THERMAL FRACTURE IN COMPOUND MATERIALS

K. Herrmann and A. Fleck*

INTRODUCTION

Composite materials, made by combining two or more materials with different thermoelastic moduli, play an important role in space travel and reactor technologies because of their favourable mechanical and other physical properties. From the standpoint of fracture mechanics, the estimation of the resistance of such materials to thermal stresses requires the treatment of the following problems:

1) Dependence of the specific fracture energy, as well as of the stress intensity factor, on the position and shape of the interfacial area between the individual phases of the composites, on the shape of the free surface, and on the crack geometry.
2) Influence of the inhomogeneity of the material on the corresponding physical quantities.
3) Calculation of the crack surface energy required for the formation of new surfaces.
4) Consideration of plastic zones at the crack tips.

At present, there exist, to the authors' knowledge, only a few investigations concerning the thermal fracture of inhomogeneous solids [1 - 5]. In reference [5], a representative element of a fibre-reinforced material with a concentric surface of discontinuity was considered. This composite had a crack in the matrix material, and was subjected to a well-defined thermal shock. By using a near-field solution of this crack-thermal stress problem, the crack edge displacement, the crack surface energy, the strain energy release rate, and the opening-mode stress intensity factor could be calculated for different material combinations and temperature distributions.

In this paper, an improved solution of the crack geometry considered in reference [5] is used to calculate the plastic zones in the vicinity of the crack tips. The validity of the DUGDALE model is assumed. Furthermore, a crack of length $a = x_2 - x_1$ in a thermoelastic two-phase solid (composite circular cylinder of infinite length) with an eccentric interface fibre/matrix is considered (Figure 1). The fibre and matrix of this composite consist of homogeneous, isotropic, and linearly elastic material, where the material properties vary discontinuously at the fibre/matrix interface from the values E_f, ν_f, α_f of the fibre to the values E_m, ν_m, α_m of the matrix, α being the linear coefficient of thermal expansion. Initially, the solid has uniform temperature which is assumed to be $T = T_0$ (temperature of the environment) which also corresponds to the temperature of the unstressed initial state. At time $t = t^*$, the cracked cylinder is subjected to a thermal shock, producing the following temperature distribution (cf. Figure 1 for notation)

*University of Karlsruhe, Karlsruhe, West-Germany.

$$T(r,\phi) = \begin{cases} T_f \text{ in } A_f \\ T_m \text{ in } A_m \end{cases} \qquad (1)$$

in which $T_f \neq T_m$, and both temperatures are constant. Moreover, the crack tips $x_t = x_1$ and $x_t = x_2$ are far from the surfaces Γ_j $(j = f,m)$.

ANALYSIS

Making use of the linear theory of quasi-static thermoelasticity for a plane strain state, and assuming temperature independence of the thermoelastic material constants E_j, ν_j, α_j $(j = f,m)$, as well as heat insulation of the solid with respect to its environment, the thermal stress field existing in the cracked inhomogeneous solids with a concentric or an eccentric interface, respectively, can be decomposed into two parts:

1) a regular stress field in the uncracked inhomogeneous solid, and
2) a corrective stress field with two singularities of magnitude $\rho^{-1/2}$ at the crack tips where ρ is a local polar coordinate with respect to the crack tip.

Considering the discontinuity of the given temperature distribution function $T(x,y)$, and using the stress function method, the regular stress field in both cases (concentric and eccentric interface) can be obtained from the solution of a boundary-value problem of the bipotential theory [6]. However, in case of an eccentric interface, a closed form expression for the AIRY stress functions $\Phi_i(x,y)$; $(i = f,m)$ was only available by consideration of the following conditions:

$$E_f = E_m = E, \quad \nu_f = \nu_m = \nu, \quad \alpha_f \neq \alpha_m, \quad T_f \neq T_m. \qquad (2)$$

The corrective stress field arises due to the tractions

$$\sigma^c_{yy}(x,0) = -\sigma^m_{yy}(x,0) \; ; \; \sigma^c_{xy}(x,0) = 0 \; ; \; (x_1 \leq x \leq x_2) \qquad (3)$$

along the faces $C^+ \cup C^-$ of the crack, where $\sigma^m_{yy}(x,0)$ represents the regular stress on the line $y = 0$ and is obtained from the solution of the boundary-value problem mentioned above. Therein, the sign of the stress $\sigma^m_{yy}(x,0)$ determines whether a crack located at $x_1 \leq x \leq x_2$, $y = 0$ will be open or will tend to close. Opening occurs only if $\sigma^m_{yy}(x,0)$ is a tensile stress over the prospective crack line which can be achieved by an appropriate choice of the thermal shock. Moreover, the corrective stress field has to satisfy the following boundary conditions on Γ_m:

$$\sigma^c_{rr}(r_m,\phi) = 0 \; ; \; \sigma^c_{r\phi}(r_m,\phi) = 0 \; ; (0 \leq \phi \leq 2\pi) \qquad (4)$$

Further, it should also not violate the conditions of the rigid contact at the discontinuity area Γ_f:

$$\left[\sigma^c_n\right] = 0, \quad \left[\sigma^c_{nt}\right] = 0, \quad \left[u^c_n\right] = 0, \quad \left[u^c_t\right] = 0. \qquad (5)$$

where n and t refer to the normal and transverse directions respectively.

