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UNSTABLE FRACTURE CRITERIA UNDER LARGE PLASTIC DEFORMATION

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INTRODUCTION

So far the ductile fracture of metals has been discussed mainly on the following three aspects: (a) the effect of some defects contained in the material on ductility of material through a continuum mechanics approach [1, 2]; (b) the development of linear fracture mechanics in reference to the stress and strain distribution near a crack tip after yielding [3, 4, 5]; (c) the generalization of ductile fracture criteria by studying failures in different kinds of material tests [6, 7, 8]. It seems, however, that ductile fracture has not yet been sufficiently studied in the aspect of the relation between a critical strain and a crack length, which might well be the most fruitful approach to the interpretation of the ductile fracture. The description of the relation is also very useful to resolve the various failure problems of engineering materials under large plastic deformation such as that in metal working, and could implicate the K_c concept in linear fracture mechanics in its extreme case.

UNSTABLE DUCTILE FRACTURE

If fracture occurs between the yield point and the load predicted by the ultimate tensile strength, then the linear fracture mechanics is no longer applicable, nor any other conventional fracture criteria. The present investigation is concerned with unstable ductile fracture criteria, which would complete a whole fracture concept along with linear fracture mechanics. In this study a crack length is limited to be (a) not so large as the plastic region does not cover entirely the test piece, and (b) not so small as a local necking takes place, providing a unfirom uniaxial plastic condition. The model for investigating the unstable fracture criterion is proposed and schematically shown in Figure 1. A curve OABC means the load-displacement relation of the specimen with an initial crack length a, and ODEF with an initial crack length $a+\Delta a$, respectively. The path $C \rightarrow F$ indicates that the state of C goes to a certain state F when a crack grows by Δa.

The following assumptions, designated KOBE-model, are postulated. (a) The state of F is independent of path. That is, the point F is also on the load-displacement curve of the specimen with initial crack length $a+\Delta a$. (b) The ratio of a load P_1 on the curve OABC to a load P_2 on the curve ODEF at any displacement is constant. Then, the load P-displacement λ relationship for the specimen with crack length a can be given by

$$P = F(a) \cdot f(\lambda) \qquad (1)$$

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MS54

and the function F(a) is the same as that in the elastic range where it is well known. The validity of (b) has been assured by the finite element analyses as mentioned in the later section.

According to the general theory conservation law, the following equation is written at the onset of crack extension:

$$\dot{W} = \dot{U} + \dot{\Gamma} + \dot{K} , \tag{2}$$

where W is the external work, U the internal strain energy, Γ the effective fracture surface energy, K the kinetic energy, and the dot denotes differentiation with respect to time. From equation (2) and the first assumption (a), a criterion for crack extension is given by

$$\frac{\partial \Gamma}{\partial a} = \frac{\partial (W-U)}{\partial a} . \tag{3}$$

Recently, the J integral [9] and the COD (crack opening displacement) are used for the estimation of the fracture toughness of notched specimens, but it is difficult to find their exact values by experiments or by calculations. Equation (3) is more useful than other procedures if the loading displacement relation is given analytically as equation (1). So equation (3) is applied to derive the unstable ductile fracture criterion in our study.

THEORETICAL ANALYSIS

Though the criterion could be induced from equation (3) for any load-displacement relationship, here we study a typical one described by a power strain hardening law such as

$$\sigma = k \cdot \varepsilon^{n} , \tag{4}$$

where σ, ε, n and k are the true stress, the true strain, the strain hardening exponent and the material constant, respectively. According to the KOBE-model, the load P-displacement λ relationship for the specimen of such material with crack length a is represented by

$$P = F(a) \cdot [\ln(1+\lambda/l_0)]^{n}/(1+\lambda/l_0) \tag{5}$$

and from the linear fracture mechanics and the equation (1)

$$F(a) = A_0 \cdot k \cdot (1-2\pi a^2/w_0 l_0) , \tag{6}$$

where A_0 is the initial area of cross-section of the specimen, l_0 the initial gauge length and w_0 the initial width.

The internal strain energy U can be given from (5) and (6) by

$$U = \int_0^{\lambda} P \cdot d\lambda = \frac{A_0 l_0 k}{1+n} \cdot (1-2\pi a^2/w_0 l_0) \cdot [\ln(1+\lambda/l_0)]^{1+n} . \tag{7}$$

When a crack grows by Δa, the change of the strain energy ΔU becomes

$$\Delta U = \frac{\partial U}{\partial a} \Delta a + \frac{\partial U}{\partial \lambda} \cdot \Delta\lambda = \frac{\partial U}{\partial a} \cdot \Delta a + P \cdot \Delta\lambda . \tag{8}$$

And in that process, the external work is also given by $\Delta W = P \cdot \Delta\lambda$.

