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STRENGTH CRITERIA FOR FIBRE-REINFORCED PLASTICS

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INTRODUCTION

Some authors (e.g. [1 - 3]) have made an attempt to formulate the rupture criterion for anisotropic non-homogeneous materials in terms of certain invariants, namely:

$$\Phi(I_1, I_2, \ldots) = 0$$
$$I_1 = B_{rs}\,\sigma_{rs},\quad I_2 = A_{ijk\ell}\,\sigma_{ij}\,\sigma_{k\ell} \qquad (1)$$

The values of I_1, I_2, ... are the convolutions of the stress tensor σ_{ij} and a number of even order tensors B_{rs}, $A_{ijk\ell}$, ..., characterising the strength properties of the composite.

Taking into account the first and second invariants only, we can assume (1) to have the following form

$$\left(B_{rs}\,\sigma_{rs}\right)^{\alpha} + \left(A_{ijk\ell}\,\sigma_{ij}\,\sigma_{k\ell}\right)^{\beta} = 1 \qquad (2)$$

For plane stress the number of constants B_{rs} is three and the number of constants $A_{ijk\ell}$ is six. The authors of the paper [1] had assumed $\alpha = 1$, $\beta = 1/2$, while in [3] $\alpha = \beta = 1$. Using any one of these criteria we have to determine experimentally a large number of material constants, and the calculation of these values using a rather restricted amount of the experimental data may be unstable. The experimental curves can be as a rule described equally well with very different sets of parameters B_{rs} and $A_{ijk\ell}$. On the other hand, equation (2) defines a smooth hypersurface in the stress space.

If we try to apply equation (2) to the case of biaxial tension, i.e. stresses σ_1 and σ_2 acting in the directions of orthogonal reinforcement, the limiting curve in the σ_1 - σ_2 plane will be an ellipse. When the matrix of the FRP is rather weak, this approximation seems to be poor. Really, neglecting the strength of the matrix we obtain an orthogonal grid which can be ruptured as a result of breaking the reinforcing fibres in one of the two directions. Consequently, the condition of rupture in the σ_1 - σ_2 plane will be represented by a rectangle. Of course, if we introduce into equation (1) the higher order invariants, we can approximate this rectangle with any desired degree of precision, but the number of the material parameters and the difficulty of their determination grow immensely.

It has been suggested [4] that various kinds of fracture occur in the composite independently. So for the unidirectional composite it is pos-

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sible to construct a set of limiting fracture surfaces for any kind of fracture: rupture of fibres, interlaminar shear or transverse rupture. If the stress direction makes a certain angle with the fibre direction, the strength of any angle will be determined by the surface corresponding to the lowest strength value. Thus the ultimate surface of rupture will be composed of sections of these partial surfaces.

Another approach to the problem of the construction of the loading surfaces for composite structures has been given in [5]. Geometrically speaking, one surface of rupture moves on the other. If a matrix obeying von Mises plasticity condition is reinforced with two orthogonal rows of fibres, the von Mises ellipse moves along the sides of the rectangle. The resulting rupture curve in the $\sigma_1 - \sigma_2$ plane has a shape like an "ottoman cushion" with rectilinear segments.

When the rupture criterion is formulated in terms of symmetric invariants, nothing can be said about the character of rupture. On the other hand, for orthogonally reinforced plastics, stretched in various directions, this character is always similar. The rupture occurs in the plane, defined by the fibres in one direction of reinforcement, the fibre layers being separated and the fibres of the orthogonal direction ruptured in the plane of the macroscopic rupture or inside the specimen, the ends of them being pulled out.

Taking into account this very specific kind of rupture of orthogonally reinforced plastics it seems reasonable to formulate the rupture criterion as a relationship connecting the values of the normal and shear stresses in one of the two planes of possible rupture. Denoting by σ_n and τ_n the corresponding values of stresses, we can write:

$$\varphi\left(\sigma_n, \tau_n\right) = C \qquad (3)$$

This condition must be fulfilled for one of the two directions of possible rupture. Equation (3) resembles the well known Mohr strength condition, according to which rupture occurs when (3) is satisfied for any one direction. In our case the direction of the vector $\bar{n}$ (Figure 1) is fixed and there are only two possible cases. In the present paper criterion (3) is applied for the interpretation of some experimental data. The simplest assumption concerning the function $\varphi(\sigma_n, \tau_n)$ being:

$$\sigma_n + M\tau_n = C \qquad (4)$$

Here M and C are experimentally determined material parameters. If the specimen is stretched in the direction making an angle θ with the fibre direction, the ultimate stress according to (4) will be: $\sigma_f(\theta) = C(\cos^2\theta + M\sin\theta\cos\theta)^{-1}$.

