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THE APPLICATION OF FRACTURE MECHANICS TO THE INVESTIGATION OF CRACKING IN MASSIVE CONCRETE CONSTRUCTION ELEMENTS OF DAMS

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INTRODUCTION

The paper describes experimental and theoretical investigations undertaken at the VNIIG on the applicability of the Griffith-Irwin theory to thermal crack history in massive concrete elements of dams.

EXPERIMENTAL INVESTIGATIONS

The K_{IC} values taken to characterize plane-strain fracture toughness of concrete were determined on concrete specimens with notches. For concrete of specific composition[1] age-dependent values of K_{IC} were established and correlated with values of tension strength σ_{rupt} (Table 1).

The K_{IC} values depend upon age in the same way as the modulus of elastic instantaneous strains or strength, they were very close to the dependence of K_{IC} on t established by Naus and Lott [1]. Estimations analogous to those performed by Kaplan [2] indicated that the strain energy release at concrete fracture was mostly spent on microcracking and to a lesser extent on the formation of a macrocrack in previously "loosened" material. In general it means that effective surface energy, γ_e, is not a material constant as is the related value of K_{IC}. It depends on the development of the "pseudoplastic" strain zone which is the zone of intensive microcracking. This may be responsible for the fact that the K_{IC} value is lower at unstable crack propagation ($\partial K_I/\partial a > 0$) than at stable growth of cracks ($\partial K_I/\partial a < 0$). For example, concrete[2] tested at an age of 150 days yields $K_{IC} \simeq 0.75 \div 0.85$ MPa$\cdot$m$^{1/2}$ in the first case and $K_{IC} \simeq 1.90$ MPa$\cdot$m$^{1/2}$ in the second one.

Data on K_{IC} available in the literature were obtained either for cement-sand concrete or concrete with aggregate of maximum size 20 - 30 mm. Studies on how these data can be applied to hydraulic concrete, i.e. to concrete with aggregate of maximum size from 80 to 150 mm, are of great practical significance. Table 2 gives the results of testing concrete

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1. The 1:2.2:4.3 concrete (by weight) with portland cement of 400 grade had the water-cement ratio 0.50. The maximum size of a crushed stone coarse aggregate was 20 mm. The tested specimens were cylinders of 0.15 m diameter and 0.10 m height with a vertical notch in a plane of compressive forces. Each point represented average of 3 - 8 tests.
2. The 1:2.7:4.6 concrete (by weight) with portland cement of 400 grade had the water-cement ratio 0.45. The maximum size of a crushed stone coarse aggregate was 30 mm. The 1.3x0.5x0.2 m specimens with a notch at the center of the longer edge were tested under central extension and eccentric compression.

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specimens at 90 days with a river gravel coarse aggregate of maximum size 20, 30 and 40 mm^3. Experiments show that K_{IC} increases with increase in the maximum size of aggregate, though at a decreasing rate.

SOLUTION OF THERMOELASTIC AND ELASTIC PROBLEMS FOR STRIPS, SLABS AND BEAMS WITH NOTCHES

Wall- or slab-type concrete structural elements are widely used in hydraulic engineering (concrete dam crests, buttress and lock walls, spillway piers, downstream apron slabs, etc.). When the K_{IC} values for concrete are known the calculation of temperature crack propagation in the above structural elements is reduced to solving problems of the theory of elasticity for strips, slabs and beams with fixed cracks (notches).

The thermoelastic problems considered are concerned with cracks on one or two sides of a constructional element, with surface and internal cracks at all possible spacings, including the extreme cases: a single crack and complete fracture of concrete in the destruction zone. Consideration is given to both "free" walls and slabs with zero principal vector and moment of stresses at each cross section and "restrained" walls and slabs, whose displacements in some cross sections normal to the surfaces equal zero.

In conformity with recently developed techniques [3 - 5], the solution of problems of thermoelasticity for slabs $-y_0 \leq y \leq y_0$ is reduced to solving the Fredholm equations of the first kind, the kernels of the equations being obtained previously. The solutions obtained are presented as tables and diagrams of dimensionless factors of stress intensity

$$\frac{K_I^{(i)}(1-\nu)}{E\alpha\sqrt{y_o}\sqrt{2\pi}}$$

for the basic temperature fields in the form of a succession of Legendre polynomials $p_i(y/y_o)$, where α is the coefficient of linear expansion of concrete. The results enable one to find the crack depth in an arbitrary one-dimensional temperature field, described by series in terms of the above mentioned orthogonal polynomials, at any moment t. The tables and diagrams of factors

