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THE EFFECT OF LOAD BIAXIALITY ON THE FRACTURE TOUGHNESS PARAMETERS J AND G^{Δ}

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INTRODUCTION

It is well known that biaxiality of loading has no effect on the resistance to fracture of an ideal linear elastic material containing a flat sharp crack. Thus, in a centre-cracked plate of linear elastic material loaded in plane strain as shown in Figure 1, the critical stress intensity factor K_{IC} is independent of σ_Q. According to the Griffith fracture criterion, the energy release rate, G, at fracture is equal to the cohesive strength G_c of the material and

$$G_c = \frac{K_{IC}^2 (1-\nu^2)}{E} \tag{1}$$

Further, for elastic materials Rice's path independent integral J is identical to G.

In elastic-plastic materials, crack tip plasticity plays an important role in enhancing the fracture toughness, i.e. the applied stress, σ_P required to cause the fracture of a ductile material with a sizeable crack tip plastic zone, is greater than the applied stress required to cause the fracture of a brittle material with similar values of, ν and G_c, but having such a high yield stress that the crack tip plastic zone size at fracture is minimal. For an ideal, unyielding elastic material, with similar elastic properties, σ_P at fracture is approximately equal to that for the brittle materials.

With elastic-plastic materials, it is not possible to use either J or G as crack propagation or fracture instability parameters [1], although J can act as a characterizing parameter. Thus G, given by the right hand side of equation (1) with K_I replacing K_{IC}, can only pertain to an unyielding elastic material. However, a Griffith type criterion can be used for quasi-brittle materials in terms of the crack separation energy rate G^{Δ} associated with a small *finite* growth step Δa where Δa is assumed to be a characteristic property of the material. This approach is described in references [2, 3] and in a paper presented at this conference [4]. The crack separation energy rate G^{Δ} is defined as $\Delta W/\Delta a$ where ΔW is the work absorbed at constant applied stress during the proportional quasi-static release of the stresses holding the surfaces together at Δa. For small scale yielding, the ratio of Δa over the crack tip plastic zone size is given by

$$r = k(\sigma_y/K_I)^2 \; \Delta a \tag{2}$$

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where k is a constant which depends on the mode of loading and σ_y is the yield stress in uniaxial tension.

By combining elastic-plastic finite element analyses on quadrant 1 of Figure 1 with a crack tip release technique, it was possible to calculate G^Δ for different values of r. Material properties used were E = 207 GN m^{-2} $\nu = 0.3$, σ_y = 310 MN m^{-2} and a linear strain hardening tangent modulus of 4830 MN m^{-2}. The results show that when r is greater than three, G^Δ is approximately equal to J (or G for the unyielding elastic material). As r tends to zero at constant applied stress σ_P, G^Δ also tends to zero, thus confirming Rice's conclusions [1] already cited. Between these two values of r there is a strong dependence of G^Δ on r.

EFFECT OF BIAXIALITY OF LOADING ON G^Δ

It has been known for some time that the sizes of the crack tip plastic zones and the resistance to fracture of elastic-plastic materials depend on the biaxiality of the mode of loading [5, 6, 7, 8]. Using the load biaxiality parameter $\lambda = \sigma_Q/\sigma_P$, λ takes the values -1, 0 and 1 for the shear uniaxial and equibiaxial modes, respectively, with σ_P always positive, i.e. in tension

Figure 2 shows plastic zone sizes for different values of λ obtained by finite element analyses at approximately the same values of σ_P. The computer drawn crack profile is that for the shear mode with displacements being exaggerated by a factor of 50. It will be noted that the zone size corresponding to the shear mode is much the largest with the equibiaxial mode being the smallest. The respective ratios of the three zone sizes are approximately 6:1.5:1 corresponding to λ = -1, 0 and 1.

Figure 3 shows the principal stress, σ_1, perpendicular to the crack surface, and ahead of the crack tip, normalized with respect to σ_P for the loads given in Figure 2. The stress patterns differ appreciably with loading mode. The combined effects of hydrostatic tension and the elastic singularity are most pronounced in the equibiaxial loading mode, for which the plastic zone size is smallest, and are almost totally absent in the shear mode corresponding to the largest plastic zone.

Figure 4 summarizes the results of this study. For all loading modes the value of G^Δ decreases with increasing crack tip ductility. This effect can be attributed mainly to the increasing plastic work of deformation required to *extend* the plastic zone as the crack advances, with little or no recovery of energy from the wake region, in spite of the unloading which has occurred in the material now in the wake. Hence, at the same value of σ_P, G^Δ must be least when λ = -1 and the plastic zone is largest. By the same token G will be greatest when λ = 1 corresponding to the smallest plastic zone. The abscissa G/G_0 of Figure 4 represents a normalized applied load. Here G_0 is the value of G at incipient yielding under uniaxial loading of the plate of Figure 1. The value of G_0 therefore depends on $\sigma_y^2\,\Delta a$ since Δa is the length of the side of the leading element at the crack tip. Hence G/G_0 is also a measure of the plastic zone size at the crack tip, i.e. the ductility of the material. The ordinates give, in the case of the top three curves, the normalized values J/G for the three modes and in the case of the lower three curves, G^Δ/G. When $G/G_0 = 1$, i.e. the truly brittle state, G, J and G^Δ are all equal since they correspond to an elastic solution. As G/G_0 increases and becomes large, G^Δ/G tends to zero in all cases, but the paths are different for the three loading modes*.

In view of the increasing use of the path independent contour integral J as a characterizing parameter it is interesting to note in Figure 4 the values G/G_o at which J begins to diverge significantly from G and which indicate states in which small scale yielding assumptions are no longer justified. In the present study J was calculated along a path always running through elastic material.