Plotting test results for various values of θ in $\sigma_n - \tau_n$ coordinates and drawing the best straight line approximation we can determine the values of M and C.

RESULTS AND DISCUSSION

The experimental data referring to the strength of tensile specimens of GFRP, CFRP, BFRP cut in various directions are plotted on Figure 1. The

width of the specimens was 10 mm, the thickness about 3 mm, they were held in the grips by friction due to the constant lateral pressure.

The quadratic formula

$$\left(\sigma_n/\sigma_o\right)^2 + \left(\tau_n/\tau_o\right)^2 = 1 \qquad (5)$$

has been used also. Here σ_o and τ_o are ultimate tensile and shear stresses for the direction along the fibres. For comparison the curve based on the "invariant" criterion is drawn in Figure 1. The picture is similar for all materials tested, and therefore we give the results for one only. Formulae (4) and (5) fit the experimental data better than (2), their degree of precision being almost the same. The curve $\sigma_f(\theta)$ is symmetric with respect to the angle $\theta = 1/4\pi$, if $\theta > 1/4\pi$, the rupture condition will be fulfilled on the second set of reinforcement planes.

The advantage of formula (4) is its relative simplicity, which enables easy determination of the material parameters. At the same time, using (4), we can find out the value of a very important composite characteristic - its shear strength, $\tau_o = C/M$. For the glass laminated plastic it was found that $\tau_o = C/M = 94 \times 10^4$ Pa, for a carbon fibre reinforced plastic $\tau_o = 55 \times 10^4$ Pa and for a boron-epoxy composite $\tau_o = 37 \times 10^4$ Pa.

The simplicity of criterion (4) makes the analytical solution of some stress concentration problems possible, compared with invariant criterion (2) or a similar one, where a rather tedious numerical procedure is needed. Consider the case of uniform tension along the fibres of a broad orthogonally reinforced plate containing a circular hole (Figure 2). The radius of the hole is assumed to be large compared with the dimension of the structural element (fibre spacing). In this case the material can be considered as homogeneous and only macro-stresses need be taken into account. It has been shown that the maximum values of σ_x, σ_y and τ_{xy} are reached on the boundary of the hole and can be expressed in terms of the circumferential stress. The corresponding analytical solution for an anisotropic plate is well known. In the vicinity of the free hole boundary the material is in a state of uniaxial tension, in a direction making an angle θ with the fibre direction, the value of the stress being σ_θ. We apply the rupture condition (4) to a certain plane, parallel to the Y-axis. Minimizing the value of rupture stress p with respect to the angle θ, using the condition $\partial p/\partial\theta = 0$, we find the point on the boundary where deterioration starts.

The condition stated above leads to an algebraic equation for $\cos^2\theta$ which can be easily solved for any particular material. The coordinate of the point of fracture initiation corresponds to the angle $\theta_o \approx 20° \pm 5°$ depending on the elastic properties of the material. For one kind of glass fibre reinforced plastic (1:1) we have found $\theta_o = 24°$. The value of load corresponding to fracture initiation in this single point is about 3.5 times less than the ultimate load for a specimen of the same minimum cross-section having no hole. But fracture initiation at a point does not mean that the bearing capacity of the specimen is exhausted. The partial fracture changes the stress field in a fashion equivalent to the reduction of the depth of the stress concentrator. Therefore the normal stress concentration at the point $\theta = 0$ will be reduced. So, on the first stage of the rupture process longitudinal cracks starting from the points $\pm\theta_o$, $\pi \pm \theta_o$ will be formed. These cracks are not dangerous, but on the contrary they unload the dangerous point $\theta = 0$, where the tensile stress in the fibres reaches the maximum value.

