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STRESS ANALYSIS OF CRACKS IN ELASTO-PLASTIC RANGE

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INTRODUCTION

The structure of the dominant singularity in strains and stresses near a crack-tip in plane problems, for hardening materials ($\varepsilon \sim \sigma^n$), has been studied by Hutchinson [1] and Rice and Rosengren [2]. The amplitude of the above singularity for pure mode problems, within the limitation of a small-strain, J_2-deformation plasticity theory, is related to the well known J-integral. Begley and Landes [3, 4] have used the J-integral as an elastic-plastic fracture criterion in the presence of large-scale yielding, with considerable success, in a recent series of experiments. Attention has also been focused recently on other concepts such as the crack-tip opening displacement, nonlinear energy release rate, etc., to deal with the problems of nonlinear fracture mechanics. In the present paper we present a rigorous computational (finite element) method to analyse general in-service situations of ductile-fracture. As an example, analysis of an actual fracture test-specimen is presented, and particular attention is paid to the details of crack-tip stress-strain field, J-integral, COD, and their correlation.

METHOD OF APPROACH

A finite deformation, embedded singularity, elasto-plastic, finite element incremental method [5, 6] using J_2-flow plasticity theory has been developed. Strain and stress singularities of the type given in [1, 2] are embedded in elements near the crack-tip, and finite geometry changes near the crack-tip have been accounted for. A hybrid-displacement finite element model [7] is employed to enforce the conditions of displacement-continuity and traction-equilibrium between the near-tip elements and the far field elements. The incremental finite element method employed is of the "tangent-modulus" type (wherein, the stiffness matrix is up-dated at each step to account for finite-deformation effects as well as plastic flow effects) with a Newton-Raphson type corrective iteration in each step.

The example problem discussed here, is of a 3-pt bend fracture test specimen, of inplane dimensions shown in Figure 3, for which elastic-plastic experimental data for thicknesses of 10 and 20 mm, and uniaxial stress-strain data for the material is reported in [3]. For the present computational purposes, this data was fitted to a Ramberg-Osgood form: $\varepsilon = (\sigma/E) + (\sigma/B)^n$ where $E = 20.7 \times 10^{10}$ Pa, $\nu = 0.3$, $B = 1.26 \times 10^9$ Pa, and $n = 22.2$.

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CRACK-TIP STRESS AND STRAIN VARIATIONS

The θ-variation of the dominant singularity was determined in [1] from a numerical solution of a nonlinear, fourth order, ordinary differential equation for each value of hardening exponent, n. In the present computations, the r-dependence of the singular solution (i.e., $r^{-n/n+1}$ in strains) is correctly built into the near-tip elements. However, θ-variation is approximated in the usual sense of the finite-element method, by using sector-shaped near-tip elements and assuming a polynomial θ-variation of singular solution in each sector. The computed θ-variation of stresses at a loading stage when the near-tip elements have yielded, is shown in Figure 1. These variations for n = 22.2 are in excellent qualitative agreement with those reported in [1] for n = 13. The r-dependence of the computed strains and stresses is shown in Figure 2, and it can be seen that in the elastic range both the stress and strain singularities are of $1/\sqrt{r}$ type, whereas under fully-developed plasticity the strain singularity is of form $r^{-0.96}$ and the stress singularity is of form $r^{-0.045}$.

