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# THE K-COD RELATIONSHIP FOR PIN LOADED SINGLE EDGE NOTCHED TENSION SPECIMENS

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## INTRODUCTION

The validity and usefulness of any fracture mechanics parameter as a suitable fracture criterion depends on the ease with which it can be calculated (like K or J), the ease and reproducibility of its measurement and its compatibility with K in the linear elastic range. The parameter can be the basis of a valid fracture criterion provided it is independent of specimen geometry and configuration. In this paper the well known K-$\delta$ (COD) relationships are examined for different widths, W and aspect ratios, a/W.

Well's analysis [1] and Burdekin et al [2] results based on Dugdales model indicate that $K^2$-$\delta$ relationship is linear.

$$\delta = \frac{K^2}{E\sigma_y} \tag{1}$$

However, Begley and Lande's [3] results and those of Anderson's [4] results indicate a parabolic relationship between $K^2/E$ (i.e. J) and $\delta$ in the linear elastic range

$$\delta \propto \sqrt{J} \propto K \tag{2}$$

Apparently equations (1) and (2) do not agree in the linear range. Finite element observations [5] indicate a linear P (load)-$\delta$ relationship in the linear elastic range. Thus

$$P \propto \delta \tag{3}$$

and

$$K = \frac{Y\,P\,a^{1/2}}{B\,W} \tag{4}$$

where $Y = f\,(a/W)$, B = thickness and a = crack length.

or

$$P = \frac{B\,W\,K}{Y\,a^{1/2}} = \frac{B\,K\sqrt{W}}{Y(a/W)^{1/2}} \tag{5}$$

Substituting equation (3) into (5) gives

$$\delta \propto \frac{B\,K\,\sqrt{W}}{Y(a/W)^{1/2}} \tag{6}$$

Equation (6) indicates a linear K-$\delta$ relationship which is similar to that in equation (2). This equation implies that for a given K and a/W ratio,

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higher widths will give higher displacement. Similarly at a given W and K higher a/W ratio should give a lower displacement. However, Well's Dugdale type K-$\delta$ relationship does not include the effect of specimen geometry and configuragion where $\delta$ is only function of K, $\sigma_y$ and E. Equation (1) has been used to support COD as a fracture criterion [6] indicating that as $K \rightarrow K_{IC}$, $\delta \rightarrow \delta_c$ and thus $\delta_c$ is a fracture characterizing parameter (for a given thickness of plate) which is independent of the width of the plate [7].

In steels, it is possible that low triaxiality induces fibrous fracture (a tough fracture and consequently a higher $\delta_c$) while high triaxiality induces cleavage. Triaxiality is a function of specimen dimensions and therefore $\delta_c$ measurements on small specimens may not correspond to the $\delta_c$ at which crack initiates in a large structure (unless the constraints in the specimen and the structure are identical).

It is expected that the state of stress at the crack tip will depend on the extent of deformation at the tip and its proximity to unnotched free edge. Thus the state of stress at the crack tip will continuously change from plane strain to plane stress as loading progresses and the crack tip deformation increases. Though this phenomenon is appreciated, the continuous change of state of stress (which can be represented by the value of constraint) is not considered in any reported calculations. Instead, it is a common practice to assume a constant state of stress throughout the loading history. It has been suggested by Hayes and Turner [5] and Egan [6] that $\delta$ in a given state of stress can be obtained using

$$\delta = \frac{K^2}{mE\sigma_y} \tag{7}$$

where m is a measure of constraint at the crack tip [8]. It is further suggested [5, 6, 9] that a value of m = 2 represents plane strain situation and is equal to 1 in case of plane stress. In this paper the results of a simple analysis developed which takes into account the continuous change of the value of m is reported. Using this analysis COD values are computed and the various K-$\delta$ relationships are examined.

