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THE J-INTEGRAL EVALUATION FOR CT SPECIMEN

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INTRODUCTION

Rice's method for the J-integral evaluation [1] is convenient for its simpleness, but the accuracy of Rice's method is not investigated sufficiently. The J-integral evaluation by Rice's method is compared to the one by the finite element method for the standard bend bar specimen (a/W=0.5). The result is that Rice's method gives the higher value than the finite element method by about 10%. For the compact tension specimen, Rice's method must be investigated on its accuracy, because it pays no consideration to the effect of axial force. In this paper, the J-integral for the compact tension specimen is evaluated by the finite element method and the accuracy of Rice's method is investigated based on the result by the finite element method.

RICE'S EQUATION AND MERKLE'S EQUATION [2]

Consider Rice's and Merkle's equations for the evaluation of J-integral. According to Rice's method, the J-integral is calculated by equation (1) based on the load-displacement curve as shown in Figure 1:

$$J = \frac{2A}{b} \tag{1}$$

where b is the ligament length of the specimen.

Merkle et al. propose the equation which considers the axial force as well as the bending force. Considering that the axial force shifts the stress reversal point by αc as shown in Figure 2, they obtain equation (2) for the J-integral evaluation for CT specimen:

$$J = \frac{\eta_A A + \eta_C C}{b} \tag{2}$$

where A is the strain energy, C is the complementary energy of the specimen and

$$\eta_A = \frac{2(1+\alpha)}{(1+\alpha^2)}, \qquad \eta_C = \frac{2\alpha(1-2\alpha+\alpha^2)}{(1+\alpha^2)^2} \tag{3}$$

$$\alpha = \sqrt{\left(\frac{a}{c}\right)^3 + 2\left(\frac{a}{c}\right) + 2} - \left(\frac{a}{c} + 1\right) \tag{4}$$

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MS134

J-INTEGRAL EVALUATION BY FINITE ELEMENT METHOD

The J-integral evaluation is carried out for CT specimen shown in Figure 3. The mechanical properties of the material are represented in Table 1. The concentrated load is applied at the top of the loading pin hole shown in Figure 4, and the 318 elements and 194 nodes are used for the calculation of plane strain.

There are three ways to evaluate the J-integral based on the result by the finite element method:
1) J-integral is evaluated by equation (1) using the strain energy obtained by the finite element method.
2) J-integral is evaluated by equation (2) using the strain energy A and the complementary energy C obtained by the finite element method.
3) J-integral is evaluated by calculating the difference of the strain energy ΔU, as J-integral is given by equation (5):

$$J = -\frac{\partial U}{\partial a} = -\frac{U(a+\Delta a)-U(a)}{\Delta a} = \frac{\Delta U}{\Delta a} \qquad (5)$$

J-integrals given by the methods (1), (2), and (3) are hereinafter referred to as J_R, J_R^*, and J_E, respectively.

As shown in the bend bar specimen, J and K_I are related by equation (6) for the elastic state. Therefore, K_I can be evaluated from J-integral of the elastic state:

$$J = \frac{1-\nu^2}{E} K_I^2 \quad \text{(for plane strain)} \qquad (6)$$

In Table 2, the values of K_I obtained from J_E are compared to the analytical values, and the good coincidence is obtained. The values calculated from J_R and J_R^* do not agree with the analytical ones. This is because of the accuracy of J_R and J_R^* is getting worse when the deformation is small. The fact that K_I values calculated from J_E coincide with the analytical solutions is just one of the bases which take account of the validity of the J-integral evaluation by equation (5).

Numerical results of J_R, J_R^*, and J_E based on both the incremental theory and the deformation theory of the plasticity are presented in Table 3. It is shown from this table:
1) J_R, not taking account of the axial force effect, underestimates the J-integral value.
2) J_R^* is about 10% higher than J_E for the wide range of δ.

CONCLUSIONS

Rice's method, paying no consideration for the axial force effect, gives lower estimation for CT specimen. Therefore, when we evaluate the J-integral value by this type of equation, it is desirable to use Merkle's equation.

Rice's equation for the bend bar specimen and Merkle's equation for CT specimen seem to give higher values than equation (5) which, in the authors' opinion, gives most accurate values of J-integral. The dependence of J_{IC} values on the specimen geometry [3] seems to be based on the evaluation method of J-integral, partially.

REFERENCES

1. RICE, J. R., et al., ASTM STP 536, 1973, 231.
2. MERKLE, J. G. and CORTEN, H. T., Trans. ASME, Ser. J. 96-4, 1974, 286.
3. KANAZAWA, T., et al., IIW X-779-75, 1975.

