THE FRACTURE MECHANISM OF MATERIALS HAVING A HETEROGENEOUS STRUCTURE

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INTRODUCTION

Mathematical modelling of the behaviour of a material subjected to loading is commonly based on two main assumptions - on homogeneity of the material on the one hand and on continuity of the medium on the other.

Basically there are hardly any objections to the bringing of these hypotheses into the study of deformation characteristics of a material, although we find a number of articles [1, 2] operating with non-homogeneity of the material in modelling these characteristics. However, after a careful examination of processes of scattered fracture, that is the fracture processes developing more or less uniformly all over the bulk of the material it is believed that the hypothesis of homogeneity is hardly admissible in this case. There is no better proof for such a statement than the fact that almost all processes associated with the accumulation of damage are qualitatively like those expressed by creep curves (with the upward bend of the curve close to fracture not necessarily taking place). To illustrate this one might point out the change of the Young's modulus in cyclic loading [3], the accumulation of submicrocracks and free radicals in polymer materials [4], and the like. An explanation of such processes is easy to find in the simple fact that at the start of testing there is rather rapid fracture of weak bonds followed by slowing down of the process until a more or less constant rate of accumulation of damage is reached.

Concerning the other assumption, in our subsequent discussion we speak of the structure as consisting of elements, i.e., of polycrystalline grains, and by elementary fracture process we mean the fracture of the grain face.

The introduction of inhomogeneity of the material in the mathematical model directly involves a statistical approach to the process of fracture. But since the classical statistical theories of brittle fracture include no time variable in the fracture process they also exclude the description of the long-term and fatigue strength of the material. This is the reason we chose a statistical kinetic model for study of the fracture mechanism. An analogous approach is used in references [5 - 9] which examine the fracture of a homogeneous material consisting of discrete elements, and also in references [10, 11] which discuss the fracture of a homogeneous medium.

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DISCUSSION OF MODEL

As stated above, the material is composed of grains, and fracture occurs only between their faces. The stresses are calculated as for isotropic elastic bodies and are averaged at the grain faces. It is assumed that the fracture of the grain face is a random process which is defined by the mean time of expectation τ

$$\tau = \tau_o \exp\left[\frac{u_o - \gamma^{\sigma}}{KT}\right] \tag{1}$$

where

τ_o = constant that equals to 10^{-13} - 10^{-12} sec,
u_o = activation energy for the fracture process,
γ = overstress factor,
σ = mean normal stress acting on the grain face.

Inhomogeneity of the structure is represented by the distribution function ψ of the factor γ [12].

For simplicity of calculation we further deal only with the fracture of oriented structures, the external loading force σ_o being applied parallel to the axis of orientation.

We introduce a definition [5]. If the fracture affects a number of adjacent faces, j, the defect obtained we call a j-defect and its area is equal to jF, where F stands for the average area of the grain face. By analogy, the non-fractured grain face assumes the name of 0-defect. In order to calculate the stress concentration round the defect we regard its form as a spheroid with radius

$$R = \sqrt{\frac{jF}{\pi}}$$

and with height H, see Figure 1. It should be noted that we neglect interactions between defects and their coalescence. Now, having assumed that fracture is caused by the mean stress acting on the grain face we have to find the mean stress acting within a ring, the width of which is the average diameter of the grain face, i.e.,

$$2\sqrt{\frac{F}{\pi}}\ .$$

On the basis of precise solutions from the theory of elasticity dealing with cavities of radial spheroid form [13], we infer the mean stress values expressed as elementary functions. Table I lists the values of $\phi_j = \sigma : \sigma_o$ for two values of r. It should be observed that for 1-defects with r = 0.0012 R we obtain H = 0.05 R, and r = 0.86 R corresponds to H = 0.99 R. Column I gives the values of ϕ_j for the faces directly surrounding the defect and column II shows the values for the faces of the next row. Poisson's ratio is assumed to be 0.3. From Table I we may conclude that:

a) the value of ϕ_j is scarcely dependent on the value of r, and as we are not quite certain about the nature of the latter it is assumed in further discussion that r equals 0, that is, we regard defects as penny-shaped cracks;

b) the mean stresses acting upon the grain faces of the second row are considerably smaller than those acting upon the grain faces of the first row, and consequently, it is reasonable that the stress concentration upon the grains of the second row be neglected.

Now let us consider the kinetics of the accumulation of defects. Through a linear transformation of the random variable γ from equation (1) and the distribution function ψ of the factor γ we come to the distribution of $\lg_e\tau$ for a definite σ :

$$f(\ell n\tau) = \frac{KT}{\sigma}\,\psi\left[\frac{(\ell n\tau_o - \ell n\tau + \frac{u_o}{KT})KT}{\sigma}\right]. \tag{2}$$

The fracture probability of the grain face with definite τ at constant σ is, by analogy with radioactive decay, as follows

$$W(t) = 1 - \exp\left(-\frac{t}{\tau}\right)\ . \tag{3}$$

However, in the case where the value of $f(\lg\tau)$ has a distribution, by generalizing equation (3) we have,

$$W(t) = \int_{-\infty}^{\infty}\left\{1 - \exp -\left[\frac{t}{\exp(\ell n\tau)}\right]\right\} f(\ell n\tau)\,d\ell n\tau\ . \tag{4}$$