## THEORETICAL CALCULATION OF COD

Dixon [10] has shown that a pin loaded SENT specimen can be represented by an axial force applied at the midpoint of a ligament and a bending moment. Richard and Ewing [11] using a similar representation have calculated yield point loads of SENT specimen, while Merkle et al [12] have used it for compact tension (CT) specimens. Dixon's work is limited to the elastic solution while the latter works do not refer to crack tip behaviour and strain hardening characteristics of the material. They assume a linear stress or strain distribution over the ligament. Liu et al [13, 14] have shown that the strain distribution ahead of the crack tip is of $1/\sqrt{r}$ type even in presence of considerable yielding. Use is made of this fact and a composite distribution comprising of $1/\sqrt{r}$ near the crack tip and linear strain distribution far away from the crack tip is assumed. For a smooth and continuous change over from the $1/\sqrt{r}$ to a -r type strain distribution, the magnitudes and slopes of the two strain distributions are matched at the change over point. The material is assumed to exhibit a linear strain hardening response. Figure 1 shows the general nature of strain distribution in the uncracked ligament with the various parameters used in the calculations. Applied axial load and moment (generated because of unsymmetric loading) was balanced with the reactive axial load and moment (generated because of the assumed strain distribution). The load and moment balance equations were simultaneously solved (using Newton-Raphsons iterative procedure) for various loads and specimen sizes. The output of this solution is R, the apparent plastic zone size ahead of the crack tip (which is defined as a point at which strain $\varepsilon_{yy}$ is equal to $\varepsilon_y$), and $X_2$, the point at which strain $\varepsilon_{yy}$ is zero, the rotation axis position.





The apparent plastic zone size was represented in the Irwin-McClintock [15] type of representation, i.e.,

$$R = \frac{1}{\pi\, m'}\left[\frac{K}{\sigma_y}\right]^2 \tag{8}$$

m' calculated from equation (8) is a measure of the constraint. It indicates the average increase of local $\sigma_{yy}$ stress at which yield occurs.

Three different $\delta$ values are calculated the procedures for which are indicated below.

(a) Compute $\delta$ using equation (1). This is termed as $\delta_D$.
(b) Compute $\delta$ from V, the crack mouth opening displacement, using Boyles et al [9] $\delta$-V relationship for SENT specimen. This is termed as $\delta_W$.
(c) Compute $\delta$ at crack tip when V is joined linearly to the point of strain reversal (Figure 1). This is termed as $\delta$.

V used in the calculation of $\delta_W$ and $\delta$ is obtained according to the following steps.

(1) Compute K using equation (4), for a given specimen W, a/W and P [16].
(2) Compute R and $r_y$ through load and moment balance equations.
(3) Compute $a_{eff}/W = (a+r_y)/W$.
(4) Compute the value of EVB/P for the $a_{eff}/W$ [16].
(5) Compute V using the results of previous step and the values of E, B and P.

## RESULTS AND DISCUSSION

Figure 2 shows the plot of m' versus K. The figure has two important features. Firstly, m' drops as K increases (i.e., loading and hence deformation at the crack tip progresses). Secondly, as the specimen width increases the m' decreases at a slower rate. This means that wider specimens maintain a higher constraint value than a narrower specimen of same a/W ratio at a given K possibly because of wider ligament. The same trends are exhibited by m'' which is obtained by dividing $\delta_D$ by $\delta_W$. Even the two constraint values m' and m'' which are obtained independently compare quite well in magnitude.

Figure 3 shows K-$\delta_W$ plots for various widths. The figure shows an interesting trend that as W decreases $\delta_W$ value increases for a given K. This trend does not agree with equation (6), because the constraint decreases more rapidly in specimens with lower widths (Figure 2). The constraint dependence of $\delta_W$ is further evident from Table 1, in which $\delta_W$ was calculated for a constant constraint m' = 2 in equation (8). It is observed that for all widths studied, the $\delta$ is almost constant for a given K value if m' is constant.

The constraint dependence of $\delta_W$ probably originates from the definition of $\delta_W$. $\delta_W$ has been defined as the resultant displacement at the crack tip when crack profiles are extended into the ligament. In this definition we approach from the crack mouth side and the overall stress-strain distribution in the ligament is not taken into account. Since the stress strain distribution in the ligament is ignored and the constraint value depends on these distributions, $\delta_W$ becomes a function of the constraint or specimen width.

