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STABILITY CONSIDERATIONS IN THE GENERALIZED THREE DIMENSIONAL 'WORK OF FRACTURE' SPECIMEN

J. I. Bluhm*

INTRODUCTION

The determination of fracture toughness in brittle metals is generally accomplished by the mechanical testing of pre-cracked specimens. In these tests the load, at spontaneous fracture, and the associated crack length are recorded and fed into appropriate energy release rate relations to determine the critical energy release rate. This latter value is the fracture toughness. Essential to the adequacy of the test is the presumption of a sharp crack. In metallic specimens this 'crack' is obtained artificially by cyclically loading a specimen with a machined starter notch until a crack of adequate length has grown from the base of the machined (and generally relatively blunt) notch.

However, in extremely brittle materials such as ceramics for example, the techniques for introducing the sharp crack are not so simple. In the usual situation a ceramic specimen containing an initial blunt starter notch will, when loaded, sustain an inordinately high load and then fail catastrophically. Attempts to introduce a sharp crack by fatiguing are generally not successful. Either loads are so small as to not initiate or propagate a crack or attempts to increase the fatigue load merely lead again to inordinately large loads at which spontaneous fracture then occurs. If this load at which fracture actually occurs is fed into the energy release rate relations a fictitiously high fracture toughness is obtained.

One promising alternative approach which provided the motivation of the present study is based upon the use by Tattersall and Tappin [1] of a relatively simple 3-point beam bending specimen having a nominally square cross section and inclined notches such that the remaining ligament is an isosceles triangle with the apex on the tension side. Those authors suggested that that specimen (with an initially blunt machined notch) tended to behave in a stable fashion during crack initiation and extension. The area under the load-deformation curve (i.e., the work) can then be related to the fracture toughness. This specimen has become identified as the 'work of fracture' (WOF) specimen.

The initial optimism with respect to the potential of the WOF specimen has been somewhat dampened in light of the observations that its stability appeared to be dependent upon crack depth and other unspecified geometric parameters. Therefore it seemed appropriate to analytically explore the stability characteristics of such a specimen with the eventual objective of identifying and optimizing a specimen configuration which might be stable throughout the possible ranges of crack depth.

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*Army Materials & Mechanics Research Center, Watertown, MA, U.S.A.

PRIOR ANALYSIS

In an earlier paper, Bluhm [2] using a slice synthesis technique developed the essential compliance relations to a generalized form of the WOF specimen. In the present paper, the form of the shear transfer coefficient which is used to compensate for the three-dimensional effects of the specimen is more fully determined; stability criteria are formulated in dimensionless form; and these stability criteria are used in a parametric optimization study.

Figure 1 shows the generalized notch/crack specimen configuration treated in the prior analysis. Figure 2 provides additional nomenclature. Note the heavy line outlining the notch/crack front.

Use of this generalized configuration permits ease of applicability to other specimen types. The original Tattersall-Tappin specimen configuration for example can be approximated from this generalized form by letting the central span $l_2 \to 0$ (3 pt. vs. 4 pt. loading)*, setting $C_o = 0$ and $\omega = W$. On the other hand by letting C_o be arbitrary and $\omega = W$ the Simpson [3] specimen configuration is obtained. Additionally by letting $C_o = \omega < W$, the conventional 'straight through' crack specimen configuration is obtained.

In the earlier work by Bluhm, the specimen compliance λ_s of this generalized work of fracture specimen was developed. The reader is referred to that paper for the form of λ_s and the related definition of terms.

THE SHEAR TRANSFER FUNCTION, $k(\phi_1\ \omega/W)$

In the earlier representation of the shear transfer function [Bluhm 2] it was recognized that k was in fact potentially a function of both ϕ, and ω/W; nevertheless because of limited experimental data it was tentatively presumed to be a function only of ϕ. That data (for $\omega/W = 1$) is shown as the circled points in Figure 3. However subsequently data for $\omega/W = 0.4$ was obtained and is indicated by the circled crosses $\oplus$. It is immediately obvious from the data that the influence of the ratio ω/W is significant indeed. In order to provide a guide for formulating a more general form of $k = k\ (\phi_1\ \omega/W)$ additional data were then obtained at $\phi = 22.5°$ (0.39 rad). That data is shown as circled $\otimes$'s. Although some experimental error is obvious, the trends are quite clear and it was possible to describe the significant effects of both ϕ and ω/W in the following form.

$$k(\phi,\omega/W) = \begin{cases} 1 + (\omega/W)^{3.12}\left(\sum\limits_{n=1}^{4} A_n\phi^n\right) & 0 \leq \phi \leq 1 \\ 1 + (\omega/W)^{3.12}\left(\sum\limits_{n=1}^{4} A_n\right) & 1 \leq \phi \leq \tan^{-1}\left(\dfrac{W-C_o}{H}\right) \end{cases} \qquad (1)$$

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*In the previous compliance derivation by Bluhm, only conditions of 4-pt. loading are generally valid since state-of-the-art analysis of cracked specimens do not yet provide satisfactory treatment of the interaction of the crack tip with the bearing load. For 3-pt. loading with deep cracks, this interaction effect may be significant.

where the following constants are for ϕ expressed in radians.

where $A_1 = +2.263$, $A_2 = -4.744$, $A_3 = +4.699$, $A_4 = -1.774$.

