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SPONTANEOUS FRACTURE OF TEMPERED GLASS

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INTRODUCTION

The spontaneous cracking of thermally toughened glasses connected with the occurrence of nickel sulfide inclusions is well-known [1, 2]. The sudden fracture is known to be brought about by a volume change of the inclusion as the result of a retarded phase transformation from hexagonal NiS at high temperatures into hexagonal-rhombohedral form at low temperatures. As the latter configuration is stable at room temperature, the gradual transformation takes place with an accompanying increase in volume and a redistribution of the state of stresses in the neighbourhood of the inclusion. When such an inclusion occurs in the high tension field, localized fractures develop. In time a critical condition is reached, spontaneous fracture of tempered glass results. This paper deals with such a situation with interpretations centred on the mechanics of the behaviour of an inclusion in glass.

THERMAL INITIAL STRESSES IN TEMPERED GLASS PLATE

Consider a uniformly heated glass plate of thickness $2z_o$ in a rectangular coordinate system Oxyz is being cooled down with a net temperature differential T_o. Depending upon the rate of cooling the temperature distribution across the thickness of the glass plate may be represented by an equation:

$$T(z) = T_o\left(1-cz^n/z_o^{\,n}\right), \tag{1}$$

where c is a characteristic constant of the glass and n is an even integer. For a symmetrical cooling process, the thermal stresses on the surfaces $z = \pm z_o$ are tensile with a magnitude of $\alpha T_o E$, where α is the linear thermal coefficient of expansion and E is the modulus of elasticity of the glass. If the glass is considered to behave elastically with geometrical constraint an adjustment of the state of stress must be accomplished in order that the net resultant force as well as the resultant moment across any unit cross-section must vanish. It is evident that because of symmetry the equilibrium condition in moments is automatically satisfied. However, the distribution of the stresses in any direction perpendicular to the z-axis must be achieved by satisfying the force equilibrium condition. This permits a uniform deformation of the specimen across the thickness of the plate. In other words, the thermal induced stress distribution during cooling without considering equilibrium condition is

$$-\alpha TE = -\alpha T_o\left(1-cz^n/z_o^{\,n}\right)E. \tag{2}$$

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This must be superimposed by a uniform opposite stress, that is, tensile stress throughout the thickness $2z_o$. Because of symmetry we may consider:

$$\frac{1}{z_o}\int_0^{z_o}\left(c\alpha T_o z^n/z_o^n\right) E\,dz = c\,\frac{\alpha T_o E}{n+1}\ . \qquad (3)$$

As a result, the remaining tensile stress is at the central plane of the plate with a magnitude:

$$c\,\frac{1}{n+1}\,\alpha T_o E\ , \qquad (4)$$

and the surface compressive stress becomes:

$$-\,c\,\frac{n}{n+1}\,\alpha T_o E\ . \qquad (5)$$

On this basis it is seen that the central plane sections are under tension and near the surfaces of the glass are under compression. The exact distribution of the stresses in the direction perpendicular to the z-axis is exponential depending upon the values of c and n. Assuming c is unity, Figure 1 shows the distribution of the locked in residual stresses as a result of cooling during the tempering processes of the glass plate.

THERMAL STRESSES SURROUNDING AN INCLUSION

Now let us consider a spherical inclusion of radius r located at the central plane of the glass plate. The centre of the sphere is placed at the origin of the coordinate system. During the process of cooling the temperature T is assumed symmetrical with respect to the centre, and thus a function of the radius r of the inclusion only. On account of the symmetry there will be three non-zero stress components, the radial component $\sigma_{rr}(r)$ and two similar tangential components $\sigma_{\theta\theta}(r) = \sigma_{\phi\phi}(r)$ in spherical coordinates. These quantities are obtainable in terms of r through solving the equations of equilibrium in terms of radial displacement u [3, 4].

