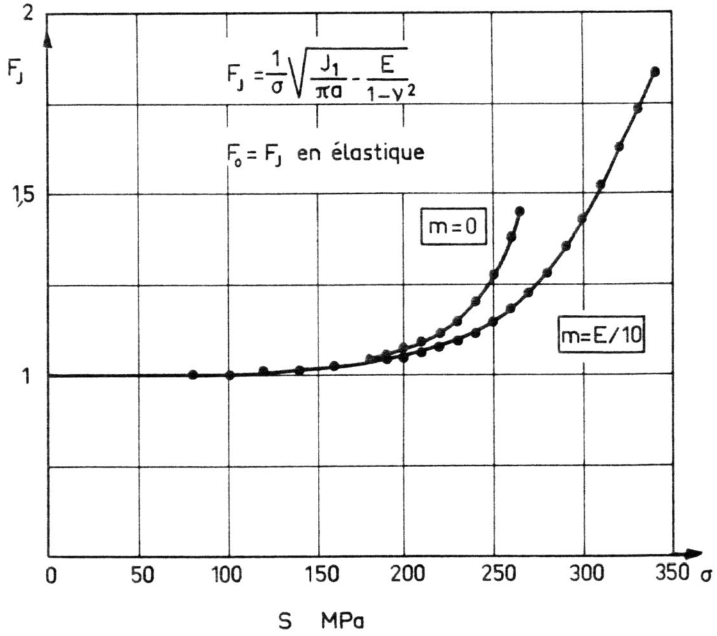

Figure 5 Reduced value of J_1 versus applied load

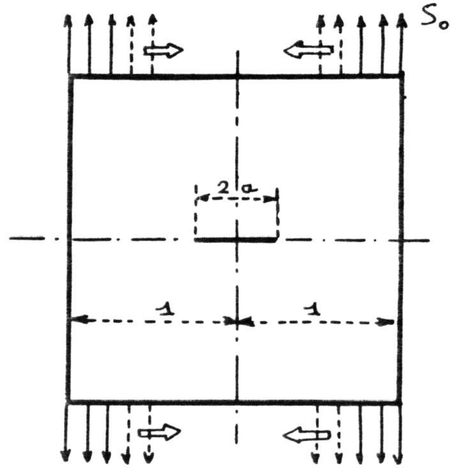

Figure 6 Non proportional loading process