The solid lines of Figure 3 represent the prediction of k using this relation. It is obvious that reasonable correlation with experiment is thus enforced.

STABILITY CONSIDERATIONS

The energy release rate G for a loaded specimen containing a crack is given by the familiar relation

$$G = \frac{1}{2} P^2\ d\lambda_t/dA \qquad (2)$$

where P is the applied load, A is the crack area and λ_t is the total system compliance, i.e. that of the testing machine λ_m as well as that of the specimen λ_s; then if for convenience the machine compliance λ_m is expressed as a factor n times the uncracked specimen compliance, one may write

$$\lambda_t = \lambda_s + \lambda_m = \lambda_s + n\ \lambda_{so} \qquad (3)$$

where λ_{so} is the compliance of the notched but uncracked specimens.

Stability of crack extension is presumed if

$$dG/dA \leq dG_{cr}/dA \qquad (4)$$

i.e. if the rate of energy release rate available for propagation is equal or less than the required energy absorption rate. In the present paper where the emphasis is principally on very brittle materials and where growth of shear lips is not a factor, it is assumed that the fracture toughness is independent of crack growth, i.e.

$$dG_{cr}/dA = 0 \qquad (5)$$

Hence, equation 4 becomes merely

$$dG/dA \leq 0 \qquad (6)$$

It is anticipated however that stability may alternatively be dictated not merely by the criteria that dG/dA be negative, i.e. equation 6, but more resolutely by the condition that dG/dA be *sufficiently* negative particularly if the initial 'crack' does not in fact simulate a theoretically sharp crack. Under this latter condition the overload to initiate crack propagation supplies a surplus of available energy which accelerates the crack after initiation. Hence only by an excessively negative value of dG/dA can one ever hope to stabilize such a specimen. Accordingly, in the present paper we assume a theoretical crack and utilize a dimensionless form of dG/dA, $\bar{S}$'s as defined later.

tend to be stable PROVIDED of course that the initial blunt starter notch does not lead to such an excessive over load that castastrophic failure occurs directly from the blunt notch.

5) Critical experimental verification of the utility of such modified WOF specimens is now called for.

ACKNOWLEDGEMENT

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2. BLUHM, J. I., Eng. Frac. Mech., 7, 1975, 593-604.
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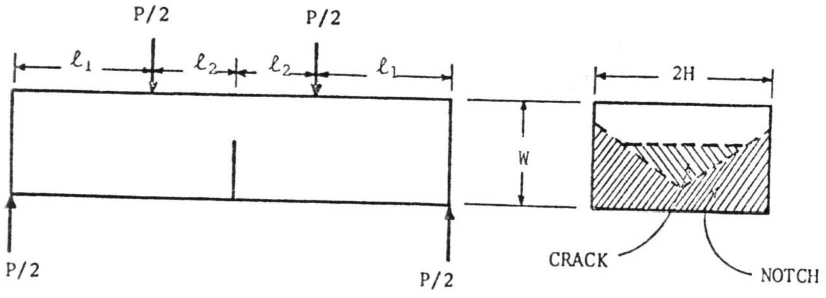

