THE EFFECT OF GEOMETRY ON CRACK FORMATION

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INTRODUCTION

In many components subjected to mechanical or thermal cycling it is frequently the case that, at a critical region, the maximum localized stress exceeds the yield strength. If continued cycling does not produce any further macroplasticity, the material will thereafter behave elastically although yielding on the first half cycle would establish a mean stress. In assessing the fatigue integrity of components, the local material behaviour must be understood, both from the point of view of the number of cycles required to initiate a crack (the crack formation life), and the subsequent crack growth whilst the crack is still within the influence of the geometry of a stress concentrator. A fundamental approach necessitates the ability to determine precisely the division between nucleation and Stages I and II crack propagation [1]. Whilst for a limited number of situations it might be possible from a diagnostic viewpoint to use such an approach, the prognostic situation is much more difficult, and the use of electron microscopy as a diagnostic tool is of only limited value to the designer. Consequently, efforts have been made to develop methods which enable crack formation in components to be predicted from data obtained from simple test pieces. An engineering crack is defined as one which can be detected using low power magnification (say X 25), and for a typical surface crack will be in the region of about 0.5mm long and 0.15mm deep.

FATIGUE LIFE OF PLAIN SPECIMENS

In order to estimate the crack formation life of a component it is necessary to relate the conditions at the critical region to known material behaviour and allow for influencing factors. If it is assumed that the conditions in a component can be represented by tests on plain specimens of the type used in obtaining fatigue data, this may provide the basis for assessing crack formation life.

There is considerable experimental support [2] to suggest that both the elastic and plastic strain range components, when plotted against cycles to failure, give approximately straight lines on logarithmic co-ordinates. Expressed mathematically,

$$\Delta\varepsilon_T = \Delta\varepsilon_p + \Delta\varepsilon_e = C_p N_f^{\alpha_1} + C_e N_f^{\alpha_2} \qquad (1)$$

where α_1 and α_2 represent the slopes of the plastic and elastic lines respectively on logarithmic co-ordinates, and C_p and C_e represent the strain range corresponding to the plastic and elastic intercept for one cycle. Equation (1) is based upon completely reversed strain cycling with zero mean stress. To allow for the effect of a mean strain the Sach's modification [3, 4, 5] is introduced into the plastic strain component, i.e.,

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$$\Delta\varepsilon_p = (\varepsilon_f' - \varepsilon_m)N_f^{\alpha_1} \qquad (2)$$

where ε_f' (= C_p) is defined as the fatigue ductility coefficient. If the elastic strain range corresponding to zero mean stress is $\Delta\varepsilon_{eo}$, then

$$\Delta\varepsilon_{eo} = C_e N_f^{\alpha_2} \qquad (3)$$

To allow for the effect of a mean stress the Goodman relationship is used, i.e., in terms of strain range

$$\Delta\varepsilon_e = \Delta\varepsilon_{eo}\left(1 - \frac{\sigma_m}{\sigma_u}\right) \qquad (4)$$

where σ_m is the mean stress and σ_u is the ultimate tensile strength. Combining equations (3) and (4) and substituting into equation (1),

$$\Delta\varepsilon_T = (\varepsilon_f' - \varepsilon_m)N_f^{\alpha_1} + C_e(1 - \sigma_m/\sigma_u)N_f^{\alpha_2} \qquad (5)$$

If an endurance limit exists for the material, say $\Delta\varepsilon_{Lo}$ at N_e cycles, then

$$C_e = \Delta\varepsilon_{Lo} N_e^{-\alpha_2} \qquad (6)$$

where $\Delta\varepsilon_{Lo}$ is the strain range equivalent of the endurance limit, for zero mean stress, i.e.,

$$\Delta\varepsilon_{Lo} = \frac{2\sigma_e}{E} \qquad (7)$$

Now it will be observed that, using an iterative procedure, equation (5) enables an estimate of crack formation life in a component to be predicted, if the conditions at the crack region can be determined.