To calculate the effective stress concentration factor we can consider a "shallow notch" according to Neuber's theory or a semi-elliptic notch. In each case the results are almost the same. The radius of curvature of the notch remains unchanged and the depth f can be easily calculated when θ_o is known. For real composite materials the values of the effective stress concentration factor fall in a very narrow range between 1.45 and 1.55. The experiments confirm this theoretical result with very great accuracy, if the radius of the hole is large enough. It should be mentioned that the theoretical stress concentration factor for a circular hole is about 3.5 if we consider the elastic anisotropic material as having the elastic moduli of a common glass laminated plastic. A strength calculation based on the traditional elastic stress concentration analysis is quite erroneous in this case.

Similar results have been obtained for other forms of stress concentrator. Both theoretical analyses and experiments have shown that if the matrix is not too strong, the local normal stress concentration before fracture cannot exceed a value of about 2.

For unidirectional plastics on the contrary the formation of a longitudinal crack is really dangerous and leads to the destruction of the structure. Therefore, in this case the strength criterion is defined by the value of the ultimate shear stress depending on the stressed volume, that is on the absolute dimensions and shape of the stress concentrator. Usually the real form of the notch is neglected and the stress concentration factor is expressed in terms of the ratio of the radius of curvature to the depth of the notch or the width of the plate. Size effects depend on the number of cut fibres and therefore on the ratio of the hole radius to the fibre spacing or diameter. When the hole is large enough one can use as a fracture criterion the value of the ultimate shear stress determined on an unnotched specimen. Neglecting the size effect ensures safety.

Neuber's method for calculation of the effective normal stress concentration factor has been used to estimate the value of the maximum shear stress on the notch surface. Two particular problems have been solved: the first for a very deep hyperbolic notch in a plate of finite width and the second for a shallow notch in a very broad plate. The real stress concentration factor is lower than either of them, but tends to their values in corresponding limit cases. The Neuber hypothesis had been expressed as: $K = K_1 K_2 (K_1^2 + K_2^2)^{-1/2}$.

Here K is the real shear stress concentration factor, K_1 and K_2 are factors determined by solving the first and second problems mentioned above. The experimental points in Figure 3 fit the theoretical curve rather well, the value of the ultimate shear stress being defined for the given radius of curvature. Unidirectional GFRP AG - 4S with side notches has been tested, the radius of curvature of the notch being 0.3 mm. Of course for such a small dimension of the stress concentration zone the results are only qualitative, but nevertheless the agreement is good. Practically, on the basis of these theoretical and experimental results we can state that if the notch is not too shallow or too deep, delamination occurs at the same level of maximum shear stress. In the same way it is possible to estimate the upper and lower bounds of the bearing capacity of more complicated structural elements.

If the notch is rather shallow it is possible to determine the optimum notch radius, at which the ultimate shear stress and tensile stress are reached simultaneously, i.e.

$$\tau_{max}/\tau_o = \sigma_{max}/\sigma_o$$

Here τ_{max} and σ_{max} are the maximum values of stresses on the notch surface. Solving the problem of the shallow notch, we find the following expression for the radius ρ in terms of notch depth f and ultimate stresses ratio:

$$\rho = 1/2\ f\left(0.65\ \sigma_o/\tau_o - 3\right)^2$$

It is to be noted that the lack of normal stress concentration does not mean that the shear stress concentration vanishes. Therefore, the traditional methods of strength estimation based on normal stress criteria can fail if the shear resistance is low. For example, consider a plate containing a narrow elliptic hole stretched along the major axis of the ellipse. If the ratio of the axes is large enough, normal stress concentration barely occurs while near the ends of the hole or crack the shear stresses are large enough to produce delamination of the composite.

For a rather short beam in three point bending a stress criterion of the type: $\sigma + M\tau = C$ is also valid. The application of this criterion explains the observed dependence of the interlaminar shear strength on the length to thickness ratio L/H. The resulting formula is the following:

$$\tau_o = \frac{MC}{M^2 + L^2/H^2}$$

It is well confirmed experimentally and enables the determination of the parameters M and C from experimental data for various length to thickness ratios.