For thermal stresses in the surrounding medium of an inclusion consider a hollow sphere of inner radius r_i and outer radius r_o, the final results describing the stresses as a function of r and T are:

$$\sigma_{rr} = \frac{2\alpha E}{(1-\nu)r^3}\left(\frac{r^3-r_i^3}{r_o^3-r_i^3}\int_{r_i}^{r_o} Tr^2dr - \int_{r_i}^{r} Tr^2dr\right)\ , \qquad (6)$$

$$\sigma_{\theta\theta} = \sigma_{\phi\phi} = \frac{\alpha E}{(1-\nu)r^3}\left(\frac{2r^3+r_i^3}{r_o^3-r_i^3}\int_{r_i}^{r_o} Tr^2dr + \int_{r_i}^{r} Tr^2dr - r^3T\right)\ , \qquad (7)$$

where ν is Poisson ratio of the surrounding elastic medium. The stress components can be calculated if the temperature distribution function is given. In the special case in which T is constant, these equations show that all stress components are zero, and that

$$u = \alpha Tr\ . \qquad (8)$$

For the solid spherical inclusion similar results may be obtained by setting $r_i = 0$. The displacement at r_o:

$$u\left(r_o\right) = \alpha T_o r_o\ . \qquad (9)$$

If the coefficient of thermal expansion α for the inclusion is the same as that for the glass, no stress adjustment will occur. Otherwise a redistribution of the state of stress around the inclusion will take place. But the relative deformation of the glass with respect to that of the inclusion may be very small. Thus there is little or no fracture problem of glass even in regions having high tensile stresses.

In a situation when thermal fluctuation exists, there is no change in the radial stresses at either r_i or r_o. The tangential stresses do fluctuate in the same manner at r_i or r_o governed by the temperature function T. This consistutes to a thermal fatigue behaviour which may be influential in the development of minute fractures in glass around the inclusion. With the thermal expansion coefficient at room temperatures $\alpha = 8.6 \times 10^{-6}$/°C and modulus of elasticity $E = 10^7$ psi and Poisson's ratio $\nu = 0.23$, the tangential stress on the inner surface of a glass hollow sphere of inner radius 100μ and outer radius approximately 0.12 in is about 10 times that of temperature variation in °C if T is a linear function of r. Therefore corresponding to a 10°C change in temperature a tangential stress of 100 psi in tension is expected at the most. However, while this effect is not very great by itself its contribution in addition to other mechanically induced tangential stresses may be quite appreciable as will be seen later.

STRESSES AROUND AN NiS INCLUSION AND FRACTURE

In the case of a sulfide inclusion the situation is quite different. During the processes of transformation or inversion of NiS from high to low temperatures that is an associated volume increase of the inclusion. This deformation acts as a prescribed pressure around the periphery of the inclusion against the surrounding glass containing it.

Consider now a spherical glass container of inner radius r_i and outer radius r_o subjected to a uniform internal pressure p_i. The radial and tangential stresses are easily found to be:

$$\sigma_{rr}(r) = p_i r_i^3\left(r_o^3-r^3\right)/r^3\left(r_i^3-r_o^3\right)\ , \qquad (10)$$

$$\sigma_{\theta\theta}(r) = \sigma_{\phi\phi}(r) = p_i r_i^3\left(2r^3+r_o^3\right)/2r^3\left(r_o^3-r_i^3\right)\ . \qquad (11)$$

The greatest tangential stress in tension is at the inner surface:

$$\sigma_{\theta\theta}\left(r_i\right) = \sigma_{\phi\phi}\left(r_i\right) = p_i\left(2r_i^3+r_o^3\right)/2\left(r_o^3-r_i^3\right) \qquad (12)$$

Since this region is already in a high tension stress field, an additional tensile stress greater than half of p_i will be created to exceed the strength of the glass viscoelastically and thereby to develop spontaneous cracking when a critical time is reached.