Secondly, it is assumed in the definition that the crack faces open by a simple hinge mechanism about an apparent axis of rotation. The position of this rotation axis is assumed to be the intersection of the extrapolated crack profile with X axis. However, the "neutral axis" position (represented here as the *point of strain reversal*) determined in the present investigation for SENT and CT specimens, as well as Merkle's analysis for CT specimens [12] are quite different from the rotation axis positions as suggested by Wells and others [17, 18]. According to the theory of bending it seems unlikely that the specimen will rotate at the apparent rotation axis position where a finite positive (tensile) strain is present. The most likely position of rotation axis is expected to be the strain reversal point.

Moreover, the different formulae for V-$\delta$ conversion (based on crack profile extension technique) given in DD 19 [19] have been analytically and experimentally verified only for SEN bend and CT specimens and no comment is made upon its usefulness to SEN, centre notched and double edge notched tension specimens. In fact it has been pointed out [5, 9] that the crack profile extrapolation technique is not suitable for SENT specimens.

Taking all these observations into account it was decided to define the crack tip opening displacement by an alternative method.

## AN ALTERNATIVE DEFINITION OF COD

The COD is defined as the resultant displacement at the crack tip when the crack mouth opening displacement is joined to the neutral axis. The results of the present investigation with this definition of $\delta$ are given below.

The $\delta E/W\sigma_y$ versus $VE/W\sigma_y$ relationship for various a/W ratios follow a trend similar to that indicated in experimental calibration given in CODA [19] and other equations [20]. However the $\delta E/W\sigma_y$ - $VE/W\sigma_y$ plot in the present study is a linear relationship while the finite element calculations report a nonlinear relationship at lower loads. Secondly for a given value of V, $\delta$ obtained here is more than the FEM $\delta$ [9] reported for SENT specimens. This is expected since the definition of the two $\delta$s are different. Secondly, the FEM results reported are valid for a constraint value of m = 2 where as m decreases continuously in the present results.

Figure 4 shows the K-$\delta$ relationship obtained in the present investigation. It is observed that K-$\delta$ obeys a linear relationship and is a strong function of specimen width and a/W ratio. Higher widths give higher $\delta$ values for a given K and a/W ratio. Similarly lower a/W ratios give higher $\delta$ values for a given K and W. These results naturally do not agree with the trends exhibited by $\delta_W$ as shown in Figure 3 but they are in agreement with the observations made in equation (6). Similar results are obtained for the CT specimens.





If the present definition of COD is adequate then according to equation (6) the plot of K and $\delta\sqrt{a}/W$ should yield a straight line independent of a/W and W. Figure 5 shows the plot of K versus $\delta\sqrt{a}/W$. It is observed that it does yield a unique straight line for all a/W ratios and widths studied. This implies that the present way of defining $\delta$ is in agreement with the proven relationships and observations of fracture mechanics. It must also be noted that this type of representation makes $\delta$ values independent of constraint.

## SUMMARY

An alternative definition of COD is given based on strain reversal point as the rotation axis. The proposed parameter $\delta\sqrt{a}/W$ appears to be an unique function of K for the widths, a/W ratios and the constraints studied in linear elastic and small scale yielding situations for the SENT and CT specimens. However the applicability of this parameter needs to be examined for other specimen geometries where rotation axis may lie outside the specimen and also in the case of large scale yielding situations where the COD application is most appropriate.