Because of the complicated shape of the boundary of the cracked solid, a closed form solution of the boundary-value problem (3) - (5) is not available. But by assuming the crack is small in comparison with the width of the matrix material (cf. Figure 1), and also that the crack tips are far from the surfaces Γ_f and Γ_m, an approximate expression of the corrective stress field can be obtained from the solution of the following mixed boundary-value problem:

$$\sigma^c_{yy}(\xi,0) = -\sigma^m_{yy}(\xi,0) \; ; \; (y = 0, \; |\xi| \leq \delta) \qquad (6)$$

$$\sigma^c_{xy}(\xi,0) = 0 \; ; \; (y = 0, \; \forall \xi) \qquad (7)$$

$$u^c_y(\xi,0) = 0 \; ; \; (y = 0, \; |\xi| > \delta) \qquad (8)$$

where

$$\xi = x - x_0, \quad x_0 = \frac{x_1 + x_2}{2}, \quad \delta = \frac{x_2 - x_1}{2} \qquad (9)$$

In addition, the crack is considered as a discontinuity in an infinite medium, at which the crack faces $C^+ \cup C^-$ experience the thermal stress field induced by the thermal shock (1). Due to the assumptions made concerning the position and the length of the internal crack, the desired corrective stress field represents a small perturbation which must be superposed on the regular stress field. A solution of the boundary-value problem (6) - (8) can be found using an analytical method from reference [7] based on the application of complex variable technique and on the method of integral equations. Therein the expressions for the stress σ^c_{yy} and for the displacement u^c_y on the line $z = \bar{z}$ can be represented by means of one complex potential $\Psi(z)$ according to

$$\sigma^c_{yy}(\xi,0) = 2\left\{\Psi'(z) + \overline{\Psi}'(\bar{z})\right\} \qquad (10)$$

$$u^c_y(\xi,0) = \frac{4(1-\nu_m^2)}{E_m} \, i \left\{\overline{\Psi}(\bar{z}) - \Psi(z)\right\}. \qquad (11)$$

Provided the following conditions hold for $y \to \pm 0$:

$$y\,\overline{\Psi}'(\bar{z}) \to 0 \; ; \; y\,\overline{\Psi}''(\bar{z}) \to 0. \qquad (12)$$

Furthermore, the following relation must hold as $|z| \to \infty$: $\Psi'(z) = O(1/z^2)$. Then the complex potential $\Psi(z)$ can be given by means of the integral

$$\Psi(z) = \int_0^\delta \frac{\Lambda_1(t) + z\Lambda_2(t)}{\sqrt{z^2 - t^2}} \, dt, \qquad (13)$$

where the functions $\Lambda_j(t)$, $(j = 1,2)$, are the solutions of a pair of ABEL type integral equations:

$$4\frac{d}{d\xi}\int_0^\xi \frac{\Lambda_1(t)}{\sqrt{\xi^2 - t^2}} \, dt = -\omega_1(\xi) \qquad (14)$$

$$; \; (0 \leq \xi \leq \delta)$$

$$+ \frac{d}{d\xi} \int_0^{\xi} \frac{\xi \Lambda_2(t)}{\sqrt{\xi^2 - t^2}} dt = -\omega_2(\xi) \; . \qquad (15)$$

The right-hand sides of the integral equations (14) and (15) (without signs) correspond to the even and odd parts of the regular stress $\sigma^m_{yy}(\xi,0)$, respectively. The solutions of these integral equations have the following form:

$$\Lambda_1(t) = -\frac{t}{2\pi} \int_0^t \frac{\omega_1(u)}{\sqrt{t^2 - u^2}} du \qquad (16)$$

$$\Lambda_2(t) = -\frac{1}{2\pi t} \int_0^t \frac{u\omega_2(u)}{\sqrt{t^2 - u^2}} du \qquad (17)$$

where the explicit expressions for the functions $\Lambda_j(t)$, $(j = 1,2)$ are omitted here because of space limitations. Finally, by means of the functions $\Lambda_j(t)$, $(j = 1,2)$ and using the formulae (11) and (13), the desired crack surface displacement of the upper face C^+ of the crack can be represented by the real integral

$$u^c_y(\xi,0) = -\frac{8(1-\nu_m^{\,2})}{E_m} \int_{|\xi|}^{\delta} \frac{\Lambda_1(t) + \xi\Lambda_2(t)}{\sqrt{t^2 - \xi^2}} dt \; ; (|\xi| \leq \delta) \qquad (18)$$