COMPLIANCE DATA AND J-INTEGRAL RESULTS

The computed load versus load-point displacement and crack-mouth-opening displacement curves are shown in Figure 3. At the non-dimensional load of 0.28, almost the entire net section ligament was found to have yielded. The linear portion of the P vs. U_L curve has the slope, $P/U_L = 1.33 \times 10^5$ N/mm, (corresponding to $K_I/P = 3137\ m^{-3/2}$) both of which are found to agree excellently with the independent linear-elastic results of [8]. The J-integral (based on the definition as applicable to the case of finite deformations, given by Knowles and Sternberg [9]) is computed on four different paths (see insert, Figure 3). The variation of the value of J between different paths was ±1.4%. The average J is plotted as a function of load-point displacement, and as a function of crack-mouth-opening in Figure 4. The experimentally determined variation of J with load point displacement from [3] is also shown in Figure 4, for identical specimens of two different thicknesses. The excellent correlation of the experimental results for J with the presently computed values based on a stationary crack model, for all load-point displacement levels (including the critical level at fracture, as determined in the experiments) suggests that there may have been no appreciable stable crack growth prior to fracture in the particular experiments. However it has been found that for some cases [6] involving compact-tension, and centre-cracked specimens, the computed critical J was higher than the experimental value, thus pointing to the effect of stable crack growth prior to fracture in small test specimens. Since J-criterion is strictly valid to define only crack growth initiation, it appears that more experimental as well as analytical studies are necessary, firstly to precisely define the J_{IC} measurement point in the experiments, and secondly to analyze the continuum mechanics problem of stable crack growth. The authors are currently completing work on finite element modelling of crack growth, involving translation of the singularity field, to obtain information on energy balances during crack growth, and, more interestingly, during the terminal loss of stability.

A detailed comparison of the present finite-deformation analysis results, for the load-point displacement, stresses and strains near the crack-tip, plastic-yield-zones near the crack-tip, and crack-surface deformation profiles, at various load levels, with those obtained using only an infinitesimal deformation theory, is presented in [5]. One interesting observation in this comparison in [5] is that, at load levels corresponding to large-scale plastic yielding, and when there is noticeable crack-tip blunting, path-independence of the computed J (with ±1.5%) was noticed even for paths closest to the crack-tip, when the appropriate definition of J [9] applicable to the finite deformations is used. However, at the same load levels, when the well-known Rice's definition of J, applicable only in the case of infinitesimal deformations, was used, the computed J for the path closest to the crack-tip was substantially lower (by about 16%) than that computed from paths in the far-field.

CORRELATION OF J WITH COD

In the literature, there appears to be no precise definition of COD as essentially a near-tip quantity. However, for test specimens such as the present, calibration relations between COD and CGD (clip gauge or crack-mouth-opening displacement) have been given. For instance, Wells [10] gives a nonlinear relation between COD and CGD in the small scale yielding range, and a linear relation in the large scale yielding range, in agreement with theoretical considerations. Using this definition for COD, the relation between COD and J in the present computations, is shown in Figure 5, which shows that $J \sim 1.44 \cdot COD \cdot \sigma_y$. This result appears to be in agreement with the slip-line theory analysis of Rice and Johnson [11], who also account for finite geometry changes. A similar relation was found to hold for other test specimens also [6]. However, efforts to correlate the above calibration values for COD with essentially near-tip-geometrical quantities, such as the diameter of a circle inscribed near the crack-tip, etc. were not successful.

CLOSURE

A computational procedure to analyze elastic-plastic fracture situations is presented. From a computational viewpoint, the J-criteria appears to be the most attractive. However, further analytical framework to analyze stable crack growth is necessary to enhance the scope of computational methods to predict not only ductile fracture initiation, but also terminal loss of stability of crack growth.

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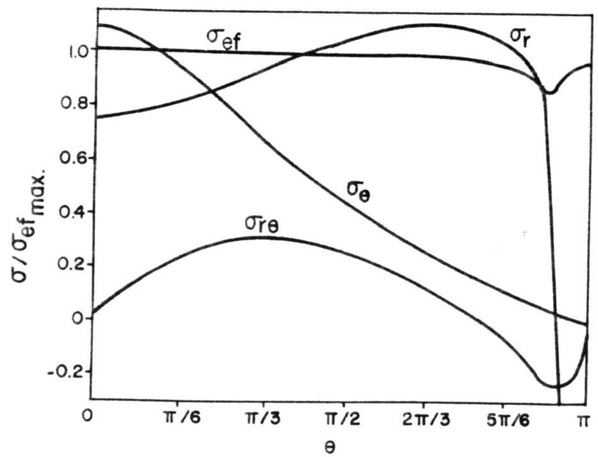

Figure 1 θ-Variation of Crack Tip Stresses in a Strain-Hardening Elastic-Plastic Material (n = 22.2)