Further we determine the probability density of the transition of a j-defect into a (j+1) - defect. If j-defects are surrounded with n 0-defects, the probability of transition from a j-defect into a (j+1) - defect is $[1-W(t)]^n$, and correspondingly the probability of transition from a j-defect into a (j+1) - defect is equal to

$$1 - [1-W(t)]^n\ . \tag{5}$$

The probability density of a transition from a j-defect into a (j+1) - defect may be derived from equations (4) and (5),

$$P_j^{j+1}(t) = \frac{d\{1-[1-W(t)]^n\}}{dt} = n[1-W(t)]^{n-1}\int_{-\infty}^{\infty}\exp[-x-t\exp(-x)]f(x)dx\ . \tag{6}$$

We introduce a definition - the value of $W_j(t)$ is the probability that an 0 - defect nucleates a defect having size $\geq$ j. Then the value of $W_1(t)$ is computed from equation (4) whereas the value of f(x) is calculated for $\sigma = \sigma_o$. The expression $W_j (j \geq 2)$ may be obtained from equation

$$W_j(t) = \int_0^t W_{j-1}(x)P_{j-1}^j\,(t-x)dx\ ,$$

where the value of f(x) of equation (6) is calculated for $\sigma = \sigma_o\phi_{j-1}$.

The probability of emergence of at least one defect having size $\geq$ j in the specimen with N-faces may be determined from the equation analogous to equation (5) $W_j^n = 1-[-1-W_j(t)]^n$. When the number of grain faces (N) is considerable then

$$W_j^n = 1 - \exp(-W_j N) \ . \tag{7}$$

RESULTS OF CALCULATION AND DISCUSSION

As reported in [14] highly-oriented capron is characterized by $u_o = 26.7.10^{-20}$J, $\gamma = 14.8.10^{-20}$mm^3. In calculations T is assumed to be 293K. We chose the Weibull distribution for overstresses $\psi(\gamma)$

$$\psi(\gamma) = \frac{h}{s}\left(\frac{\gamma-\mu}{s}\right)^{\eta-1} \exp\left[-\left(\frac{\gamma-\mu}{s}\right)^{\eta}\right] \qquad \gamma \geq \mu$$

$$\psi(\gamma) = 0 \qquad \gamma < \mu$$

with the following parameters $\mu = 6.5.10^{-20}$, $s = 10^{-29}$, $\eta = 0.07$. See Figure 2.

Results of calculations are given in Figures 3 - 6. Figure 3 shows the probability of the transition of a (j-1) - defect into a j - defect. It is apparent that with growth of j the curve tends to the right and that considerable growth of j results in an almost immediate enlargement of the defect. This means that the limit curve, as shown in Figure 4, shows the probability that the grain face will nucleate a defect which causes the ultimate fracture of the specimen. We define this probability as W_f, and similarly W_f^N stands for the probability of emergence of a critical defect in a specimen having N-faces.

The mean size of the structural element of oriented capron is 10 - 25 mm [15]. By assuming the grain face to be 20 mm, a specimen with a volume of 10^3mm^3 will possess $1.25.10^{17}$ faces exposed perpendicular to the applied force. The fracture probability of the whole specimen is assumed to be $W_f^N = 0.5$. Then from equation (7) we determine the probability that any grain face will nucleate a critical defect, i.e., $\lg W_f = -17.3$. Figure 3 allows us to derive the logarithm of the fracture time as given in Figure 6. The value of U_o resulting from Figure 6, $U_o = 25.5.10^{-20}$ J, closely approximates to that obtained experimentally, $U_o \cong 26.7.10^{-20}$ J. As noted in [16] the value of U_o agrees well with the value of activation energy for the thermodestructive process and might be easily determined from independent physical tests. This means that only constants of distribution, $\psi(\gamma)$, remain undefined. Figure 5 depicts the curves that show accumulation of defects at different stresses. As it stands, the concentration of 1 - defects is approximately 10^6 higher than the concentration of 4 - defects and hence the difficulty arises of revealing large defects.

As seen from graphs of Figure 5 the probability of emergence of critical defects in the wide region of probability might be well characterized by a straight line $\ell n\, W_f = m\ell nt + B$ that corresponds to $W_f = \exp(b)t^m$. By replacing W_f in equation (7) we come to the Weibull distribution

$$W_f^N = 1 - \exp\,[-\exp(b)t^m N]$$

for the fracture time. The above formula and Figure 4 give the size effect.

In conclusion it should be mentioned that the model discussed embraces the whole fracture process and presents a natural transition from the process of scattered fracture to the process of propagation of the macro-crack.