STRAIN DISTRIBUTION IN A COMPONENT

In most practical designs the local plastic strains will usually be sufficiently contained to limit the plastic zone to only a small region. The local behaviour will be dependent upon a number of factors, such as the relative magnitude of the plastically to elastically strained material, the strain distribution, the materials cyclic strain hardening or softening characteristics, and the effect of environment. Further even though the component may be subjected to constant amplitude cyclic loading, the material in the vicinity of the concentration feature and which is locally plastic, will experience a variation of strain range with cycles [6]. It is the behaviour of this local material which governs crack formation.

Excluding finite element methods, two possible avenues are currently available for evaluating the strain at a concentration feature, namely the Neuber [7] and the Hardrath-Ohman or modified Stowell [8] methods. The total strain at a concentration feature can be estimated using an analysis suggested by Zwicky [9]. Based on the modified Stowell method:

$$\Delta\varepsilon_T = \frac{m\,\Delta\sigma_o(K_t - 1)\Delta\sigma}{E(\Delta\sigma - m\Delta\sigma_o)} \qquad (8)$$

where K_t is the theoretical stress concentration factor; $\Delta\sigma_o$ is the nominal stress range; $\Delta\sigma$ is the local stress range and m is a factor which relates the effective stress to the principal stress. Using the Neuber method:

$$\Delta\varepsilon_T = \frac{m\,g\,K_t^2\,\Delta\sigma_o^2}{\Delta\sigma} \qquad (9)$$

where g is a factor which relates the effective strain to the principal strain.

For plane stress,

$$m = 1, \qquad g = 1.$$

For plane strain,

$$m = (1 - \nu' + \nu'^2)^{1/2} \qquad (10)$$

$$g = \frac{m}{1 - \nu'^2} \qquad (11)$$

where ν' is defined as the pseudo-Poisson's ratio, obtained from

$$\nu' = 0.5 - (0.5 - \nu')(E_s/E) \qquad (12)$$

and E_s is the secant modulus defining the local material behaviour.

Accepting the assumptions inherent in the derivation of equations (8) and (9) either the modified Stowell or the Neuber rule may be used to estimate the total strain range at a critical section in a component.

ESTIMATING CRACK FORMATION

If the behaviour of the material at a critical region in a component can be predicted from smooth specimen behaviour, then we have a ready method for assessing crack formation life. This consists essentially of using the known material cyclic behaviour, as expressed by equation (5) with the total strain range. The total strain range can be determined using an iterative procedure with either the modified Stowell or Neuber method, applied first to the loading half-cycle and then to the unloading half cycle. It would seem reasonable to assume, for the first half cycle, that the material will follow the monotonic stress-stress curve. On unloading various possibilities are likely, depending upon the magnitude of strain attained on the loading half cycle and the overall stress ratio. Thus the local material behaviour may be such that either (i) yielding in compression occurs; (ii) no yielding in compression but residual compressive stress is obtained; or (iii) residual tensile stress is achieved due to high normal stress ratio. In any real component the strain distribution will be such that strain gradients exist and the volume of plastically strained material and the strain exponent may be significantly different from that of the plain test piece where the strain gradient will usually be essentially zero. A recent study by Leiss *et al* [10], suggests that the theoretical stress concentration factor K_t, should be replaced by an experimentally determined value for K_f (termed the "fatigue notch factor"). Investigations conducted by the present authors suggest that if K_t is replaced by K_f in the relationship for estimating strain range, then closer predictions to crack formation lives are obtained. However, the values for K_f were obtained using the usual definition involving notch sensitivity index q, i.e.,

$$K_f = 1 + q\ (K_t - 1) \qquad (13)$$

has been used to replace K_t in the method for calculating total strain range, and q was expressed in terms of the Neuber material constant [6]. Once this local material behaviour has been established, the mean strain may be determined from

$$\varepsilon_m = \frac{\varepsilon_{max}}{2}\ (1 + r_\varepsilon) \qquad (14)$$

where r_ε is the local strain ratio. To extend the crack formation model to include the effect of bulk stress ratios, the nominal stress range in equations (8) and (9) has been modified by including the stress ratio for the unloading half cycle. Thus if σ_o is the nominal stress amplitude attained on the loading half cycle, then for the unloading half cycle

$$\Delta\sigma_o = \sigma_o\ (1 - R) \qquad (15)$$

where R is the bulk stress ratio. This allowance for bulk stress ratio would not be expected to be applicable if time dependent influences, such as creep or stress relaxation are involved, without some modification.