Putting (7), (8) and ΔW into (3), the criterion equation can be obtained:

$$\frac{\partial \Gamma}{\partial a} = \frac{2\pi a k}{1+n} \cdot [\ln(1+\lambda/l_0)]^{1+n} . \tag{9}$$

A crack growth occurs when the left-hand term in equation (9) reaches a constant value G_c, which is defined as the critical strain energy release rate. The unstable ductile fracture criterion is finally reduced to the form

$$\varepsilon_f \cdot a^{\frac{1}{1+n}} = \left[\frac{(1+n) \cdot G_c}{2 \cdot \pi \cdot k}\right]^{\frac{1}{1+n}} = \text{constant} , \tag{10}$$

where $\varepsilon = \ln(1+\lambda/l_0)$, and ε_f denotes a uniform strain enough away from notches at failure. The relationship (10) implicates the well known relations in linear fracture mechanics as its particular case, n = 1 ,

$$\varepsilon_f \cdot a^{1/2} = \left(\frac{G_c}{E \cdot \pi}\right)^{1/2} = \text{constant} \tag{11a}$$

or,

$$\sigma_f (\pi a)^{1/2} = K_c = \text{constant} , \tag{11b}$$

where E is the Young's modulus.

EVALUATION OF THE THEORY BY NUMERICAL ANALYSIS

The purposes of the numerical analysis are to examine the assumption (b) in the previous section and to calculate the values of the J integral and the COD. The finite element method based on the infinitesimal incremental theory by Y. Yamada [10] was used for this analysis. The specimen is divided into about 300 finite elements and the ratio of crack length to width varied from 0.01 to 0.1. Calculations were conducted for the double edge notched plates under plane stress condition. The maximum increment at each step was limited below 0.2% for strain increment or 0.1 times yield stress for stress increment. A uniform displacement was sequentially applied on both ends of the plate. The stress-strain behaviour of material was assumed to be $\sigma = E \cdot \varepsilon$ below the yield stress σ_y, and $\sigma = k \cdot \varepsilon^n$ over σ_y.

The values of $-(d \ln\varepsilon_f/d \ln a)$ obtained by these analyses on n = 0.2635 material are compared, that is, a theoretical prediction 1/1+n = 0.7915, while 0.86 by the J integral, 0.85 by the COD criterion and 0.75 by the G criterion [11]. The theoretical value falls among three calculated values and seems reasonable taking account of cumulative errors of numerical analysis. It is also confirmed in course of calculation that the slope of the $\ln\varepsilon_f$-ln a curve is almost equal to -1/2 in the elastic range, which corresponds to the theoretical value of linear fracture mechanics.

EXPERIMENTATION AND EXPERIMENTAL RESULTS

The uniaxial tensile tests of 0.2 mm thick saw cut notched specimens of normalized 0.80% C steel were conducted. The test pieces were 46 mm wide, 7 mm thick and 100 mm wide, 6 mm thick, and the notch depths at both edges of the specimens were 0.2 mm to 32 mm. The variation of hardness throughout the thickness of a specimen was within Hv 10. The mechanical properties of non-notched specimen were as follows; ultimate tensile strength 91.9 kg/mm^2, elongation 12.9%, yield stress 44.5 kg/mm^2 and reduction of area 20.7%. The Young's modulus of the material is 20500 kg/mm^2 and the strain hardening exponent n is about 0.31 on true stress-true strain basis. Three quantities were measured in the test; first one is a uniform uniaxial strain away from notches by a plastic strain gauge, second one, an elongation of the gauge length 220 mm and third one, an applied load.

The experimental results are shown in Figure 2. Below about 0.2%, i.e., before overall yielding, the slope of $\ln\varepsilon_f$-ln a relationship is -1/2, while the slope is about -1.0 over 0.2% strain possibly due to unstable excess yield strain and the effect of the ratio of crack length to specimen width. On the other hand, in the range beyond 5% uniform strain, the slope tends to be flat due to the local necking of specimen. Between 0.2 mm and 1.0 mm in crack length, stable uniform strain conditions are satisfied, where the $\ln\varepsilon_f$-ln a curve has the slope of about -0.76 predicted by the present theory for n = 0.31 of 0.80% C steel. The agreement indicates that the fracture criterion represented by equation (10) is appreciably reasonable in case of unstable ductile fracture, so long as uniformity is kept and materials obey a power strain hardening law.

