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THE RELATION BETWEEN CRACK OPENING DISPLACEMENT AND FLOW STRESS

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INTRODUCTION

The width of a crack in a material about to fracture, and in a material in which fracture is taking place is an important parameter in fracture analysis; in particular the change in width as the applied stress is raised from zero to the critical stress for fracture is related to the crack extension force (or fracture toughness) of the material [1]. Crack width has been estimated indirectly in the case of very brittle materials [2, 3] but the analysis of the results needs further clarification [4]. Crack opening displacement has been calculated using numerical techniques for an aluminum alloy [5], and has been measured for a number of steels [6]; however, care is required in the detailed interpretation of the results according to the position on the crack where the measurement is made [7].

It has been suggested that only limited flow at a crack tip (and hence limited widening of the crack) can take place without fracture being prevented altogether. Two approaches taking this into account have led to criteria to brittleness [8, 9] which depend in a simple way on the flow stress of the material. From these theories it follows that crack width should depend on flow stress, and indeed Wells obtained a relation between crack opening displacement and flow stress of the form [1]

$$\delta = \frac{\pi\sigma_c^2 a}{E\sigma_y} \qquad (1)$$

for the C.O.D. occurring during the initial deformation of a material under an applied stress σ_c, with a flow stress σ_y, modulus E, and having a crack length 2a at its centre. Two materials were chosen to investigate this relation, a carbon steel which work hardness to a limited extent, and a brass which work hardness very considerably. Thin sheets of the material were used (i.e., plane stress conditions were maintained).

EXPERIMENTAL DETAILS

The steel used was a high carbon spring steel (SAE-1090). It was tested after various heat treatments to obtain different hardnesses. The brass was 70/30 alloy (SAE 70) and was work hardened and annealed to obtain the different hardnesses. The materials were in the form of strip 50 mm wide, and 0.38 mm thick in the case of the steel, and 0.50 mm thick in the case of the brass.

A crack 10 mm long and about 0.17 mm wide was formed in the strip by spark machining. In order to reduce the risk of micro cracks induced by the spark

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machining, to retain a fine finish on the crack surfaces, and to minimize the thickness of the disturbed layer at the crack surface, a very low cutting rate was used. No microcracks were observed on the test specimens.

The specimens were tested on a Hounsfield Tensometer, the crack tip was observed using a microscope, and the crack size was recorded on film by a 35 mm camera attached to the microscope. C.O.D. was determined from prints of the photographs obtained, by measuring the distance between easily recognizable features on either side of the crack tip within 0.1 mm of the tip. The estimated error of the measurement was ± 1 micron (two prints of each photograph were made, and agreement between measurements on them was better than ± 1 micron). The applied stress was increased in steps of 53 $\text{MPa}\cdot\text{m}^{1/2}$ (100 kg) in the case of the steel, and 10 $\text{MPa}\cdot\text{m}^{1/2}$ (25 kg) in the case of the brass. The time at a given stress was kept as near to 30 s as possible, and the rate of applying stress was kept as constant as possible.

EXPERIMENTAL RESULTS

It was possible to obtain hardnesses ranging from about 150 to 250 VHN in samples of the steel by heat treatment. Stress-strain curves for the material are shown in Figure 1. C.O.D. is plotted as a function of stress for a number of hardnesses in Figure 2. The value of C.O.D. is given as a percentage of the undeformed crack length. There is a nearly linear region in the curves for all hardnesses; this ended at about 80% of the stress at which unstable propagation of the crack occurred. The results for hardnesses of 175, 200 and 250 VHN were not significantly different in the linear region.

In the case of the brass it was possible to vary the hardness over the range 40 to 100 VHN by rolling and annealing. Stress-strain curves for the material are shown in Figure 3. The observed C.O.D.s were much larger and more variable than in the case of the steel. Three specimens of each hardness were therefore tested and a typical set of results is shown in Figure 4. The mean results for each hardness were used for the plot shown in Figure 5. The curves relating C.O.D. to stress have two regions, an approximately linear one at low stress, and an approximately quadratic curve (C.O.D. $\propto$ (stress)^2) at high stress.