AN ACTUAL EXAMPLE AND DISCUSSION

As an example a spontaneously fractured 1/4" glass plate was examined. Figure 2 shows the site midway between the plate glass surfaces where an ellipsoidal inclusion was found. Using scanning electron microscopic method, the actual inclusion was seen partially as shown in Figure 3. From x-ray critical-absorption and emission measurements the inclusion was determined to contain nickel and sulpher with many other impurities such as iron, aluminum and calcium embedding the core containing nickel and sulphur. Fine fracture lines were found to occur around the near spherical inclusion in glass metrix as shown in Figure 2. The fracture surfaces around the inclusion is shown in Figure 3. The patterns of the spontaneous fracture of this plate glass is shown in Figure 4 and Figure 5. This example was one of a number of spontaneous glass fractures which occurred months after manufacture caused by nickel sulfide inclusions located very close to the central tension region of the full tempered glass plates. The small inclusion near the corner of a fracture line was also examined by x-ray diffraction method. With copper $K\alpha$ radiation for approximately 72 hours both $\alpha Ni_{1-x}S$ where x may attain as high as 0.1 and βNiS diffraction lines were found. The initial crystallization of NiS was in α phase which partially inverted to the β phase upon cooling. This $\alpha \to \beta$ phase inversion may take place at much lower temperatures and has the unusual characteristics of volume expansion [2, 5, 6, 7, 9]. The crystal volume of αNiS is 26.99Å^3 and the same in βNiS is 28.05Å^3. This represents a 3.93% volume increase. Iron content throughout the inclusion was also detected. It is probable that the substitution of iron for nickel can be expected in αNiS but not in βNiS. However, this does not alter the property of increasing volume in case of nickel deficiency.

When a partial volume increase of the inclusion occurs, a change in volume of, say, 2% corresponds to an enlarged radius of $r = (1.02)^{1/3} r_o$ if the original radius is r_o. This means that for an inclusion of 200μ or 0.02 mm in diameter the enlarged diameter becomes 201.4μ which corresponds to an overall radial displacement of about 0.7μ. To create such a deformation in a hollow glass sphere with modulus of elasticity $E = 10^7$ psi, and Poisson's ratio $\nu = 0.23$, an internal pressure of $p_i = 11{,}382$ psi is required. This is based upon the analysis of the stresses in the inner surface where $\sigma_{rr}(r_i) = -p_i$ for a hollow sphere pressured by an inclusion and the stress strain relations of the glass. Consequently using (12) again, the maximum tangential stress is tensile and its magnitude is half of p_i on the basis of elasticity theory, which is

$$\sigma_{\theta\theta}\left(r_i\right) = \sigma_{\phi\phi}\left(r_i\right) = p_i/2 = 5{,}691 \text{ psi}$$

For a full tempered glass the internal tensile stress is approximately 10,000 psi. This together with the inclusion induced additional stress results in a high internal stress of 15,691 psi immediately adjacent to the periphery of the inclusion. As a result minute fractures are likely to be initiated viscoelastically. Without considering other factors such as stress concentration and temperature and fatigue influence, the tangential stresses will increase if the creep mechanism and anisotropy of

the glass are represented by a simple Voigt model [8]. In this case the circumferential stresses in the inner surface of the glass increases as time increases. A greater tangential stress than the internal pressure in magnitude is possible. This means that after a little more than 50% of the αNiS has inverted into βNiS, about 2% volume increase would result. The tangential stresses in tension may increase from 17,967 psi gradually to a possible value of 23,374 psi according to the analysis shown in [8]. Therefore a critical failure condition will be attained and spontaneous fracture of the whole tempered glass will eventually result. Furthermore, consider the time dependent phase transformation if all the αNiS is assumed to have inverted into βNiS a net increase in volume will be 3.93%. Corresponding to this amount of volume change, the radial displacement becomes 1.3μ for an approximate spherical inclusion of 100μ in radius. With this displacement the internal pressure $p_i = 21{,}138$ psi. This means that $\sigma_{rr}(r_i) = -21{,}138$ psi and $\sigma_{\theta\theta}(r_i) = 10{,}569$ psi will be the additional stresses in the immediate internal surface of the glass cavity. Thus the total tangential stress becomes 20,569 psi in tension whereas the radial stress is - 11,138 psi. Taking into account the viscoelastic creep kinetics and certain anisotropic considerations the variation of the total tangential stress can be from 24,797 psi initially to 34,837 psi eventually according to Figure 3 in [8]. These values are certainly excessive even for full tempered glass as far as fracture is concerned.