$$\frac{K_I^{(i)}(1-\nu)}{E\alpha\sqrt{y_o}\sqrt{2\pi}}$$

for the surface cracks in "free" slabs are presented in [4, 5]. Figure 1 illustrates the values of these factors for a "free" slab (wall) with a single surface crack. In practice crack growth is frequently determined not only by temperatures but also by forces, it should be noted that due to non-linearity of the problem both factors must be considered simultaneously. So the problems of the isothermal theory of elasticity for notched slabs and beams under bending, central extension, eccentric compression, hydrostatic pressure in joints, etc. were also analyzed. Some results are presented as tables, diagrams and rather simple equations in [3].

3. Cylinders of diameter 0.4 m and length 0.4 m with a vertical notch in a plane of compressive forces were tested. Each point represents average of 5 - 11 tests.

THE PRACTICAL APPLICATION OF THEORETICAL AND EXPERIMENTAL DATA

Theoretical and experimental results were applied to various practical problems, first to determining the depth of the crack or set of cracks which forms in a plain concrete wall in a one-dimensional temperature field. Effects of the wall thickness, heat insulation, the kind of crack (on one or two sides of a wall), air temperature variation amplitude and dead load contribution, etc. on the kinetics and depth of cracks were studied. An example of the analysis is given in Figure 2, a diagram of depth variation of a crack in a concrete wall with a notch of the depth a_o in the process of getting colder. K_{IC} is assumed to be independent of a.

The stress analysis of crests of a number of concrete dams (the Ust-Ilim dam on the Angara River, the Toktogul dam on the Naryn River) with due regard for horizontal and vertical joints opened on the upstream and downstream faces were performed in such a way.

Another procedure, also based on ideas of brittle fracture mechanics was developed for the stress analysis of lower, massive parts of dams which cannot be calculated as a wall or a strip. This procedure is discussed below.

OPENING OF HORIZONTAL JOINTS ON A DOWNSTREAM FACE OF A DAM

The stresses in concrete dams which are in operation in regions with severe climate are profoundly affected by variation in temperature of the ambient air. In winter low air temperatures produce tensile stresses exceeding those of compression due to hydrostatic pressure and the dead weight of the structure. The fact is confirmed by field observations of dams. Stresses in the dam are affected by three factors: the dead weight, the hydrostatic pressure on the upstream face and the mean monthly temperature variation.

The variations in ambient air temperatures on the downstream face during a year are approximated by:

$$T = T_o \cos 2\pi\omega t \ , \tag{1}$$

where T_o is the amplitude dependent on a dam location and ω is the variation frequency equal to 0.0317μHz. These inside the dam have the form of:

$$T(x,y,t) = T^{(c)}(x,y)\cos 2\pi\omega t + T^{(s)}(x,y)\sin 2\pi\omega t \ , \tag{2}$$

where $T^{(c)}(x,y)$ and $T^{(s)}(x,y)$ are functions estimated by solving the problem of thermal conductivity.

A temperature field (2) in a dam is assumed to contain no cracks induces stresses σ_x, σ_y, τ_{xy} of the form:

$$\sigma_x(x,y,t) = \sigma_x^{(c)}(x,y)\cos 2\pi\omega t + \sigma_x^{(s)}(x,y)\sin 2\pi\omega t \ . \tag{3}$$

The terms with superscripts c and s are obtained by solving the two-dimensional static problem of the theory of elasticity for a dam with temperature fields $T^{(c)}(x,y)$ and $T^{(s)}(x,y)$, respectively. Temperature and stress fields written as two terms determined by $\cos 2\pi\omega t$ and $\sin 2\pi\omega t$ enable the stress distribution to be readily established for any month of the year.

The crack depth on the downstream face is calculated on the assumption that the K_I value is limited by the value of K_{IC} on crack extension. Due to the probability of many repeated openings of cracks the effective energy of fracture and the stress intensity factor in the region of the crack tip are taken as zero.

