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STRESS AND CRACK-DISPLACEMENT INTENSITY FACTORS IN ELASTODYNAMICS

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This paper deals with several remarks concerning stress-intensity factors in elastodynamics, for the steady-state and the transient problems of a moving crack with a constant velocity V. It discusses the equivalence between dynamic fracture criterions and introduces the new J-integral for moving crack. The present paper will bring out the same importance of the stress-intensity factor and the crack-displacement intensity factor, in the presentation of fracture criterions.

In the plane strain condition, the components of the displacement fields u,v, with respect to the fixed cartesian axes Oxy, can be expressed in terms of two scalar functions $\phi(x,y,t)$ and $\psi(x,y,t)$ which satisfy respectively the two-dimensional wave equations with velocities $c_L=\{(\lambda+2\mu)/\rho\}^{1/2}$ and $c_L=\{\mu/\rho\}^{1/2}$ where λ,μ are the elastic Lamé's constants and ρ is the mass density. Expressions of the displacement and the stress fields in terms of ϕ and ψ or their well known complex representations can be found in [1]. We introduce the velocity parameters:

$$\beta_1^2 = 1 - \left(\frac{V}{c_L}\right)^2 \qquad \beta_2^2 = 1 - \left(\frac{V}{c_T}\right)^2$$

THE STEADY-STATE PROBLEM

The steady-state problem of a moving crack with constant length has solutions which can be found in [1]. The load is assumed to be symmetric with respect to the crack line Ox. In the moving axes x'=x-Vt, y'=y with polar coordinates, r,θ the displacement and stress fields near the crack-tip are known. We report here only the expressions for v and σ_{yy} with a change on notations from those given in [1]:

$$v = \frac{K_I}{\mu}\left(\frac{r}{2\pi}\right)^{1/2} \mathrm{Im}\left\{-2\beta_1(1+\beta_2^2)(\cos\theta+i\beta_1\sin\theta)^{1/2} + 4\beta_1(\cos\theta+i\beta_2\sin\theta)^{1/2}\right\}\left\{4\beta_1\beta_2-(1+\beta_2^2)\right\}^{-1} \qquad (1)$$

$$\sigma_{yy} = \frac{K_I}{(2\pi r)^{1/2}} \mathrm{Re}\left\{-\frac{(1+\beta_2^2)^2}{(\cos\theta+i\beta_1\sin\theta)^{1/2}} + \frac{4\beta_1\beta_2}{(\cos\theta+i\beta_2\sin\theta)^{1/2}}\right\}\left\{4\beta_1\beta_2-(1+\beta_2^2)^2\right\}^{-1} \qquad (2)$$

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Equations (1) and (2) are Yoffé's work [2]. Instead of K_I, introduce the notations K_I^σ for the dynamic stress-intensity factor. It is defined as in the static case by the asymptotic expression of the normal stress $\sigma_{yy}(\theta=0)$:

$$K_I^\sigma = \lim_{r\to o} \sigma_{yy}(r,o)(2\pi r)^{1/2} \tag{3}$$

If we represent the crack-opening displacement $v(r,\pi)$ by similar formula known from the plane strain static case, we must introduce a crack-displacement factor:

$$K_I^V = \lim_{r\to o} \frac{2\mu}{\varkappa+1}\left(\frac{2\pi}{r}\right)^{1/2} v(r,\pi) \tag{4}$$

where $\varkappa=(\lambda+3\mu)/(\lambda+\mu)=3-4\nu$ (ν = Poisson ratio). From (1)-(4) it results:

$$K_I^V / K_I^\sigma = \frac{4}{\varkappa+1}\;\frac{\beta_1(1-\beta_2^2)}{4\beta_1\beta_2-(1+\beta_2^2)^2} \tag{5}$$

This ratio varies monotonically from unit, at V=0, to infinity at the Rayleigh velocity c_R (defined by the vanishing of the denominator of (5)).

A TRANSIENT PROBLEM

Let us consider the particular problem of a small cut which extends symmetrically with the velocity V. The crack tip's coordinates in the fixed axes are x=Vt, y=0 (and x=-Vt, y=0). The crack is subject to opposite normal impulses $\sigma_{yy}=\delta(x)\delta(t)$. This problem is solved by Afanasev and Cherepanov [3] who gave the stress-intensity factor, as defined by equation (3) in the following form:

$$K_I^\sigma(t) = -S(1/V)V^{1/2}(V^{-2}-c_L^{-2})^{-1/2}(c_T^{-2}-c_L^{-2})^{-1}\;\frac{1}{2t^{3/2}\pi^{1/2}} \tag{6}$$

where:

$$S(\tau) = (c_T^{-2}-2\tau^2)^2 + 4\,\tau^2(c_L^{-2}-\tau^2)^{1/2}(c_T^{-2}-\tau^2)^{1/2}$$

Here, the Rayleigh velocity is defined by the root of the equation S(1/V)=0. Afanasev and Cherepanov did not calculate the crack displacement factor. But from their solution, we may obtain the latter quantity as:

$$K_I^V(t) = \frac{2}{\varkappa+1}\,V^{-1/2}c_T^{-2}(c_T^{-2}-c_L^{-2})^{-1}\;\frac{1}{t^{3/2}\pi^{1/2}} \tag{7}$$

From (6) and (7) the ratio K_I^V/K_I^σ is found to be the same as in the steady-state case. This agrees with the general result obtained by Achenbach and Bazant [4] who stated that the near-tip fields are of the same form for steady-state and transient crack-propagations. The result (5) can be

found in many works. Here, we only introduce new notations and terminologies in order to make more distinction between the two intensity factors. Remark that for $V=c_R$, the stress-intensity factor (6) vanishes, while the crack-displacement factor (7) does not. So, the latter factor has some importance on the characterization of the crack-tip just when the notion of stress-intensity factor falls.