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REFERENCES

1. WOHLLEBEN, K., WOELK, H. and KONOPICKY, K., Glastechn. Ber. 39, 1966, 329.
2. MERKER, L., Glastechn. Ber. 47, 1974, 116.
3. BOLEY, B. A. and WEINER, J. H., "Theory of Thermal Stress", John Wiley, N.Y. and London, 1960.
4. TIMOSHENKO, S. P. and GOODIER, J. N., "Theory of Elasticity", 3rd Ed., McGraw Hill, 1970.
5. KULLERUD, G. and YUND, R. A., J. Petrol. 3, 1962, 126.
6. Mineral Soc. Amer. Short Course Notes 1, Sulfide Mineralogy, 1974.
7. STRUNZ, H., Mineralocishe Tabellen, Adad. Vol. 4th ed. 1966.
8. MOGHE, S. R. and HSIAO, C. C., J. Appl. Mech. 32, 1965, 31.
9. ZOLTAI, T., private communication.

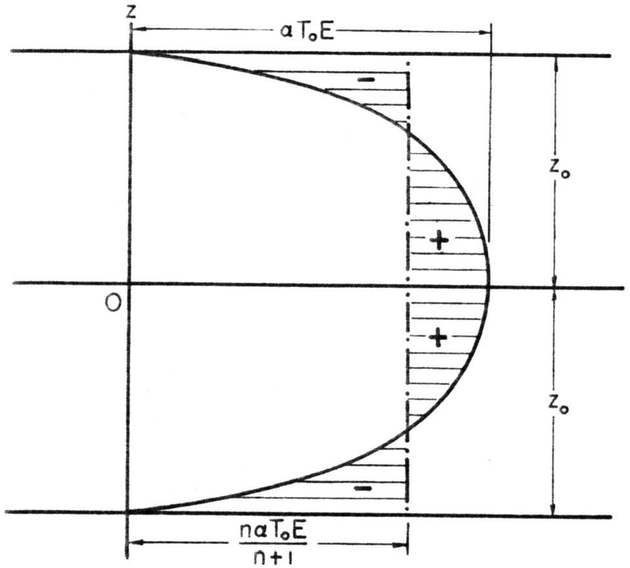

Figure 1 Distribution of Thermal Initial Stresses in a Fully Tempered Plate Glass

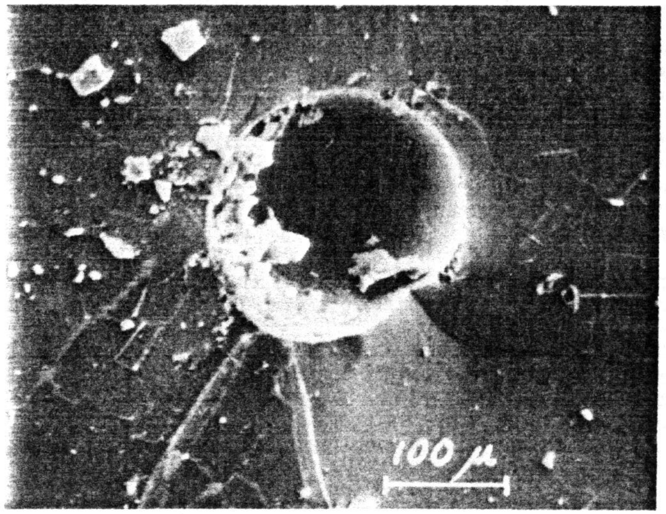

Figure 2 Scanning Electron Micrograph of Inclusion Cavity Showing Minute Fractures Resulted from Excessive Tangential Tension

