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THE CONCEPT OF MATERIAL DIVAGATION AND ITS APPLICATION TO FRACTURE

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INTRODUCTION

Material Divagation is an expression used to describe processes or material behaviour that are characterized by changes or "wandering" of the material functionals that are used to characterize the mechanical properties of the material in the reference configuration.

The most applicable aspect of the theory of material divagation lies in the interpretation of material divagation as damage. The formalism of the theory provides the machinery necessary for understanding and predicting the (macroscopic) failure of materials.

Measures of time-to-failure and cycles-to-failure are the most popular methods used to predict the failure of materials [1]. The relationship between these measures and the mechanical behaviour of a material has never been adequately established. Thus the use of these measures as a fundamental criterion is not based on rational mechanical principles. Mechanics researchers have taken note of this and have started to work on the problem [2-7].

THE CONCEPT

The basic concept underlying the theory of divagation can be presented by a simple experiment. Consider two test samples that are physically and chemically identical in every aspect. Subject one sample to a cyclic deformation process so that after the process is complete the sample is in the original geometric configuration but with an altered microstructure. At this point subject both samples to identical deformation histories. The observed response of the test samples would, in general, be different. This difference reflects the change in the material properties induced by the cyclic deformation history applied to the first test sample. This observed divagation of the material response properties is identified as the mechanical damage or enhancement due to deformation.

The measure of divagation is developed directly from the constitutive equation that characterizes the response of the material as a functional of the history of the deformation. To this end, let us assume that the first Piola-Kirchhoff Stress Tensor, [8] $\tilde{S}(\tilde{x},t)$, at position $\tilde{x}$ and time t is given by a functional, $\tilde{Q}$, of the history of the deformation gradient, $\tilde{F}^t(s) = \tilde{F}(t-S)$; ie,

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$$\underset{\sim}{S}(\underset{\sim}{x},t) = \underset{\sim}{Q}[\underset{\sim}{F}^t(S)] \tag{1}$$

for S in $(0,\infty)$.

To measure the divagation of the material properties using $\underset{\sim}{Q}$, it is necessary to separate the direct effect of the deformation from the intrinsic material properties characterized by the functional $\underset{\sim}{Q}$. To accomplish this, let $\underset{\sim}{F}_1(\tau)$ represent some reference deformation gradient history applied at time $\tau=0$ with the properties that $\underset{\sim}{F}_1(\tau) = \underset{\sim}{I}$ for τ in $(-\infty,0)$ and $\underset{\sim}{F}_1(\tau) \neq \underset{\sim}{I}$ for τ in $(0,t)$. The stress $\underset{\sim}{S}_1$ at time t resulting from $\underset{\sim}{F}_1(S)$ is determined from (1) as

$$\underset{\sim}{S}_1(\underset{\sim}{x},t) = \underset{\sim}{Q}[\underset{\sim}{F}_1{}^t(S)] \tag{2}$$

Denote a second deformation gradient $\underset{\sim}{F}_2(\tau)$ with the property that $\underset{\sim}{F}_2(\tau)$ is arbitrary for τ in $(-\infty,0)$ and such that the stress

$$\underset{\sim}{S}_2(t) = \underset{\sim}{Q}[\underset{\sim}{F}_2{}^t(S)] = \underset{\sim}{0} \text{ for } t > 0. \tag{3}$$

Further we require that full recovery occurs by time $\tau=0$, that is $\underset{\sim}{F}_2(0) = \underset{\sim}{F}(t)$ and is constant for all τ in $(0,t)$.

Define a deformation history $\underset{\sim}{F}_c{}^t(S)$ resulting from the composition of $\underset{\sim}{F}_1{}^t(S)$ with the 'preworking' deformation history $\underset{\sim}{F}_2{}^t(S)$ that is adjusted to $\underset{\sim}{0}$ at time t; that is, let

$$\underset{\sim}{F}_c{}^t(S) = \underset{\sim}{F}_1{}^t(S) + \underset{\sim}{F}_{d2}{}^t(S) \tag{4}$$

for all s in $(0,\infty)$. The quantity $\underset{\sim}{F}_{d2}{}^t(S) = \underset{\sim}{F}_2{}^t(S) - \underset{\sim}{F}_2(t)$ is the difference history and represents a measure of the deformation relative to the current configuration; ie, $\underset{\sim}{F}_{d2}(t) = 0$ for $t \geq 0$.