Knowledge of the crack surface displacement (18) also allows calculation of the elastic surface energy required for the formation of new surfaces $C^+ \cup C^-$ in a specimen of unit thickness according to the formula

$$U = -\int_{-\delta}^{\delta} \sigma^c_{yy}(\xi,0)\, u^c_y(\xi,0)\, d\xi . \qquad (19)$$

Finally, using the value of the strain energy release rate for the Mode I displacement which is defined by the integral

$$G_I = -\frac{1}{2}\frac{\partial}{\partial\delta} \int_{-\delta}^{\delta} \sigma^c_{yy}(\xi,0)\, u^c_y(\xi,0)\, d\xi \qquad (20)$$

the opening-mode stress intensity factor K_I can be calculated according to IRWIN's formula:

$$K_I = \left(\frac{E_m}{1 - \nu_m^{\,2}}\right)^{1/2} G_I^{\,1/2} \qquad (21)$$

The expressions for the strain energy release rate G_I and for the stress intensity factor K_I become functions of the quantities a (the crack length), x_0 (position of the centre of the crack), s (eccentricity of the interface fibre/matrix), and T (temperature). Thereby, both formulae mentioned above are related to the crack tip which is most vulnerable to propagate. The latter behaviour is dependent on the position of the corresponding crack tip relative to the fibre/matrix interface and on the stress distribution σ^m_{yy} acting on the prospective crack line. Besides,

the respective second value of the stress intensity factor can be obtained by an appropriate variation of the parameter x_0, keeping the crack length unaltered.

Finally, by consideration of plastic zones at the crack tips in accordance with the DUGDALE-model [8], the corrective stress field (10) must be superposed on a stress field resulting from a crack subjected to tractions σ_y (yield stress) in a small region in the neighbourhood of the crack tips. The complex potential belonging to the latter stress field for the region $z = \bar{z}$ can be obtained using the technique of reference [7]. By using the condition of vanishing stress singularities at the crack tips $\xi = \pm a/2$ of the true crack, the length of the plastic zones evolves after some lengthy manipulations. Furthermore, preliminary results concerning plastic zones in materials showing strain hardening were obtained.

NUMERICAL RESULTS AND DISCUSSIONS

The Figures 2 - 5 show the results of the numerical evaluation of the formulae (18) - (21). Figure 2 gives the value of the crack surface energy $\hat{U}$ for several crack lengths in dependence on the quantity $\tilde{x}_0 = x_0/r_f$. It can be seen that the value of $\hat{U}$ increases with increasing crack length. Furthermore, the Figures 3 - 4 show for the cases of a concentric and an eccentric interface fibre/matrix, the opening-mode stress intensity factors $K_I(x_1)$ and $K_I(x_2)$ for several material combinations. These stress intensity factors were obtained by consideration of the temperature dependence of the elastic and thermal material constants as functions of crack length and temperature T_m of the matrix and the quantity $\tilde{x}_0$. The graphs mentioned above were given for a negative matrix temperature, because according to the assumption $T_f = 0$, a cooling of the matrix only leads to tensile stresses along the crack line. The stress intensity factor $K_I(x_1)$ increases strongly with increasing crack length and with decreasing distance of the crack tip, relative to the interface fibre/matrix, whereas the stress intensity factor $K_I(x_2)$ is nearly constant with increasing crack length. Extended numerical calculations are in progress. Moreover, Figure 5 shows the length of the plastic zone at the right crack tip according to the DUGDALE-model as a function of the quantity $\tilde{x}_0$ and the different stress distributions over the length of the plastic zone. The smaller plastic zone in the case of material showing strain hardening can be explained physically as due to dislocation interactions. The full length of the plastic zone may consist of two particular parts, the first one showing strain hardening and the second one occurring softening. Then the length calculated from softening dominates over that of strain hardening. There is an evident difference. Some experimental results concerning fatigue strength of steel CK 15 [9] allow a physical interpretation of the mathematical facts.

CONCLUSIONS

From a physical point of view, the crack-thermal stress problem treated above is of significance for the study of the behaviour of small initial cracks in a brittle or semi-brittle material stressed by well-defined macroscopic thermal stresses. Hence, to gain an understanding of the strength of such a material containing subcritical cracks, knowledge of the elastic crack surface energy is important. This energy represents a part of the elastic self-stress energy stored in the originally uncracked specimen. In the case of a crack formation in a primarily uncracked solid, a relaxation takes place up to the point where the crack is arrested due to shortage of the elastic self-stress energy. Consequently, self-stress fracture, with regard to the velocity of crack propagation, represents the converse case of fracture under the influence of external loading. Further, it should be noticed that in case of cracks where the surfaces, Γ_f and Γ_m, strongly influence the stress concentration at both crack tips $x_t = x_1$ and $x_t = x_2$, the analytical method described above cannot be used. At present, calculations using the finite element method are in progress.