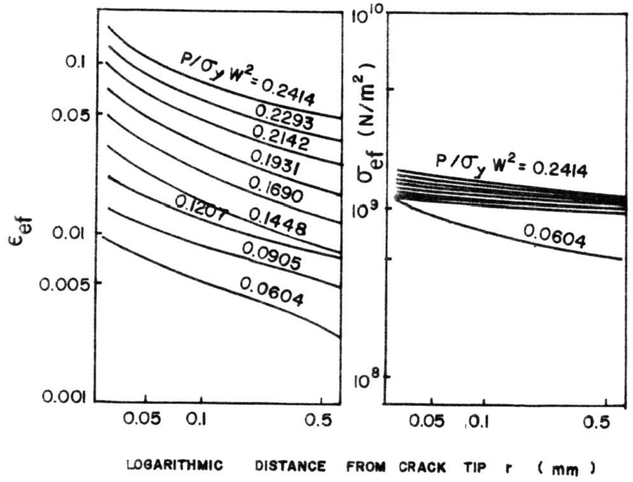

Figure 2 Strain and Stress Singularities Near the Crack-Tip in a 3-Point Bend Bar

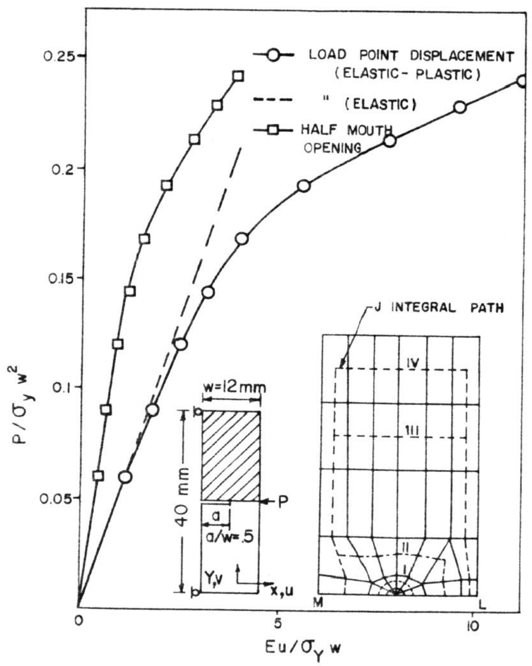

Figure 3 Load versus Load-Point and Crack-Mouth Opening Displacement Variations for a 3-Point Bend Bar

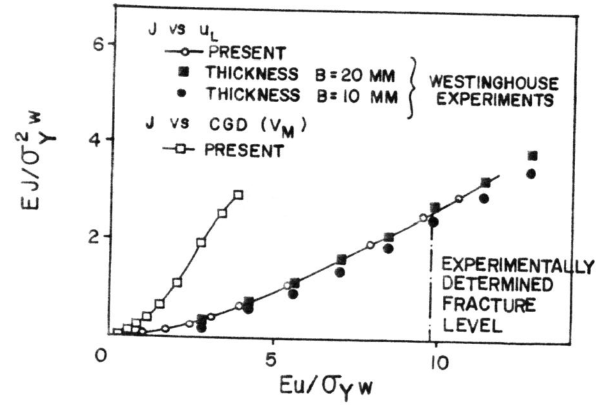

Figure 4 J-Integral Variation with Load-Point and Crack-Mouth Opening Displacement

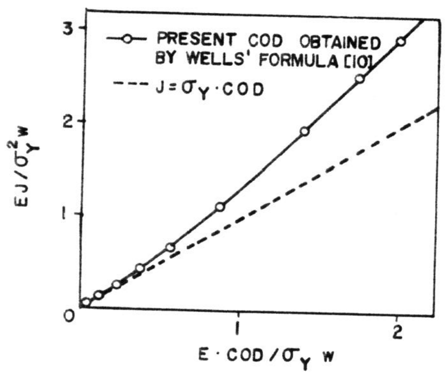

Figure 5 Correlation of J-Integral with Crack-Tip-Opening