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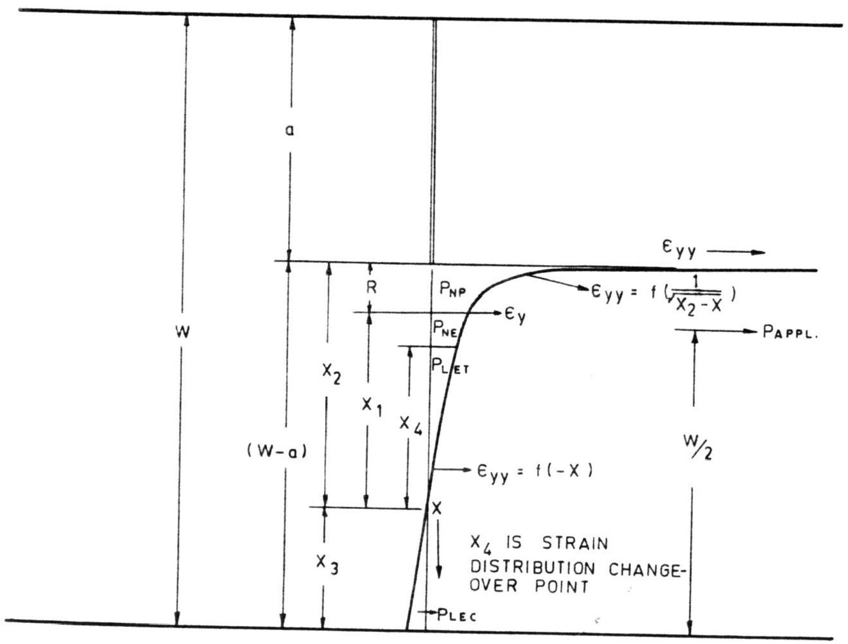
Table 1 $\delta$ Obtained at Constant Constraint m' = 2 using FEM V-$\delta$ Relationship for a/W = 0.5 [9]

| Width mms | K $MPa \cdot m^{1/2}$ | $\delta$ mms |
|---|---|---|
| 15.0 | 24.5 | 0.00432 |
| 30.0 | 25.0 | 0.00410 |
| 60.0 | 25.05 | 0.00393 |
| 120.0 | 25.20 | 0.00389 |
| 240.0 | 24.90 | 0.00376 |




LOAD BALANCE EQUATION:

$P_{APPL} = P_{NP} + P_{NE} + P_{LET} - P_{LEC}$

MOMENT BALANCE EQUATION:

$P_{APPL}\ (A + X_2 - W/2)$ = SUM OF MOMENTS DUE TO $P_{NP}, P_{NE}, P_{LET}$ AND $P_{LEC}$ ABOUT STRAIN REVERSAL POINT

Figure 1 Strain Distribution in the Ligament of a SEN Tension Specimen





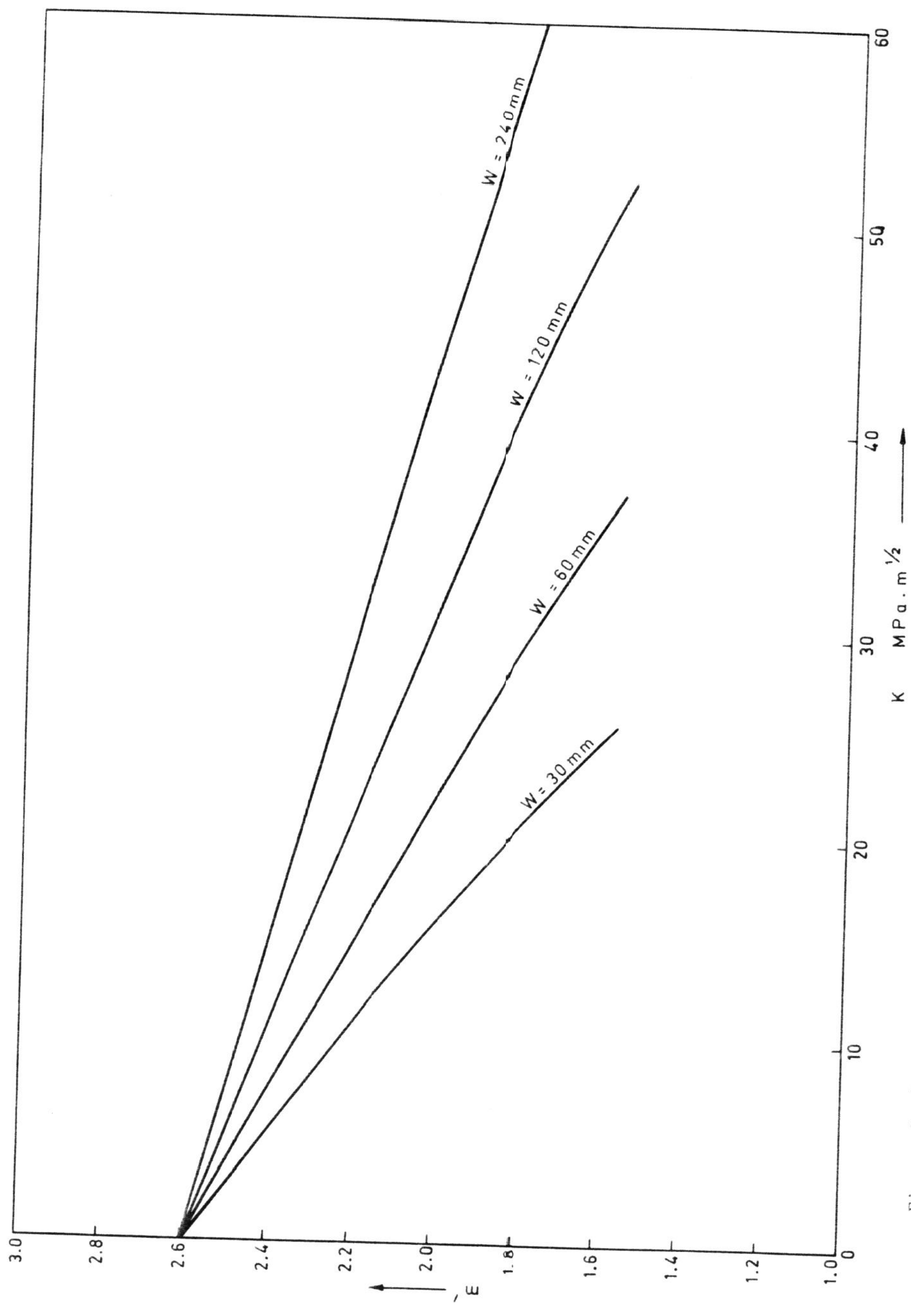