CORRELATION OF EXPERIMENTAL DATA WITH MODEL

To investigate the validity of the proposed model, experimental data for fatigue crack formation has been obtained for a variety of materials, geometries, bulk stress ratios and elevated temperature. Laboratory fatigue tests were conducted on SEN bend specimens having different notch configurations and tension plates with central holes; the onset of fatigue crack propagation was located using the electric potential method [2]. Aero-engine model discs were tested by cyclic spinning, and the cracks detected by NDT methods. Values for K_t varied from less than 2 up to about 14. Table 1 summarizes the material properties, and for identification purposes the materials are designated, A, B and C. Predictions for crack formation life have been made using the model represented by equation (5). The total strain range $\Delta\varepsilon_T$ has been calculated using both the modified Stowell and the Neuber methods. For each situation it is necessary to know the service conditions (K_t and $\Delta\sigma_o$) and the appropriate material properties obtained from monotonic tests and fully reversed strain cycling fatigue tests on plain specimens (σ_u, E, ν, ε_f', α_1 and α_2). The material behaviour may also be predicted from a deformation model as suggested by Proctor and Ayers [11]. The iterative procedure required to solve equations (5) (8) and (9) make a computer solution desirable, and a suitable programme has been developed to facilitate the analysis. Strain measurements made using miniature electrical resistance strain gauges indicated good correlation with the predicted strains, except that for large plastic strains, (in excess of 1%) the assumption that the material behaviour during the unloading half cycle is similar to the loading half cycle was found to lead to an overestimate of the residual strain. The accuracy of the Neuber and modified Stowell methods is found to depend upon the shape of

the cyclic stress strain curve and the degree of plastic strain. Strain conditions are dependent upon geometry and specimen or component dimensions, and strictly speaking a three dimensional finite element analysis is required to establish the precise conditions. If complete restraint exists at the notch plane strain will be obtained, and this condition is approached if the ratio of notch radius (ρ) to specimen on component thickness (B) is small. As (ρ/B) increases, so the conditions approached are those of plane stress. Figure 1 indicates a restraint factor (F), allowing for notch configuration, to be applied to the total strain range calculated on the basis of a plane stress analysis. Figure 2 shows the correlation between experimental and predicted results for crack formation lives obtained on the above basis, using the modified Stowell method of calculating local material behaviour. For the majority of results the scatter is within the tolerance band of ± 25%, thus suggesting that the predictive method presented has a most acceptable accuracy. Both Neuber and the modified Stowell methods were found to overestimate actual strain range but, Stowell method was generally more accurate. Further studies are continuing to extend the work to include creep-fatigue interactions.

ACKNOWLEDGEMENTS

This research has been undertaken in the Department of Mechanical Engineering, Portsmouth Polytechnic, in collaboration with Rolls-Royce (1971), Limited, Derby Engine Division, under Contract No. URC 104.

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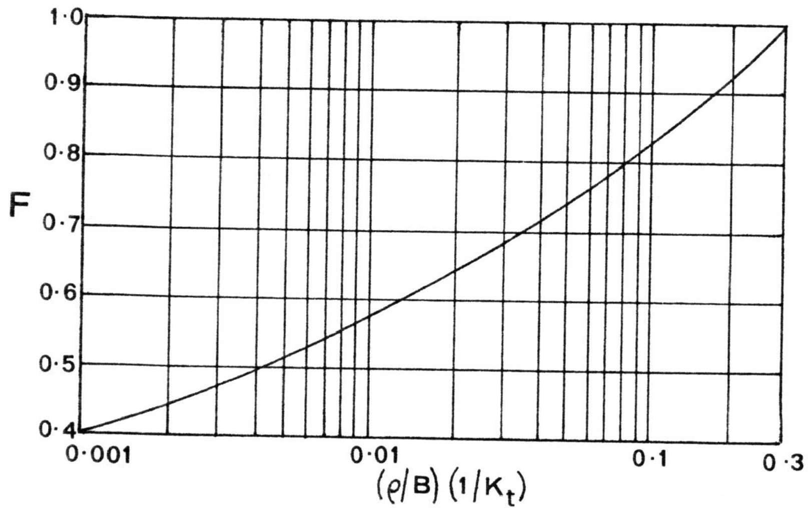