Figure 1 Generalized Notch/Crack Configuration

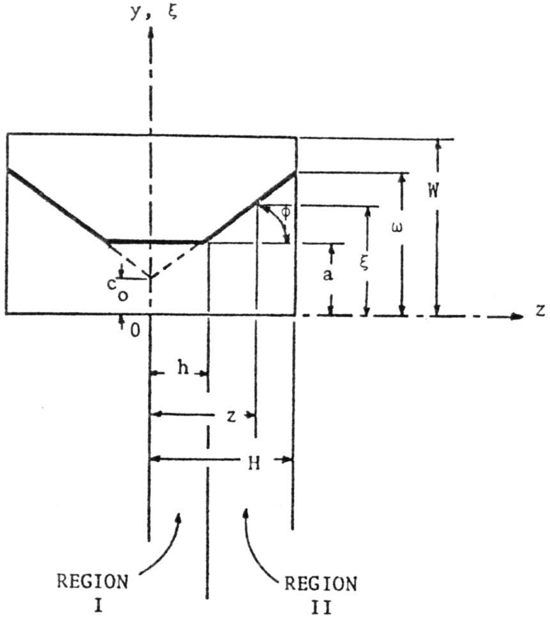

Figure 2 Nomenclature of Notch/Crack Cross Section

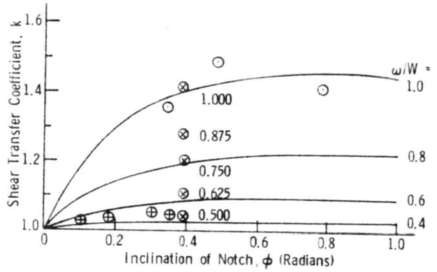

Figure 3 The Shear Transfer Coefficient k

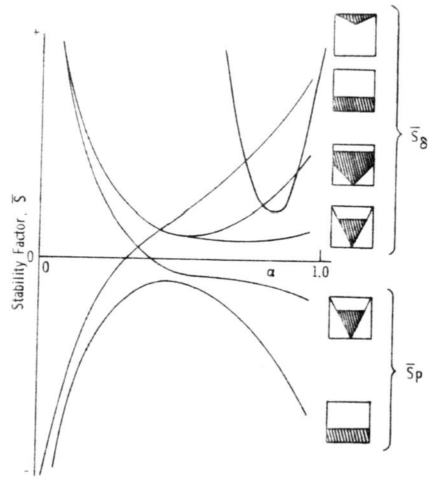

Figure 4 Schematic Showing Stability Tendency As a Function of Crack Length for Various Notch/Crack Configurations and Load Conditions

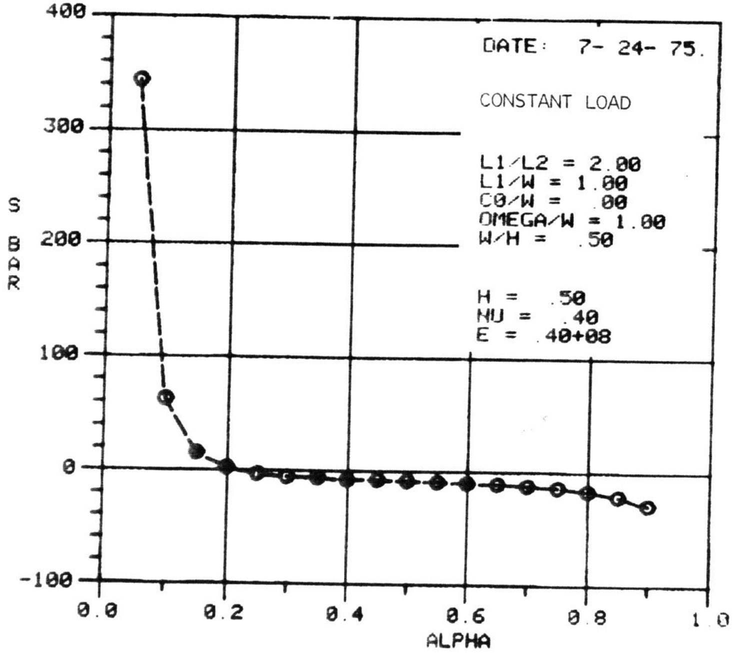

Figure 5 Typical Computer Output Showing Stability Factor vs. Crack Length (at constant load)

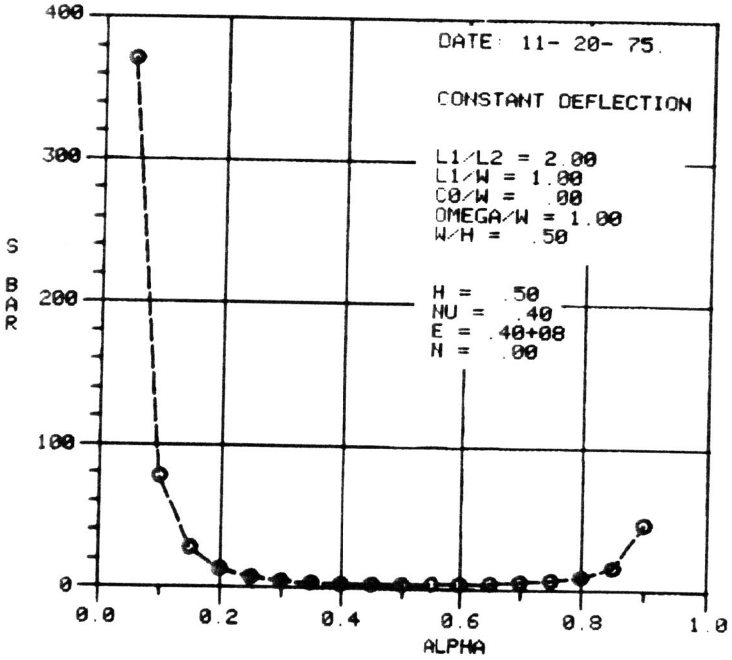

Figure 6 Typical Computer Output Showing Stability Factor vs. Crack Length (at constant deflection)

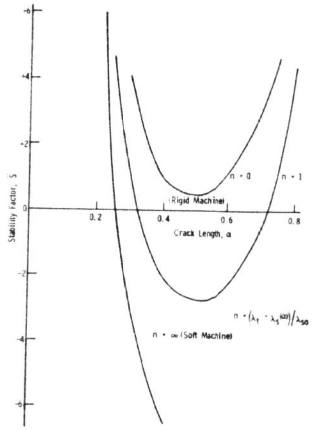

Figure 7 Influence of Machine Stiffness on Stability Factor