To find the crack depth a in the horizontal cross-section under consideration the total coordinate system thus:

$$\sigma_y = \sum_{i=0}^{m} B_i x^i , \quad \tau_{xy} = \sum_{i=0}^{m} c_i x^i . \tag{4}$$

Since a number of horizontal cracks occurs (or all the construction joints open) on the downstream face the stress intensity factors are evaluated for a periodic system of cracks. The value of K_I can be written as follows

$$K_I(a) = \sum_{i=0}^{m} \left[B_i R_{11}^{(i)}(s/a) + c_i R_{12}^{(i)}(s/a) \right] a^{i+1/2} ,$$

$$R_{11}^{(i)}(s/a) = n_{11}^{(i)} \; \phi_i(s/a) , \quad R_{12}^{(i)}(s/a) n_{12}^{(i)} , \tag{5}$$

where s is the crack spacing; $n_{11}^{(i)}$, $n_{12}^{(i)}$ are the stress intensity factors for a single crack of depth a = 1 propagating at an angle θ to the face of the dam when the loads on the crack contour are $\sigma_y = x^i$, $\tau_{xy} = 0$ or $\sigma_y = 0$, $\tau_{xy} = x^i$, $\phi_i(s/a)$ are the functions taking the periodic pattern of cracks into account.

The values of $n_{11}^{(i)}$, $n_{12}^{(i)}$ are derived according to [6]. The functions $\phi_i(s/a)$ are assumed to be similar to those derived for periodic cracks normal to the boundary [7], their spacing coinciding with that of inclined cracks. Coefficients $R_{12}^{(i)}(s/a)$ are very small so they are taken to be the same as for a single crack.

Figure 3 shows normal stresses over horizontal cross sections of the downstream and upstream faces of gravity dams as well as diagrams of variation in crack depth during a year.

Cracking of the downstream face of a dam gives a decrease in the compressive stresses (or an increase in tensile stresses) due to hydrostatic pressure and dead weight, as well as in tensile thermal stresses, as against the same stresses in a monolithic dam. The total effect is determined by the mean monthly temperature amplitude, the downstream face slope, the cross-section position over the dam height and other factors. According to calculations performed for existing dams with the opening of crack considered, the total tensile stresses at the upstream face within the middle part of the height of the Ust-Ilim dam decrease during winter (Figure 3a), while the compressive stresses at the upstream face of the Krasnoyarsk dam (Figure 3b) increase, the amplitude of stress variation being ∿ 0.4 to 0.6 MPa in the cross-section analysed during a year.

As a rule, cracks open in November-December and close in March-April. Thermal stresses are found to achieve maximum values just at the moment of crack closing. The longest horizontal cracks occur in the middle portion of a dam in January-February.

Thus, the application of fracture mechanis to concrete permits the analysis of massive concrete dams in operation in cold regions, regarding the processes associated with ambient air temperature variations, as well as making it possible to consider more completely the factors affecting dams at the design stage.

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Table 1

Criteria of Concrete	Test Age, Days				
	3	7	14	28	90
K_{IC}, $MPa \cdot m^{1/2}$	0.152	0.219	0.440	0.491	0.609
σ_{rupt}, MPa	0.64	1.08	1.52	1.86	2.70

Table 2

Criterion of Concrete	Maximum Size of Aggregate, mm		
	20	30	40
K_{IC}, $MPa \cdot m^{1/2}$	0.563	0.946	1.005

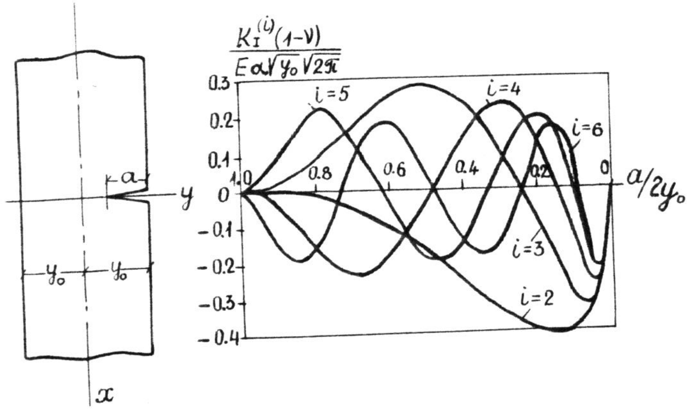

Figure 1 Diagrams of Dimensionless Factors of Stress Intensity $K_I^{(i)}(1-\nu)/E\alpha\sqrt{y_0}\sqrt{2\pi}$ for the Basic Temperature Fields in the Form of a Succession of Legendre Polynomes $p_i(y/y_0)$. (A "Free" Infinite Strip with a Single Surface Crack)