REFERENCES

1. BROWN, E. J. and ERDOGAN, F., Int. J. Engng. Sci. 6, 1968, 517.
2. BRURY, E., Ph.D. Dissertation, University of Stuttgart, 1973.
3. BREGMAN, A. M. and KASSIR, M. K., Int. J. Fracture 10, 1974, 87.
4. KASSIR, M. K. and SIH, G. C., Mechanics of Fracture, Vol. II, Noordhoff International Publishing, Leyden, 1975, 211.
5. HERRMANN, K., a) Mech. Res. Communications 2, 1975, 85.
   b) EUROMECH 64, Stockholm, August 20-22, 1975.
6. HERRMANN, K., Beiträge zur Spannungs- und Dehnungsanalyse VI, 21,41, Akademie-Verlag, Berlin, 1970, (in German).
7. ENGLAND, A. H. and GREEN, A. E., Proc. Camb. Phil. Soc. 59, 1963, 489.
8. HAHN, H. G., Fracture Mechanics, B. G. Teuber, Stuttgart, 1976, 67. (in German).
9. FLECK, A., unpublished results, University of Karlsruhe, 1976.

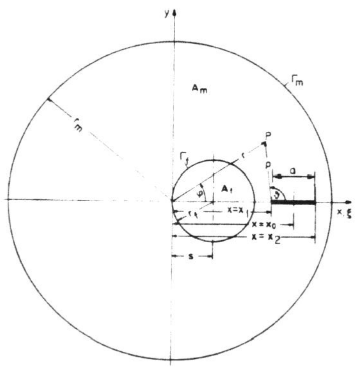


Figure 1 Crack Configuration

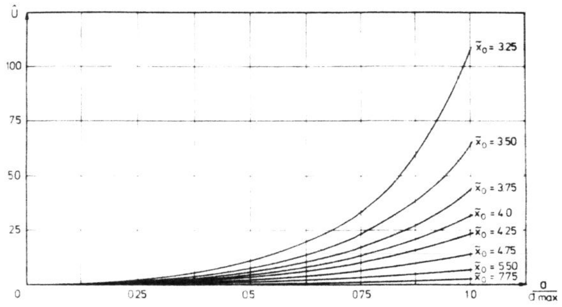
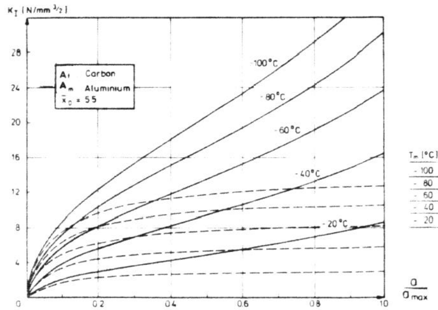


Figure 2 Crack Surface Energy (Concentric Interface)



T_m [°C]	E_m [N/mm²]	α_m 10^{-6} deg^{-1}	v_m
-100	76047	17.15	0.32
-80	74458	18.24	0.32
-60	73669	19.18	0.32
-40	73281	20.05	0.32
-20	72692	21.08	0.32

Figure 3 Opening Mode Stress Intensity Factor

——— Left Crack Tip $x = x_1$
----- Right Crack Tip $x = x_2$
(Concentric Interface)

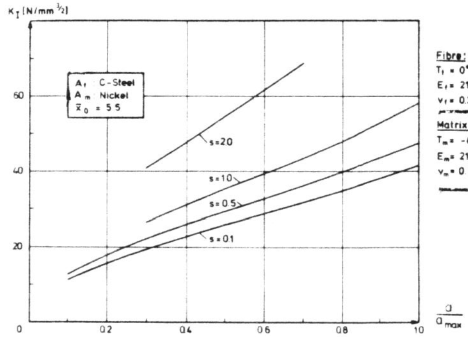


Figure 4 Opening Mode Stress Intensity Factor Right Crack Tip (Eccentric Interface)

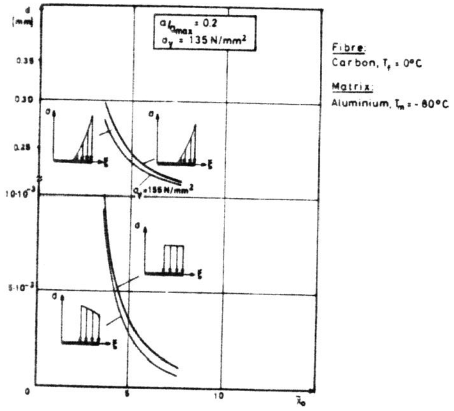


Figure 5 Length of the Plastic Zone (Concentric Interface)