DISCUSSION AND IMPLICATION

(1) In order to get a better fit for various engineering materials, other expression of a stress-strain relation will be applied:

$$\sigma = Y + H \cdot \varepsilon_p^{n'}, \tag{12}$$

where Y and n' are material constants and ε_p is the plastic strain. In this case, by the same procedures as in the former, the following representations are obtained:

$$P = A_0(1-2\pi a^2/w_0 l_0)(Y+H\varepsilon^{n'})(1-Hn'\varepsilon^{n'-1}/E)/(1+\lambda/l_0) \tag{13}$$

$$a \cdot [\varepsilon_f(Y + \frac{H}{1+n'}\varepsilon_f^{n'}) - \frac{H}{E}\varepsilon_f^{n'}(Y + \frac{H}{2}\varepsilon_f^{n'})] = \text{constant} \tag{14}$$

or approximately

$$\varepsilon_f \cdot a^{\frac{1}{1+n'}} = \text{constant.} \tag{15}$$

Putting the value of each parameter into (14) and (15), the relationship ε_f-a is obtained for 0.80% C steel as shown in Figure 3. The former theoretical result and the experimental result are also shown. There is little difference among them and there comes a more simple conclusion that since the strain hardening exponents of most metals are usually between 0.25 and 0.35, the product of a critical strain and the 0.75 ~ 0.8th power of a crack length is almost constant, whichever representation is used for the stress-strain relationship.

(2) The present theory has the close relation with the Griffith criterion, and also throws light on the physical meaning of reduction of area in conventional tensile test, the historical basic ductility measure. If a material shows an ideal power strain hardening characteristics and a specimen has infinite width, an ideal $\ln\varepsilon_f$-ln a diagram will be drawn as Figure 4. Obviously, the unstable ductile fracture criterion is rewritten with reference to the conventional K_c value,

$$\varepsilon_f \cdot a^{\frac{1}{1+n}} = \frac{\sigma_y}{E}\left[\frac{K_{Ic}^2}{\pi \cdot \sigma_y^2}\right]^{\frac{1}{1+n}} . \tag{16}$$

While the true fracture strain ε_n derived from reduction of area in conventional tensile testing is plausibly given by

$$\varepsilon_n \cdot a_i^{\frac{1}{1+n}} = \text{constant}, \tag{17}$$

where a_i is the effective inclusion size of a particular material. Though the $\ln\varepsilon_f$-ln a relation over maximum uniform strain ε_u is somewhat ambiguous due to necking of specimen, for 0.80% C steel ε_n is about 0.19 as shown in Figure 2 and corresponding a_i is estimated about 0.04 mm which is reasonable value as the size of inclusions in the steel. So a whole physical interpretation is obtained throughout ductility by tensile test, i.e., unstable ductile fracture and brittle fracture. Another engineering application to estimate the K_c value from ε_n of tensile test is available on these lines. The relationship should be

$$K_c = (\pi \cdot a_i \cdot E^{1+n} \cdot \varepsilon_n^{1+n} \cdot \sigma_y^{1-n})^{1/2} . \tag{18}$$

CONCLUSION

1) A fracture criterion for unstable fracture under uniaxial tension of notched plates is proposed. The criterion for a power strain hardening material is represented by:

$$\varepsilon_f \cdot a^{\frac{1}{1+n}} = \text{constant.}$$

2) The theory has been approved by means of two procedures; the numerical analysis by the finite element method and the experiments with notched plates.

3) The present criterion is identical with that of linear fracture mechanics for n = 1, elastic body. In view of this theory, one can have better understanding about the reduction of area in conventional tensile test.

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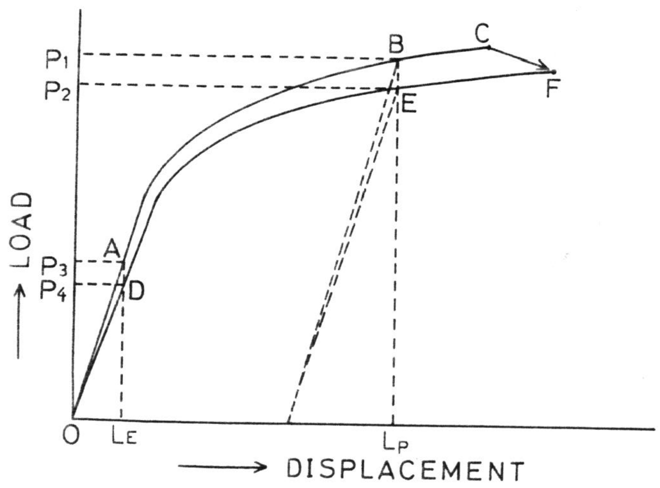