Figure 2 Constraint m' - K Plots for SENT Specimens a/W = 0.5

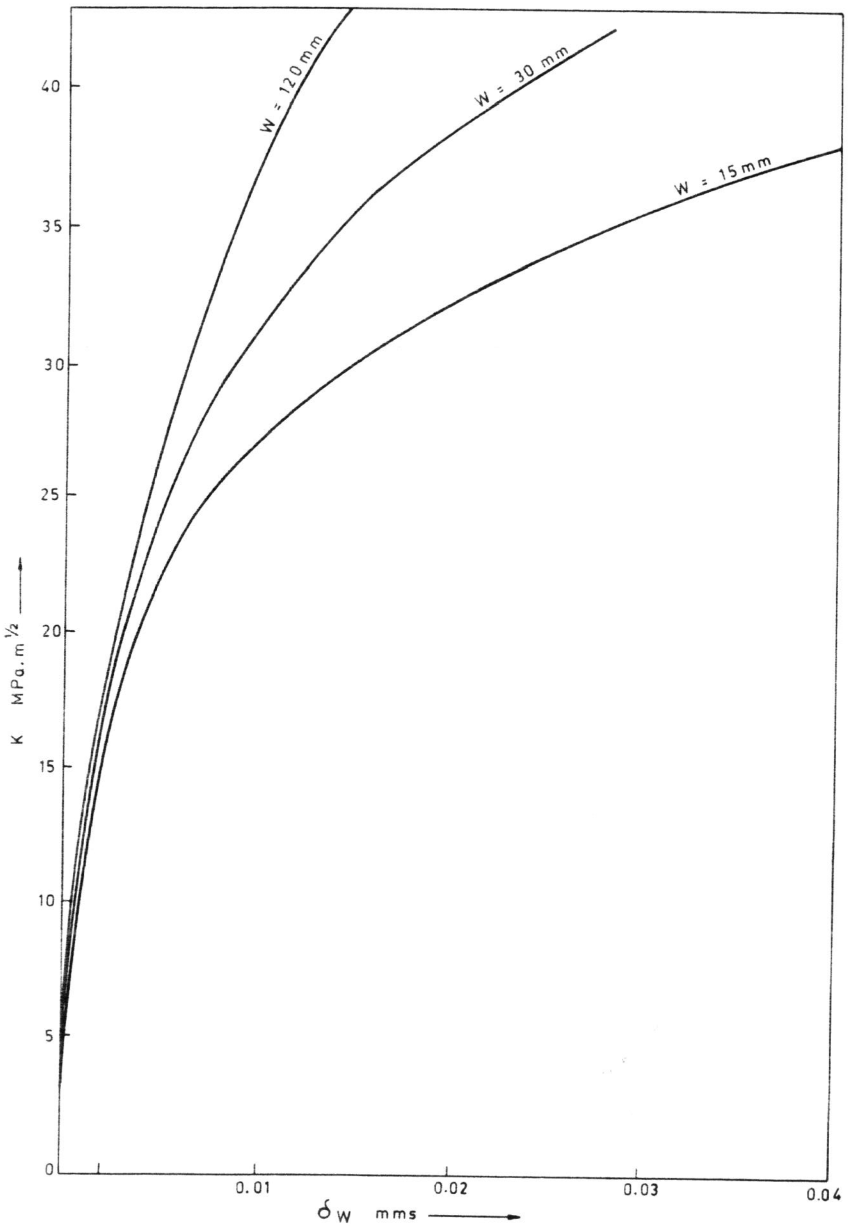



Figure 3  K versus $\delta_W$ Plots for SENT Specimens [9]a/W = 0.5





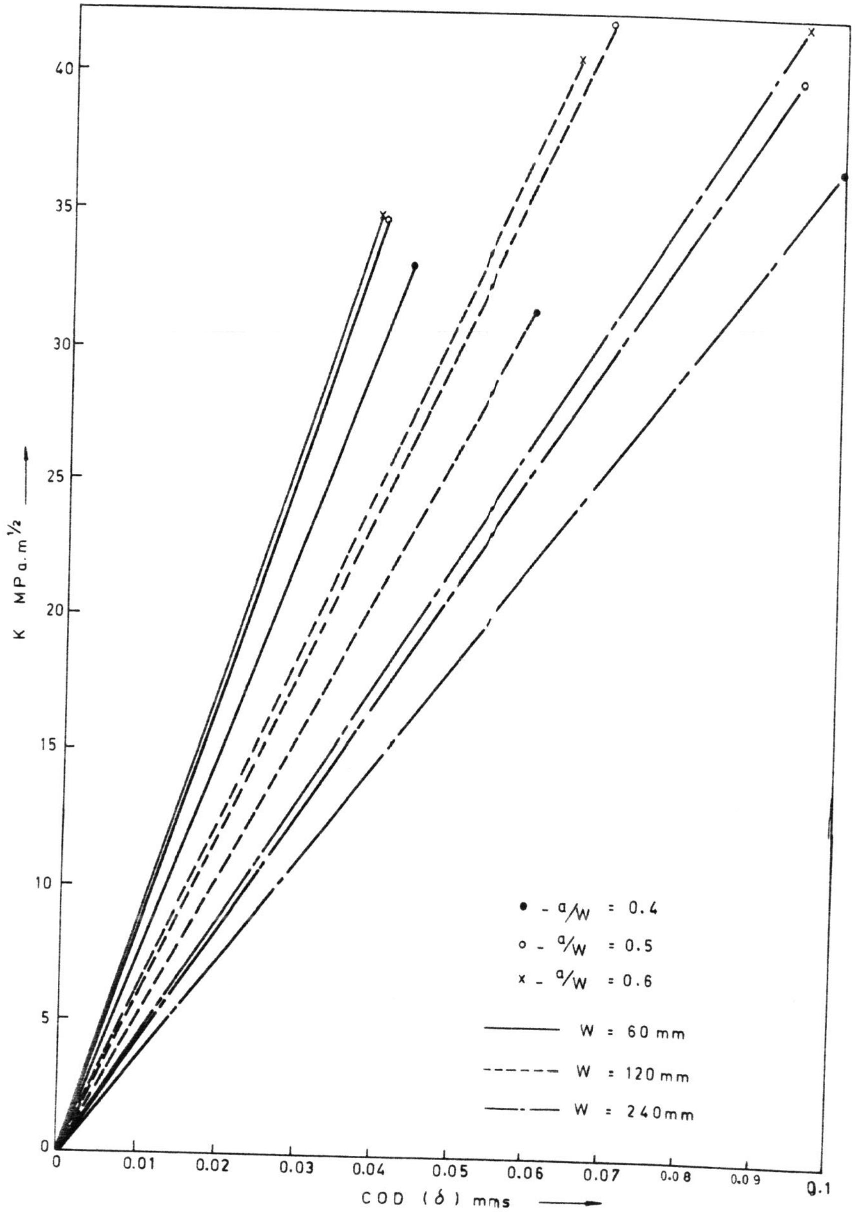