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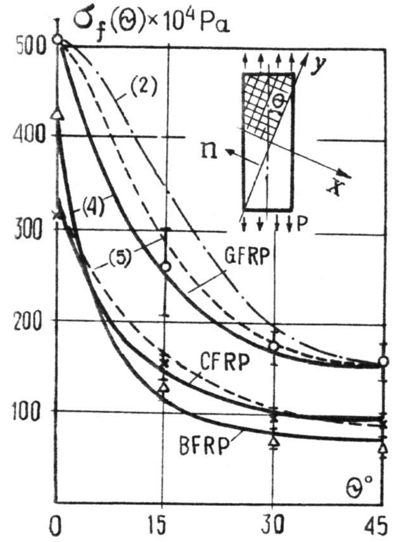

Figure 1 The Strength of Tensile Specimens of Glass Fibre Reinforced Plastic (1:1), 0-90°, Carbon Fibre Reinforced Plastic (1:1) 0-90°, Boron Fibre Reinforced Plastic (1:1) 0-90° Cut in Various Directions. Theoretical Curves Have Been Plotted According to equations (2), (4) and (5).

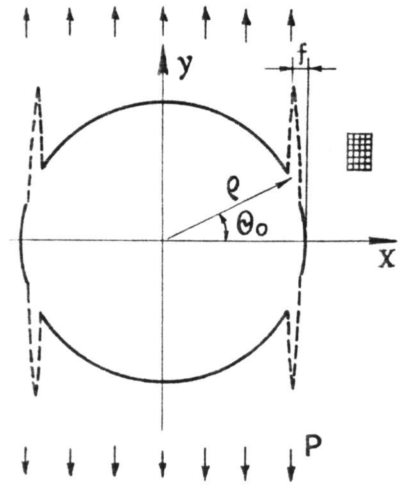

Figure 2 Longitudinal Crack Initiation Near a Circular Hole in a Fibre Reinforced Plate

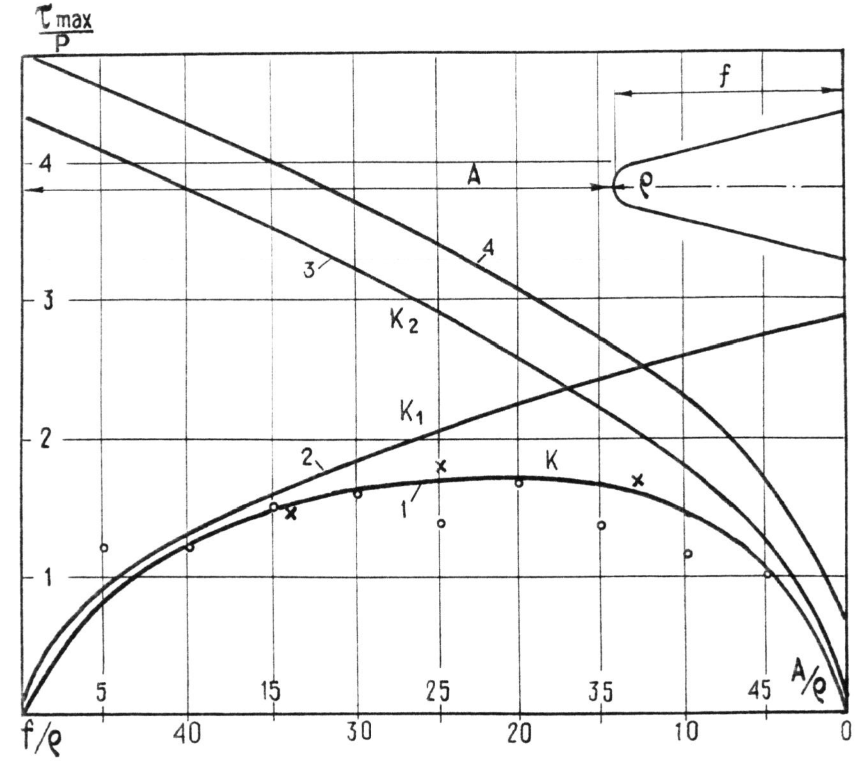

Figure 3 The Ratio of the Maximum Shear Stress on the Notch Surface to the Tensile Stress p at Infinity. (1) "Real" Stress Concentration Factor K; (2) K_1 - for Hyperbolic Notches; (3) K_2 - for Shallow Notch; (4) K_2 - Value for Semi-elliptic Notch. The Small Circles and Crosses Represent the Experimental Data for Unidirectional GFRP AG - 4S