Figure 1 Schematic Diagram of the Model for the Load-Displacement Relation

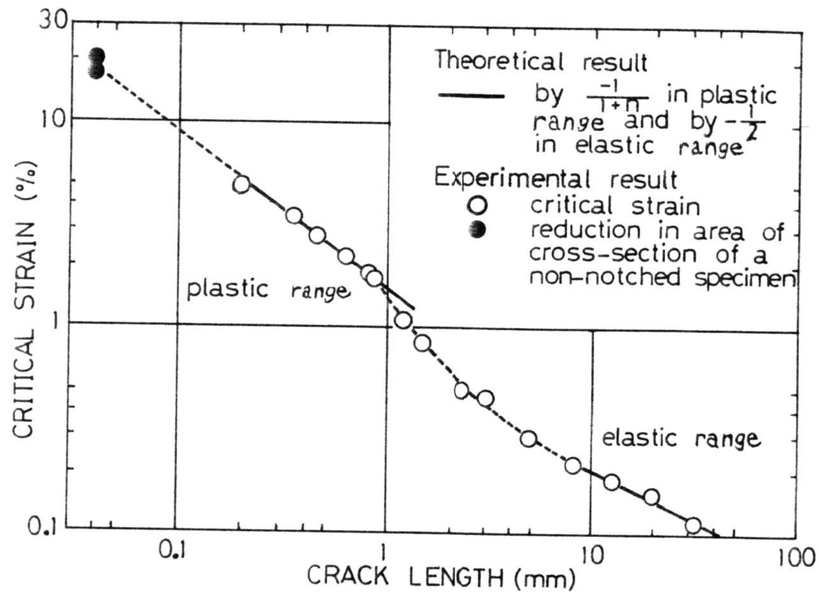

Figure 2 Theoretical and Experimental Relationship Between Strain to Fracture and Crack Length for .80%C Steel

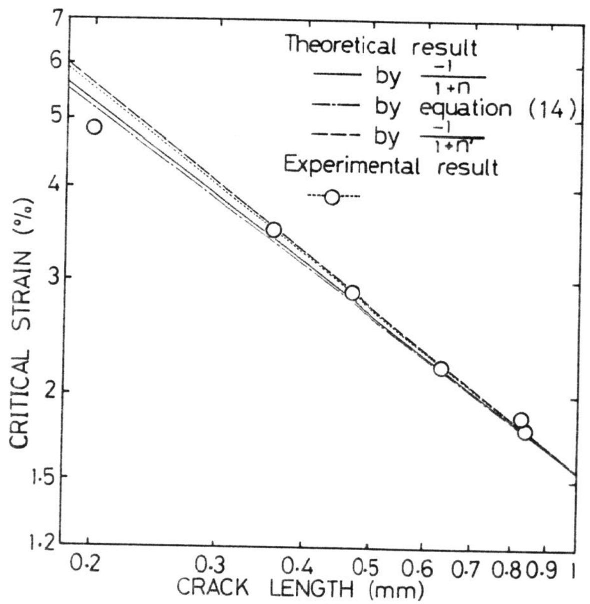

Figure 3 Theoretical and Experimental Relationship Between Strain to Fracture and Crack Length for .80%C Steel

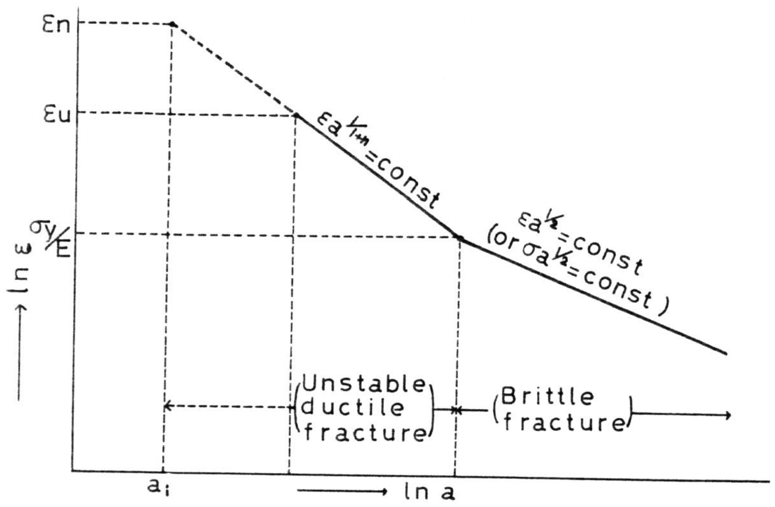

Figure 4 An Ideal Crack Length-Fracture Strain Relationship