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STRESS INTENSITY FACTORS AT THE TIPS OF KINKED AND FORKED CRACKS

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INTRODUCTION

Amongst the fundamental problems that form the theoretical basis for studies of the paths of crack propagation and of the stability of cracks are the problems of the elastic fields around a long or semi-infinite crack with either a single kink or a fork at its tip (to discuss the onset of deviation or branching, the case where the main crack is long is of most interest). We present here briefly some of our computations of the relevant stress intensity factors for these two problems. A number of workers have considered problems of this kind; for some references, see [1 - 10]. In making available our own results, which sometimes differ from those previously published by other workers, we hope to contribute to an ultimate consensus of agreement, for these problems have a considerable history of published error, some of which has been acknowledged. In the text we give some indications of why we think this has come about.

THE KINKED CRACK

We image that a semi-infinite crack has, at its tip, a kink of unit length making an angle α with the main crack (Figure 1). The loading is specified by the stress intensity factors K_1 and K_2 of the main crack without the kink. The analysis of [14] enables the stress intensity factors k_1 and k_2 at the tip of the kink to be computed in terms of K_1 and K_2 by quadratures. We find [1]:

$$k_1 = K_{11}(\alpha)\ K_1 + K_{12}(\alpha)\ K_2\ , \qquad (1)$$

$$k_2 = K_{21}(\alpha)\ K_1 + K_{22}(\alpha)\ K_2\ , \qquad (2)$$

where the functions $K_{ij}(\alpha)$ are displayed in Figure 1. (A table of $K_{ij}(\alpha)$ allowing interpolation to within 1% accuracy is available upon request to the authors). We have checked the accuracy of the numerical procedures whereby $K_{ij}(\alpha)$ are computed, and we believe that the results upon which Figure 1 is based are an accurate solution to the problem. Where comparisons can be made, we agree with the results of Chatterjee [5], but disagree slightly with [2]. The appropriate curves in our Figure 1 and the Figure 4 of [2] look very much the same, but our results differ from those of [2] by as much as 20%. We are unable to explain this difference on the basis of an error in our calculation, and suggest that the method of conformal transformation used in [2] may be less reliable than is usually supposed. This supposition is partly born out by the difficulties we experienced in

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attempting to make a related method work for the problem of the forked crack.

THE FORKED CRACK

We have solved this problem by two methods.

(a) Firstly, we used a method of conformal transformation which maps a finite crack with a forked tip into the unit circle. The appropriate stress functions are found by inverting an infinite system of algebraic equations, and we found that the simplest formulation of the problem was that of [9]. Although the method produced results which agree qualitatively with those of [2] we had to do very large amounts of computation to achieve them, and we suspect that the rate of convergence of the method was slow enough for simple numerical rates of convergence to be misleading.

(b) Secondly, we represented the cracks by continuous distributions of dislocations and we solved the resulting singular integral equations by a method similar to that reported in [11]. All our attempts to represent a semi-infinite crack by a continuous distribution of dislocations were unsatisfactory, and so we performed the computations for a finite, but long, main crack. Figure 2 shows the normalized stress intensity factors k_1/K_1, k_2/K_1 at the tip of the upper fork when the main crack is 40 times as long as the kink. K_1 is the stress intensity factor at the tip of the main crack without the kink. Our method is numerically unstable for small α, and we have no reliable results, as yet, for $0^0 < \alpha < 5^0$.

The most important feature of Figure 2 is the zero of k_2 at about 18^0, slightly larger than the value read from Figure 14 of [2]. The existence of this zero is used in theories of the crack forking which occurs both with fast moving cracks [12], and in stress corrosion [2]. This zero appears to arise because of the presence of the fork, and the computations of Kalthoff [12] for a fork without a main crack clearly show its presence. A recent paper [10] reports the results of computations on a forked crack where the main crack is four times the length of a fork; k_2 has no zero. We have repeated out computations for this particular geoemtry and clearly discern a zero somewhere between 10^0 and 20^0. Finally, we note that our Figure 2 disagrees with the results of [4]. We believe that this is because the formulation of the problem given in [4] is incorrect. We find that proper application of the boundary conditions leads to a coupled pair of Wiener-Hopf equations, rather than the separated equations of the authors. The point is a subtle one and has not been noticed by a recent reviewer [13].

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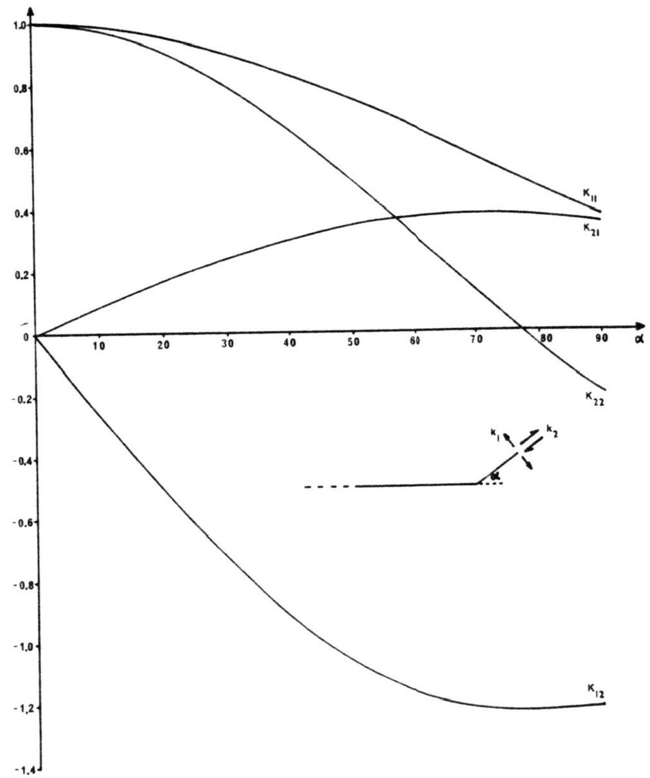
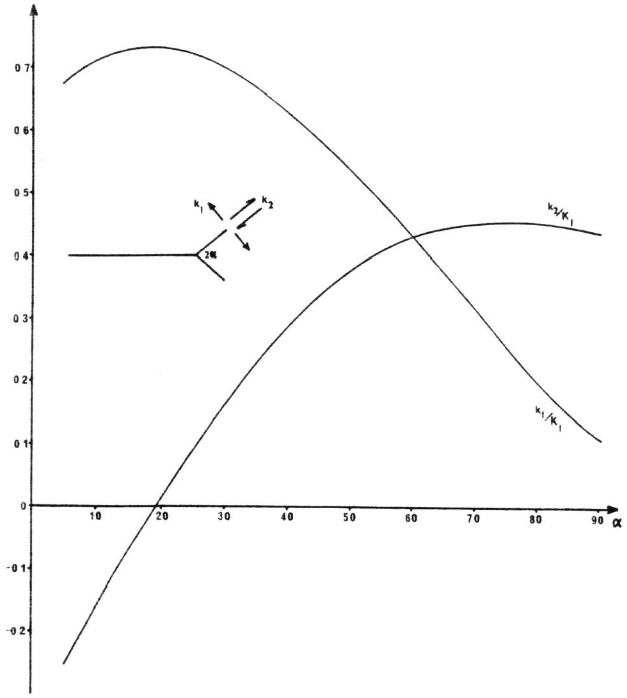


Figure 1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0 5
0 4
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0 2
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-0 1
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10
20
30
40
50
60
70
80
90
α
k1
k2
2α
k2/K1
k1/K1

Figure 2