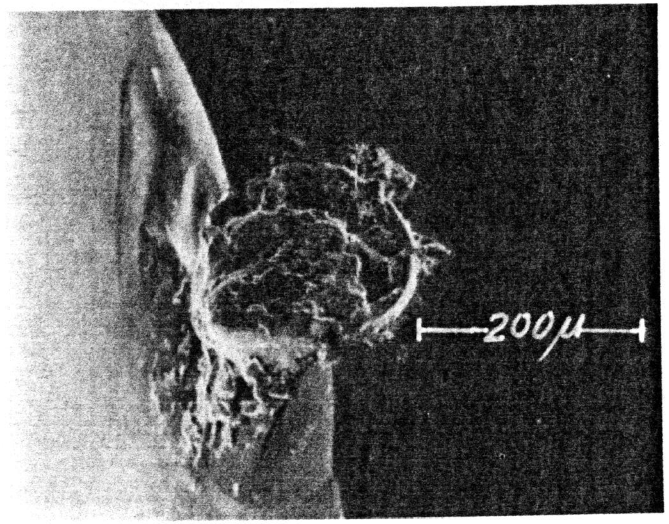

Figure 3 Scanning Electron Micrograph of Inclusion Located in the Central High Tension Plane Region and Its Adjacent Fracture Surfaces

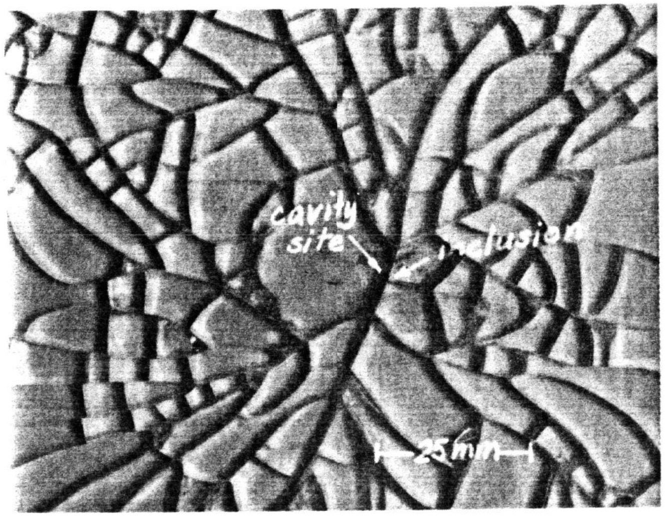

Figure 4 Spontaneous Radial Fracture Pattern of Fully Tempered 1/4" Plate Glass in the Vicinity of the Origin of the Fracture

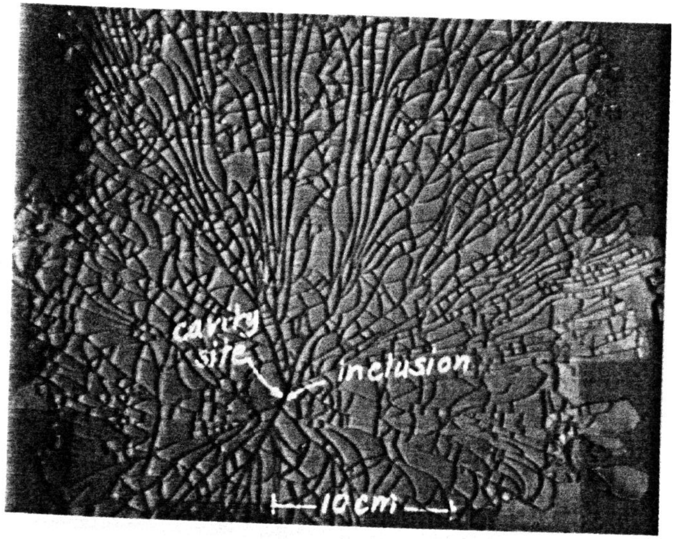

Figure 5 Spontaneous Radial Fracture Pattern of Fully Tempered 1/4" Plate Glass Originated from a Nickel Sulfide Inclusion in Tension Zone