Table 1 Mechanical Properties of A533B Steel

Young's Modulus (MPa)	Poisson's Ratio	Yield Stress (MPa)	Hardening Rate (MPa)
205800	0.3	480	2060

Table 2 $K_I^2B^2W/P^2$ for CT Specimen

a/W	Analysis	F.E.M.
0.50	92.16	91.59
0.52	104.24	102.47
0.54	118.59	117.31

Table 3 J-integral for CT Specimen (a/W=0.52)

Disp.	Incremental Theory			Deformation Theory		
δ	J_E	J_R	J_R^*	J_E	J_R	J_R^*
0.10	3.09	1.93	2.43	3.09	1.93	2.43
0.15	5.36	4.99	6.12	5.36	4.99	6.12
0.20	7.85	8.34	10.32	7.85	8.34	10.32
0.25	14.81	12.83	15.92	14.76	12.71	15.79
0.30	20.33	18.60	23.01	20.40	18.46	22.86
0.35	28.55	25.40	32.59	28.36	25.29	31.22
0.40	37.26	33.26	40.95	37.01	33.17	40.84
0.45	46.71	42.20	51.81	46.43	42.07	51.64
0.50	56.98	51.94	63.59	56.88	51.80	63.39

(δ:mm, J:kPa·m)

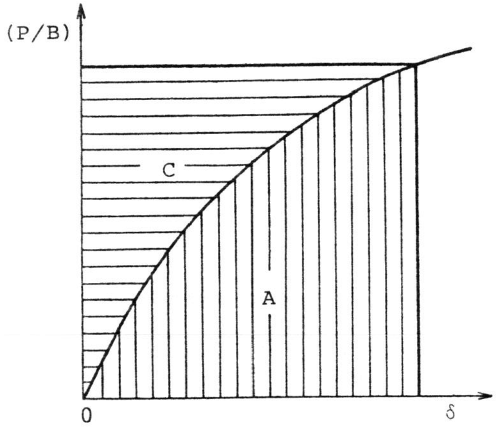


Figure 1 Load-Displacement Curve

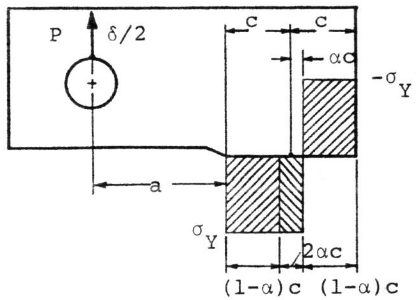
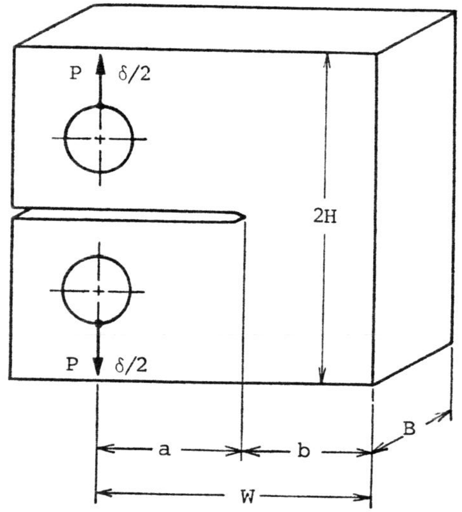


Figure 2 Reversals Point of Stress



W=50mm, a/W=0.52

B/W=0.50, H/W=0.30

Figure 3 Geometry of CT Specimen

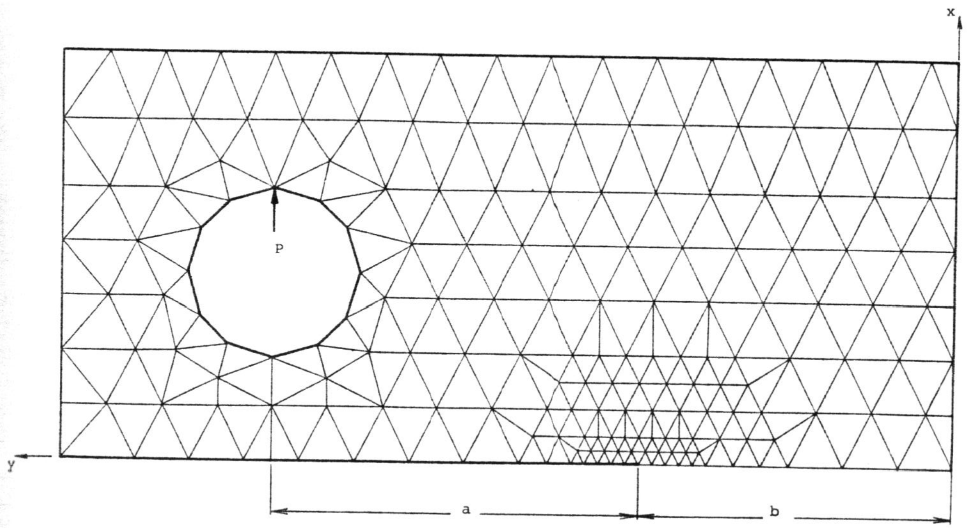


Figure 4 Nodal Breakdown of CT Specimen