ACKNOWLEDGEMENTS

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*G/G_o and G^Δ/G_o are related to the ϕ, ψ parameters of reference [3] by equations

$$G/G_o = \psi$$
$$G^\Delta/G_o = \phi$$

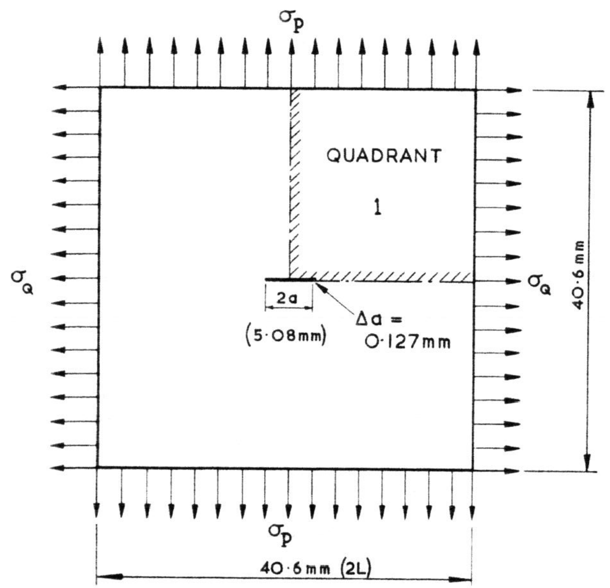

Figure 1 Centre-Cracked Plate in Plane Strain

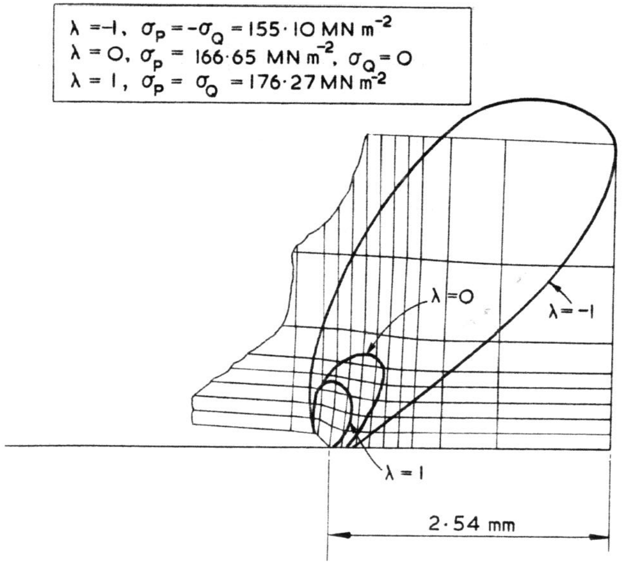

Figure 2 Crack Tip Plastic Zone Sizes for Different Biaxial Modes of Loading

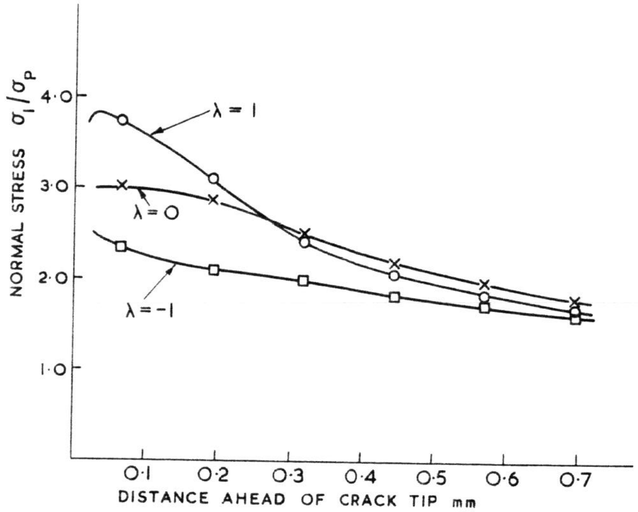

Figure 3 Normal Principal Stresses Ahead of the Crack Tip for Different Biaxial Modes of Loading

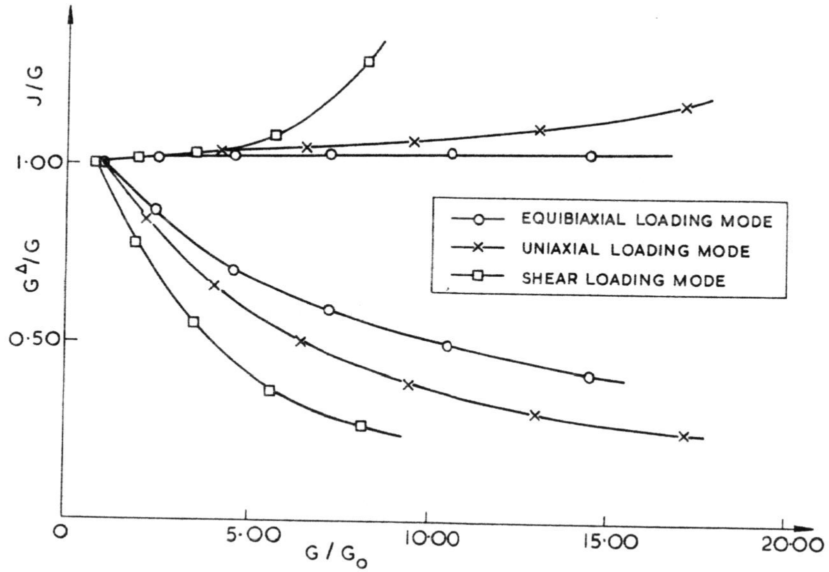

Figure 4 The Effect of Various Biaxial Modes of Loading on the Fracture Toughness Parameters J and G^Δ