Figure 4  K versus δ Plot SEN Tension Specimen for W = 60,120,240 mm

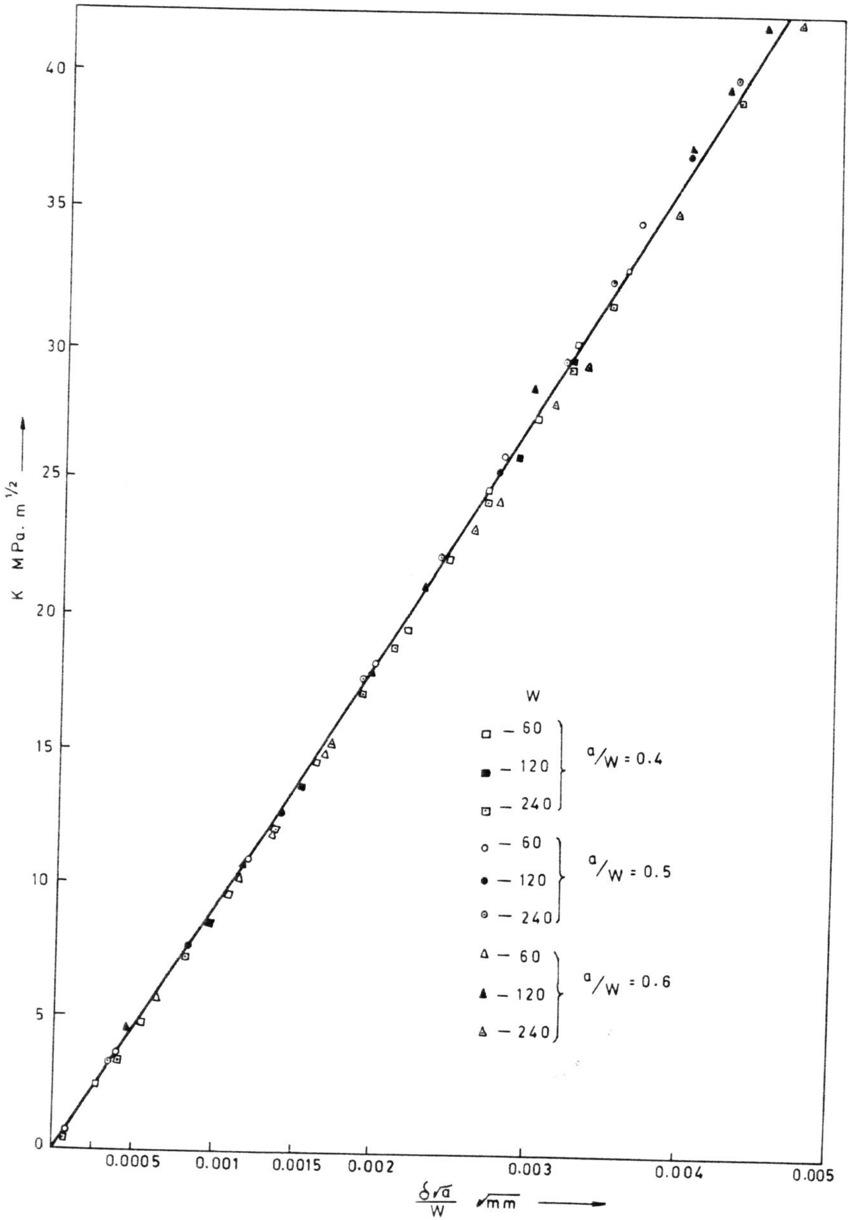



Figure 5 K - $\frac{\delta\sqrt{a}}{W}$ Plots for SENT Specimens with a/W = 0.4 , 0.5 and 0.6 and W = 60, 120 and 240 mm