The stress $\underset{\sim}{S}_c(t)$ due to the deformation history $\underset{\sim}{F}_c{}^t(S)$ can be determined from (1) as

$$\underset{\sim}{S}_c(t) = \underset{\sim}{Q}[\underset{\sim}{F}_c{}^t(S)] \tag{5}$$

for t in $(0,t)$. In general, $\underset{\sim}{S}_c$ is different from the stress $\underset{\sim}{S}_1(t)$ due to the prior deformation history $\underset{\sim}{F}_{d2}{}^t(S)$. The difference in the response as observed by $\underset{\sim}{S}_c(t)$ and $\underset{\sim}{S}_1(t)$ for $t > 0$ the MATERIAL DIVAGATION TENSOR $\underset{\sim}{V}(t)$; ie,

$$\underset{\sim}{V}(t) \equiv \underset{\sim}{S}_c(t) - \underset{\sim}{S}_1(t) = \underset{\sim}{Q}[\underset{\sim}{F}_1{}^t(S) + \underset{\sim}{F}_{d2}{}^t(S)] = \underset{\sim}{Q}[\underset{\sim}{F}_1{}^t(S)] \tag{6}$$

The tensor $\underset{\sim}{V}(t)$ is a measure of the relative change in the material properties due to the predeformation $\underset{\sim}{F}_{d2}{}^t(S)$. Observe that the deformation $\underset{\sim}{F}_{d2}{}^t(S)$ does not contribute directly to the divagation $\underset{\sim}{V}(t)$ (since $\underset{\sim}{S}_2(t) = 0$ for $\tau > 0$) but only through history effects which are manifested by changes in the material microstructure. It should be noted that the material divagation $\underset{\sim}{V}$ is a tensor valued functional. This reflects the fact, for example, that plastic tensile and shear deformations would induce different microstructure changes in the material.

REPRESENTATIONS

A representation for $\underset{\sim}{V}(t)$ can be written down immediately by direct application of Taylor's theorem to functionals [11]. However, the Frechet derivative, $\delta\underset{\sim}{Q}$, of the functional $\underset{\sim}{Q}$ with respect to the norm $||\underset{\sim}{F}_{d2}(\ )||_h$ is given by

$$\underset{\sim}{Q}[\underset{\sim}{F}_1{}^t(S) + \underset{\sim}{F}_{d2}{}^t(S)] = \underset{\sim}{Q}[\underset{\sim}{F}_1{}^t(S)] + \delta\,\underset{\sim}{Q}[\underset{\sim}{F}_1{}^t(S)]$$
$$|\underset{\sim}{F}_{d2}{}^t(S)] + o(||\underset{\sim}{F}_{d2}{}^t(\ )||_h) \tag{7}$$

where $\delta\underset{\sim}{Q}$ is linear in $\underset{\sim}{F}_{d2}{}^t(S)$ and continuous in both hostories [9,10]. An alternative formulation for the divagation tensor, $\underset{\sim}{V}(t)$, can be developed by observing for any time $t > 0$ that

$$\underset{\sim}{V}(t) = \int_0^t \frac{d}{d\tau}\underset{\sim}{V}(\tau)d\tau$$
$$= \int_0^t \{\delta\underset{\sim}{Q}[\underset{\sim}{F}_c{}^\tau(S)\ \dot{\underset{\sim}{F}}_c{}^\tau(S)] = \delta\underset{\sim}{Q}[\underset{\sim}{F}_1{}^\tau(S)\ \dot{\underset{\sim}{F}}_1{}^\tau(S)]\}d\tau \tag{8}$$

since $\underset{\sim}{V}(0) = \underset{\sim}{0}$ and $O(||\underset{\sim}{F}_{d2}{}^t(\ )||_h)$ vanishes by definition.

RESULTS

The above definition and representation of the divagation tensor $\underset{\sim}{V}$ is used to demonstrate the following:

1) A material is designated as ideal if $\dot{\underset{\sim}{V}}(\tau) = \underset{\sim}{0}$ for all τ in $(0,t)$. The consequences of this statement are sufficient to show that any viscoelastic constitutive equations, with integral kernel functions that depend only on time, cannot predict material divagation.

2) For an isothermal cyclic process the observed change in dissipation due to the 'preworking' deformation history $\underset{\sim}{F}_{d2}{}^t$, is linear in the material divagation tensor $\underset{\sim}{V}(t)$.

3) If the constitutive functional $\underset{\sim}{Q}$ is extended to include an arbitrary number (i) of independent histories $\underset{\sim}{N}_i{}^t(S)$, then the material damage or enhancement due to a particular history $\underset{\sim}{N}_\alpha{}^t(S)$ will vanish if $\dot{\underset{\sim}{N}}_\alpha$ vanishes for all t in $(-\infty,\infty)$. However, the constant value of $\underset{\sim}{N}_\alpha$ does remain coupled to the response.

4) If the response $\underset{\sim}{S}$ of the functional $\underset{\sim}{Q}$ is related to a second parameter $\underset{\sim}{M}$ by $\underset{\sim}{M} = \underset{\sim}{f}(\underset{\sim}{S})$, where $\underset{\sim}{f}$ is a constitutive function. The divagation in $\underset{\sim}{M}$, denoted by $\underset{\sim}{V}_M$, is related to the divagation in $\underset{\sim}{S}$, denoted by $\underset{\sim}{V}_S$, by

$$\underset{\sim}{V}_M = \int_0^t (\underset{\sim}{\nabla}_S \cdot \underset{\sim}{f})\ \dot{\underset{\sim}{V}}(\tau)d\tau$$

where $\underset{\sim}{\nabla}_S$ is a generalized gradient operator.

5) Finally, the divagation tensor can be used as a method of developing the constitutive functional Q, since previous history effects are included through the Frechet derivative of Q.

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