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Table I

j	$r = 0.0012\sqrt{F/\pi}$		$r = 0.86\sqrt{F/\pi}$	
	I	II	I	II
1	1.21	1.01	1.21	1.01
4	1.51	1.03	1.48	1.04
9	1.80	1.07	1.74	1.09
16	2.07	1.12	1.98	1.15
25	2.31	1.17	2.20	1.21
36	2.55	1.22	2.41	1.28
49	2.77	1.27	2.60	1.34
64	2.97	1.32	2.78	1.41
81	3.17	1.38	2.95	1.47
100	3.36	1.43	3.11	1.53

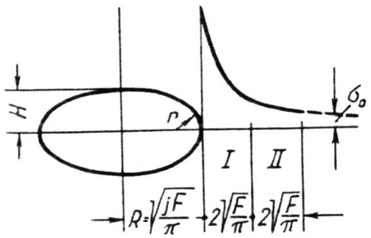


Figure 1 Stress Distribution at the Model Defect

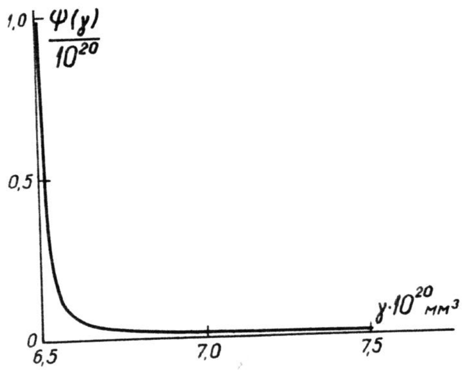


Figure 2 Distribution of Factor γ as Assumed in Calculation

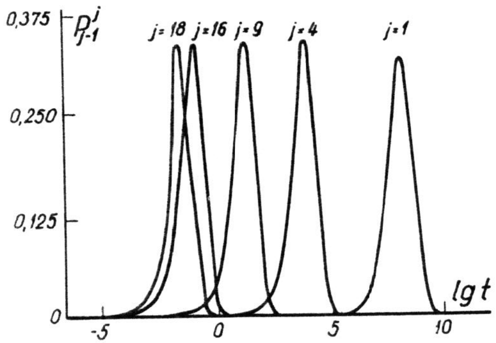


Figure 3 Probability Density of the Transition of a (j-1) - Defect into a j-Defect $\sigma_0 = 9.8 \cdot 10^8$ Pa

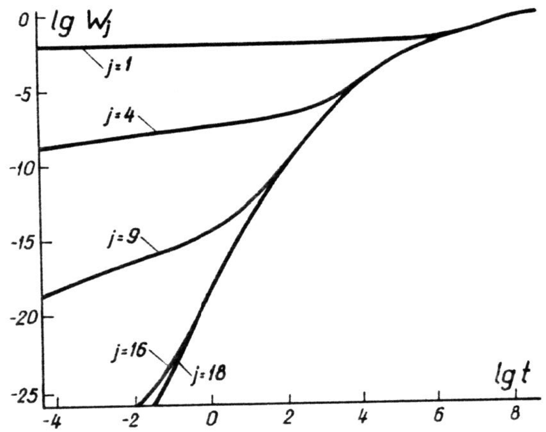
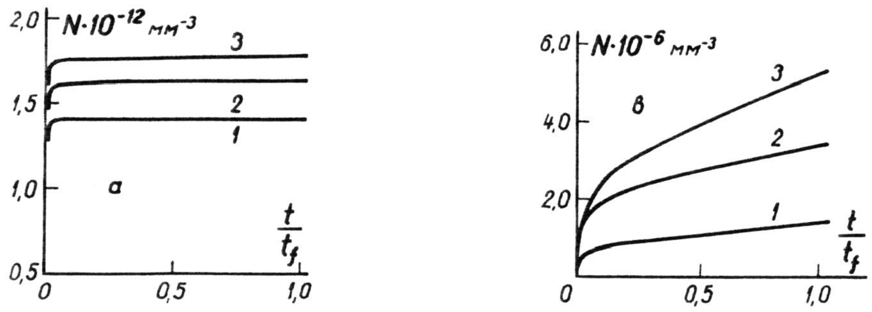


Figure 4 Probability that the Grain Face will Nucleate a Defect of Size of j $\sigma_0 = 9.8 \cdot 10^8$ Pa



(a) (b)

Figure 5 (a) Accumulation of 1-Defects
(b) Accumulation of 4-Defects

t_f - time prior to fracture 1 - σ_o = 4.9 . 10^8 Pa
2 - σ_o = 6.4 . 10^8 Pa 3 - σ_o = 9.8 . 10^8 Pa

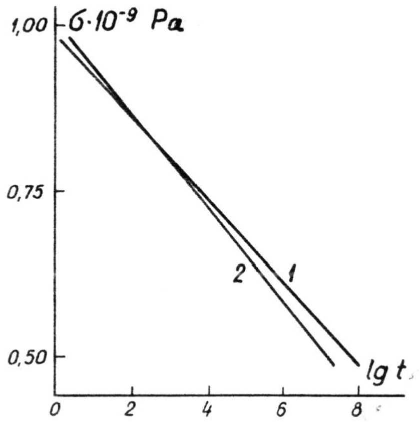


Figure 6 Curves of the Long-Term Strength of Oriented Capron
1 - Experimental Data [14], 2 - Calculation Data