DISCUSSION

C.O.D. in the steel and brass used in this investigation does not appear to depend on the square of stress at low stress as required by equation (1). (The effect of the finite specimen width [10] is neglected, since it is small (3%) compared with experimental error.) The relation appears to be much closer to a direct proportionality between C.O.D. and stress. This type of behaviour would be expected if the material were behaving elastically. However, the C.O.D. for the hardest brass and steel used is more than an order of magnitude greater than the elastic displacement at the region of the crack tip where the measurements were made. In addition, the dependence of C.O.D. on flow stress indicates that some plastic flow is taking place.

The linear dependence of δ on σ_c is in agreement with one of the theories for a brittleness criterion [9]. In the examples discussed in that paper, the crack opening displacement increases with stress, and is given by

$$\delta = \frac{\alpha\sigma_c a^2}{\beta a\sigma_y - \pi\sigma_c{}^2 a/E + 2\gamma} \tag{2}$$

where α and β are dimensionless constants, and γ is the surface energy of the material. The surface energy can be neglected compared with $\pi\sigma_c{}^2 a/E$ for moderate stresses, with crack lengths of 10 mm, so that curves exhibit the behaviour expected, i.e., a linear function of σ_c at low stresses, and a nonlinear deviation, above the line, at higher stresses.

At low stresses the expression reduced to

$$\delta = \frac{\alpha\sigma_c a}{\beta\sigma_y} \tag{3}$$

The experimental results only agree with this equation if $\alpha/\beta \propto (E/\sigma_y)^2$. Figure 6 shows the slopes, S, of the linear regions in Figure 2 and 4, plotted as a function of hardness. We assume hardness is proportional to flow stress, after Tabor [11]. The ordinate is $(S/E^2)^{1/3}$ and the abscissa is $(\text{VHN})^{-1}$. The points for both steel and brass are reasonably close to a straight line going through the origin. Thus

$$\delta = c_1 a\sigma_c E^2/\sigma_y{}^3 \tag{4}$$

where c_1 is a dimensionless constant. Thus, comparing equations (3) and (4)

$$\alpha/\beta = c_1(E/\sigma_y)^2 \tag{5}$$

This suggests that the work in the plastic zone at the tip of a crack is a function of the yield strain. The theory attempted to separate the elastic and plastic components of the work at a crack tip. Equation (5) suggests that this may not be possible. However, the result is not at variance with the conclusion that metals obey a brittleness criterion of the form: flow stress/modulus > 7 x 10^{-3}.

CONCLUSION

Crack opening displacement does not appear to behave as indicated by simple theory. With relatively small plastic zones at the crack tip it obeys an expression of the form $\delta = c_1\sigma_c E^2/\sigma_y{}^3$ where c_1 has the same value for both brass and steel. At higher stresses, where the plastic zones is a significant fraction of the specimen width, the crack opening displacement increases more rapidly with applied stress.

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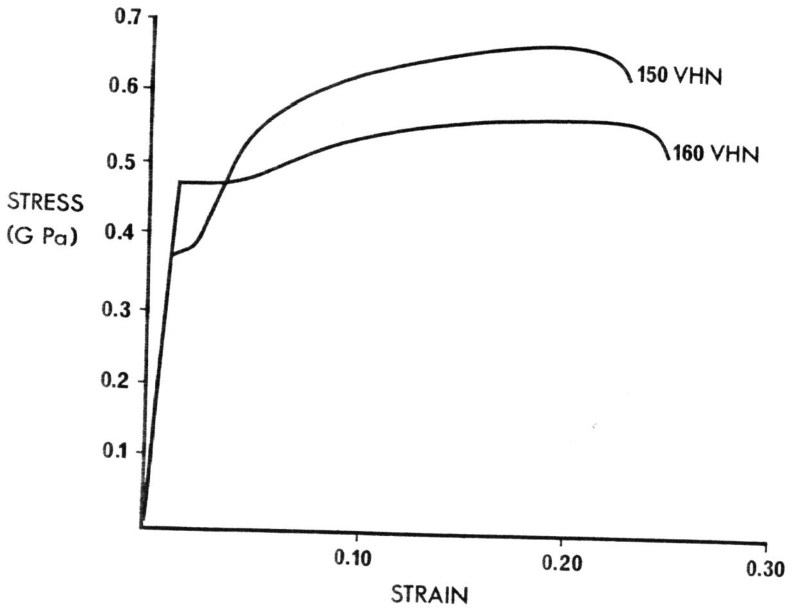