Figure 1 Variation of Material and Restraint Factor (F) with Geometric Parameter (ρ/b) $(1/K_t)$

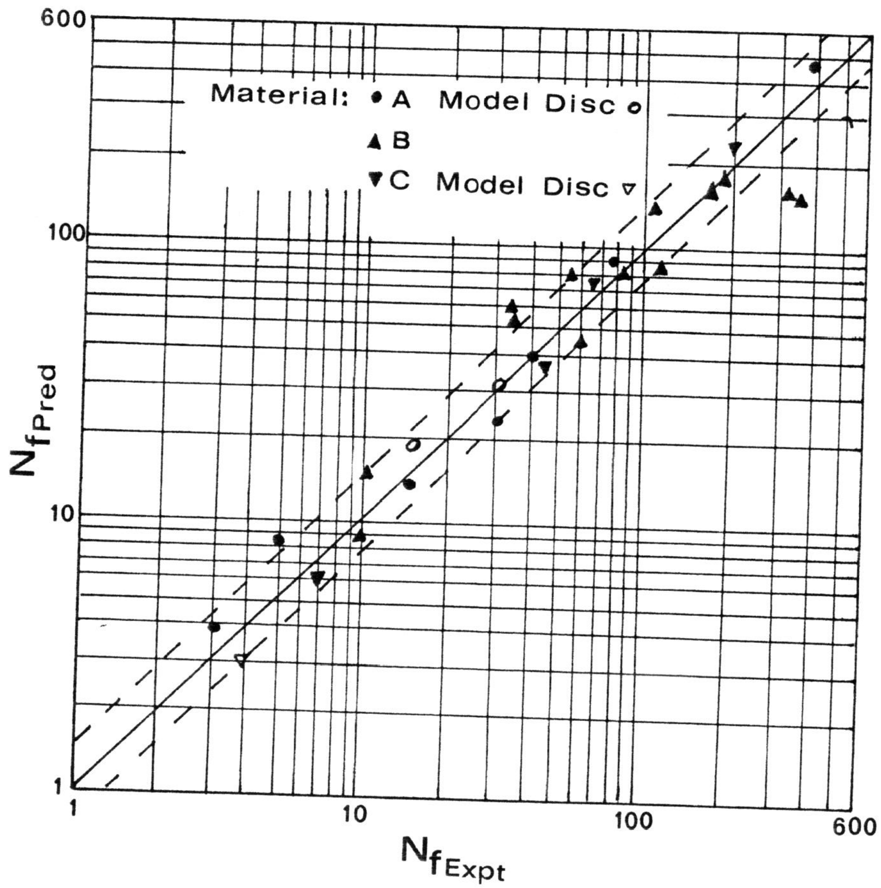

Figure 2 Correlation Between Predicted and Experimental Crack Formation Lives

Table 1 Summary of Mechanical Properties of Materials Studied

Material	A		B		C	
Temp °C	RT	500	RT	450	RT	500
σ_u MN/m^2	1095	817	900	693	1175	1027
σ_y MN/m^2	919	645	768	535	860	769
σ_e MN/m^2	600	-	464	-	310	277
E GN/m^2	214	173.9	205.4	182.0	179.6	166.2
α_1	-0.616	-0.617	-0.610	-0.596	-0.461	-0.439
ε_f'	1.046	1.138	1.127	1.076	0.214	0.194
α_2	-0.134	-0.138	-0.138	-0.137	-0.092	-0.092
C_e	0.0214	0.0207	0.0194	0.0167	0.0169	0.016