SOME REMARKS ON FRACTURE CRITERIONS IN SYMMETRIC LOADING

In the static case, the usual criterions can be reduced to the K_I criterion. Whatever is chosen as a criterion, the stress criterion, the COD criterion, the G-theory and the J-integral are equivalent. Let us state that the equivalence between two criterions means that the relationship between their critical values involves only the material's properties, not the velocity dependence. For example, the equation $J_{Ic}=(1-\nu^2)K_{Ic}^2/E$ (E: Young modulus) establishes the equivalence between the J-integral and the K_I criterion, for opening mode in static case. In dynamic crack-propagation, we can see that the K_I^σ criterion has no equivalence with the usual other criterions. As a first example, suppose that the critical value K_{Ic}^σ is a material constant independent of the velocity, then the value K_{Ic}^V computed from K_{Ic}^σ through equation (5) is not, and vice versa. Thus, the criterions K_I^σ and K_I^V are not equivalent in the sense stated above.

Let us consider other parameters.
The G-parameter is defined by the Griffith's energy balance (See Erdogan [5] and Achenbach [6]):

$$G\delta a \equiv -\delta W_{elas} + \delta W_F - \delta W_{kin} = 2D\delta a \tag{8}$$

where $\delta a=V\delta t$, δW_{elas} is the elastic energy variation, δW_F is the work done by given external forces, δW_{kin} the kinetic energy variation, and D the dissipative energy rate in fracture. For brittle material: $D=\gamma_s$ (Specific surface energy). The computation of the left side of equation (8) requires the knowledge of the dynamic fields in the whole body. If G has a representation by mean of some path-independent integral, the computation would be possible with the near-tip fields. If not, the direct computation of G is not easy. Nevertheless, the relationship between G and K_I^σ is expected to involve the velocity dependence.

The G'-parameter is the crack-closure energy (See [5] and [6]):

$$G' = \lim_{\delta a\to o}\frac{1}{2\delta a}\int_{crack}\sigma_{yy}(a)\;v(a+\delta a)\;ds \tag{9}$$

This parameter is obviously easy to compute once the near-tip fields (1) and (2) are known. It may be obtained as:

$$G'(t) = \frac{1-\nu^2}{E}\,K_I^\sigma(t)K_I^V(t) \tag{10}$$

With different notations, equation (10) can be found in Atkinson and Eshelby [7], in Freund [8] and also in [4]. The interpretation of (9) as the dissipative energy rate is given in [5]. (See in the latter reference the comparison between G' and the strain energy release rate $G_e = -dW_{elas}/da$; in dynamic crack-propagation $G' \neq G_e$). We remark again that the fracture criterions based upon G,G' or any other parameter derived by the energy's consideration are not equivalent to the stress criterion, due to the velocity dependence. For example, if the critical value G'_c is independent of the velocity, the value K^{σ}_{Ic} computed from G'_c through equation (1) is not.†

In what follows, we consider another parameter given by a path-independent integral.

THE J-INTEGRAL FOR MOVING CRACK

Let us consider the fixed axes Ox_i ($x_1=x$, $x_2=y$, $u_1=u$, $u_2=v$). The conservation law given by Fletcher [10] is:

$$\frac{\partial}{\partial t}(\rho \dot{u}_j u_{j,i}) + \frac{\partial}{\partial x_k}\left\{-u_{j,i}\sigma_{kj} + \left(W - \frac{1}{2}\rho \dot{u}_h \dot{u}_h\right)\delta_{ik}\right\} = 0 \tag{11}$$

where W is the elastic energy density, $\dot{u}_i = \partial u_i/\partial t$. Consider a contour Γ_V joining two points on opposite sides of the crack's surface while going around the tip, and moving with the velocity V, and let $A(\Gamma)$ be the area within the contour Γ_V. The Rice's J-integral [11] is extended to moving crack in transient loading as follows:

$$J = \int_{\Gamma_V}\left\{Wn_1 - \sigma_{jk}n_k u_{j,1} - \frac{1}{2}\rho \dot{u}_h \dot{u}_h n_1\right\}ds + \frac{d}{dt}\int_{A(\Gamma)} \rho \dot{u}_j u_{j,1} d\omega - \int_{\partial A} \rho \dot{u}_j u_{j,1} V n_1 \, ds \tag{12}$$

where n_i is the unit outward normal to the contour and d/dt is the time derivative of integral over moving domain $A(\Gamma)$. The J-integral is not a line integral, due to the second term. However, it results from (11) that J is independent of the path. The value of the J-integral may be obtained by a contour flattened on the crack line. It is easy to see that only the second singular term in the first integral contributes to J. Thus, it is sufficient to consider the near-tip fields (1) and (2) for the computation. For the steady-state case, we find a very simple formula:

$$J = \frac{1-\nu^2}{E} K^{\sigma}_I K^V_I \tag{13}$$

†In fact, for some material, the experimental value of K_{Ic} depends on the velocity [9]. This raises the question as to the validity of the K_I criterion in such a case. Perhaps, a better choice would be some parameter X such that the theoretical ratio X/K_I multiplied by $K_{Ic}(V)$ has a nearly constant value.

Equation (13) is the generalization of that obtained by Rice for his J-integral in the static opening mode. It has the same form as equation (10). Consequently, there is equivalence between the G' and the J criterions. The result (13) is exactly the flux of energy into the crack tip, as discussed in [6], [7] and [8].

It should be noted that equation (13) can be extended to transient crack-propagation by the use of the near-tip fields obtained by Achenbach and Bazant. Another proof of the above extension to transient loading can be found in [12].

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