Figure 1 Stress-Strain Curves for Steel

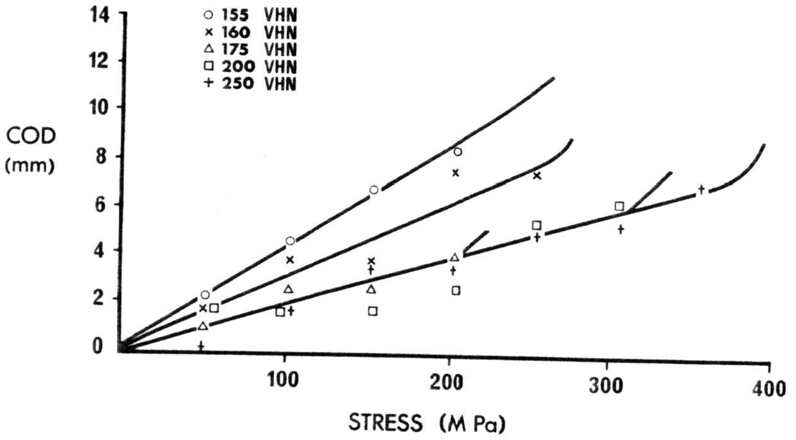

Figure 2 C.O.D. Measurements on Steel

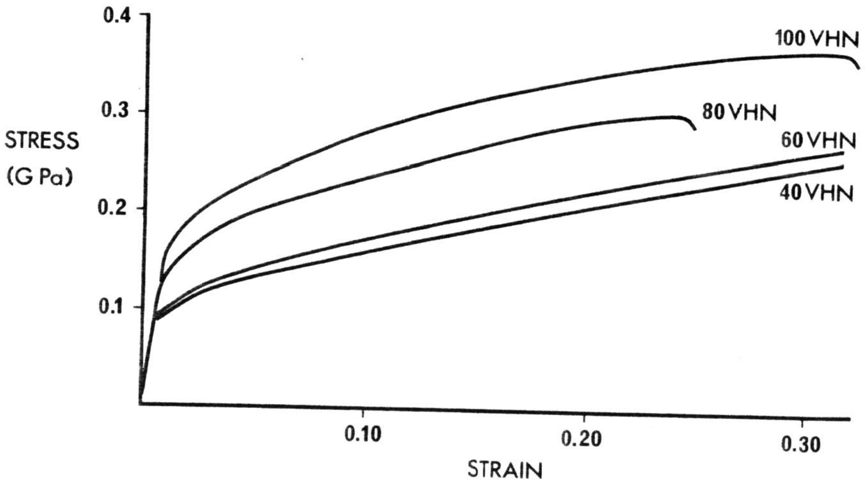

Figure 3 Stress-Strain Curves for Brass

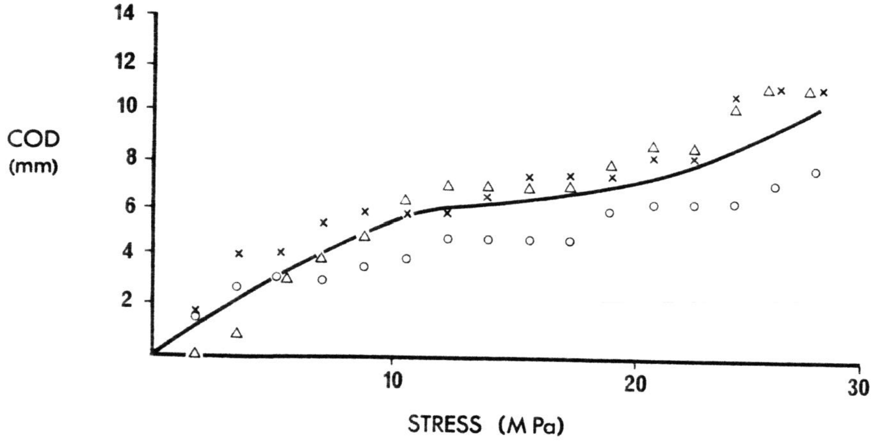