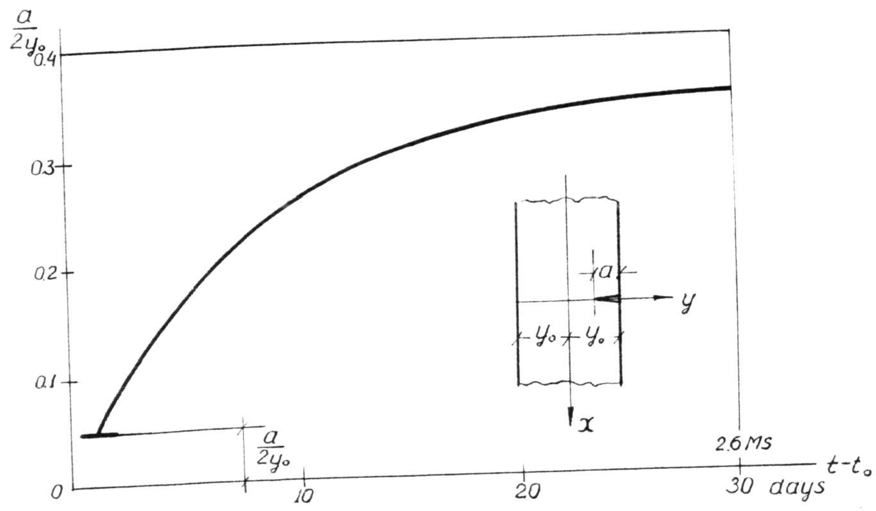

Figure 2 Variation in the Depth of a Crack Opened in a Concrete Wall $-y_0 \leq y \leq y_0$ with an Initial Notch $a_0/2y_0 = 0.05$, put at the Temperature T_0 into the Medium with the Temperature T_A; $2y_0$ being Equal to 10 m, and the Coefficient of Thermal Conductivity to $k = 1.11 \cdot 10^{-6}$ m^2/s; $K_{IC}(1-\nu)/E\alpha(T_0-T_A)\sqrt{y_0} = 0.227$.

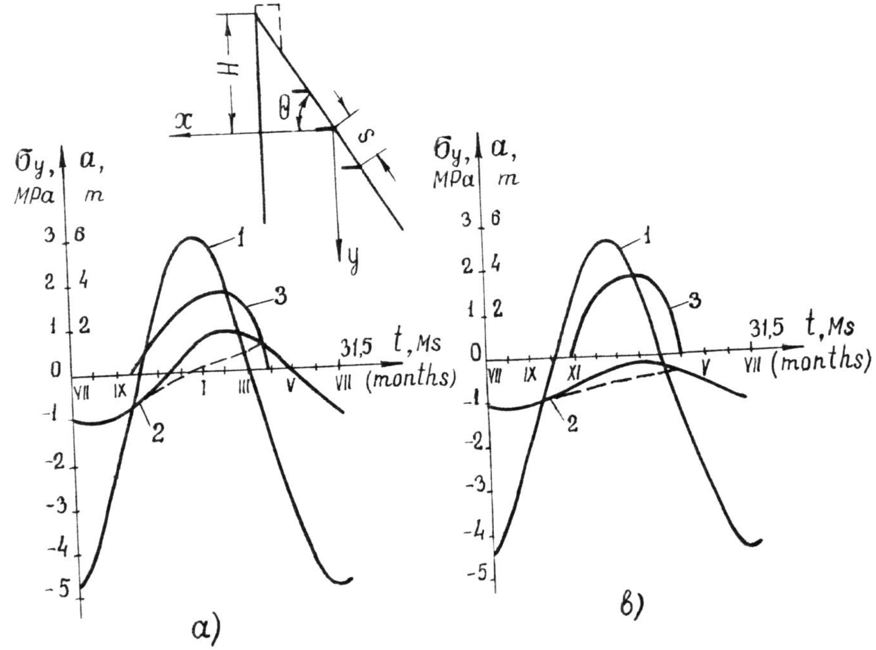

Figure 3 Variation in the Total Stresses and in the Depth of Cracks Opened in Horizontal Cross-Sections of

a) The Ust-Ilim Dam (H = 48 m, θ = 55°, $E\alpha T_0$ = 7.15 MPa)
b) The Krasnoyarsk Dam (H = 64 m, θ = 51°, $E\alpha T_0$ = 6.86 MPa) at Water Density - 1000 kg/m³ and Concrete Density - 2400 kg/m³).

1 - stresses σ_y at the downstream face
2 - stresses σ_y at the upstream face (the dashed curves show the stresses calculated with opening of cracks taken into consideration)
3 - crack depth on the downstream face