Figure 4 C.O.D. Measurements on Brass with Hardness 50 VHN

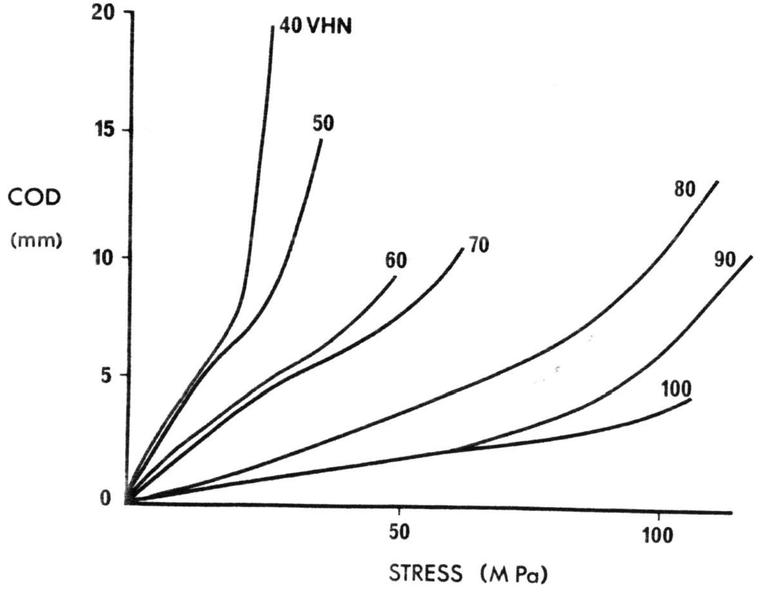

Figure 5 C.O.D. Measurements on Brass with Different Hardness Values

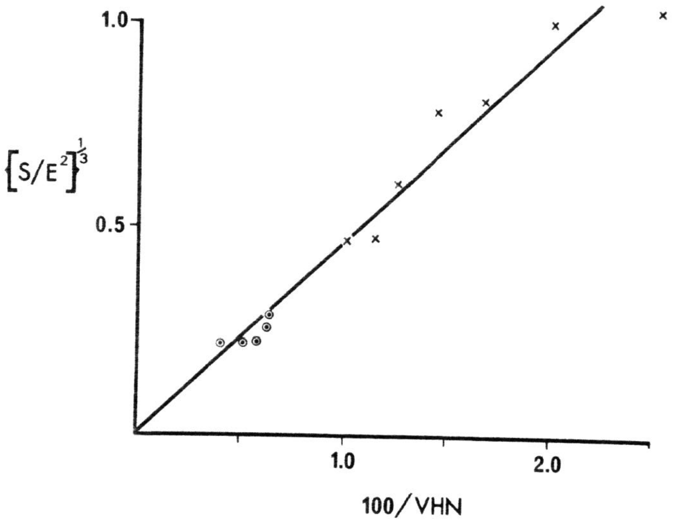

Figure 6 Data from the Approximately Linear Regions of the Curves in Figures 2 and 5