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ON CONSTITUTIVE PROPERTIES AT SINGULAR CRACK BORDERS

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INTRODUCTION

Is the tip of a sharp notch always a crack tip? Or, more precisely, is the singular field of stress and/or strain at the border of a crack-like surface in a body always modelling a crack border, which implies that this border line is able, under specific conditions, to propagate in the body? If not, this singular line simply constitutes a localization of stress and/or a strain concentration and does not deserve attention from the fracture mechanics point of view.

Linearly elastic materials possess features that can be interpreted within a theory of fracture. The first crack problems solved were also formulated for Hookean materials and the stress intensity factors were interpreted as measures of stress at singular crack borders. During the last decades attention has turned considerably to materials capable of plastic deformation. Much effort has been made to describe fracture properties by formal mathematical generalization to infinite strain of constitutive equations which have proven useful for a description of plastic behaviour at small and moderate strains. One main problem discussed in this regard is which measure of stress intensity in plastic materials should replace the stress intensity factors for elastic materials in a fracture criterion. However, the still more important question of whether the resulting theories also fit into a sound fracture theory has not been properly considered.

In this paper, consequences for constitutive properties of materials containing cracks bounded by singular lines will be discussed. For this purpose the general thermomechanical theory of fracture in simple solids developed by Strifors [1] is employed. For convenience the discussion will be based on the fracture theory resting upon use of the linearized strain tensor, ε_{ij}, and the stress tensor, σ_{ij}, which is symmetric by definition. Both tensors are referred to the undeformed configuration according to classical fracture mechanics theory. The qualitative results of the discussion, however, remain valid also within the physically more reasonable theory that duly accounts for large strain.

SURFACE ENERGY

The observation that energy is required to fracture real bodies is accounted for by introduction of the concept of surface energy. The rate of change of surface energy at a propagating singular crack border can be derived by consideration of the energy flow through a control volume in the shape of a narrow tube surrounding the crack border and moving with the same propagation velocity. The net supply of energy through the tube in the limit when the diameter of the tube approaches zero is defined as the rate of change

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of surface energy. Recently, using a similar technique Freund [2] considered a special case of crack extension in elastic materials.

Without going into mathematical details we formulate the equation for the energy balance at a smooth singular line modelling a propagating crack border. The rate of change of surface energy per unit length of the border line at a particle instantaneously located at a point on that line is given by

$$\dot{\gamma} = \lim_{d\to 0} \oint_{c(d)} \left\{ \left(W + \frac{1}{2} \rho\, u^i{}_{,j}\, u_{i,k}\, w^j w^k \right) w^l - w^i u^k{}_{,i}\, \sigma_k{}^l - h^l \right\} n_l \, dc \, , \qquad (1)$$

where W is the internal energy per unit volume, ρ is the mass density, $\underline{u}$ is the displacement field and $\underline{h}$ is the heat flux. The integration path, c, is a closed curve surrounding the singular line in the normal plane of the line in which the outward unit normal $\underline{n}$ and the propagation velocity $\underline{w}$ of the crack border are also situated, and d denotes an upper bound for the distance between points on the integration path and the singular line.

Since heat, as well as mechanical work, may contribute to the surface energy, it is necessary to be able to specify the heat content in the surface energy in a consistent thermomechanical theory. Analogous to the case of internal energy, surface entropy is introduced as the heat specifying parameter for the surface energy. By consideration of the entropy flow through a narrow tube surrounding the crack border it can be shown that the specific production of surface entropy, β, per unit length of the crack border is given by

$$\beta = \dot{\zeta} + \lim_{d\to 0} \oint_{c(d)} \left(\frac{h^i}{\theta} - \rho\, \eta\, w^i \right) n_i \, dc \, , \qquad (2)$$

where ζ is the surface entropy, η the internal entropy, i.e., the heat specifying parameter for the internal entropy, and θ is the temperature.

The equations given should be considered as definitions of $\dot{\gamma}$ and β. Then the question arises as to whether the defining intergrals exist and are unique.

If a circular cylindrical coordinate system (r, ϕ, z) is introduced in such a way that the z-axis is directed along the tangent of the border line, it is readily shown that the integrands must be of the order of r^{-1} to give non-trivial and bounded contributions to the integrals. As regards established fracture mechanics one apparently non-trivial general conclusion follows. The existence of (1) implies that a stronger singularity than that of $r^{-1/2}$ is impossible in the displacement gradient, or, equivalently, in the strain tensor.

From here on we will consider the consequences of the requirement of uniqueness of the given definitions. Uniqueness means that the integrals must yield the same value independent of the shape of the curve surrounding the crack tip. The integrals must in a specified sense be path-independent in the singular region, i.e., in the region where the functions can be approximated by the leading terms of their series expansions. The most far-reaching consequence arising from this requirement combined with the dissipation principle is that the production of internal entropy,

(i.e., the rate of change of internal entropy that is not due to radiation or conduction of heat), must be of inferior order to r^{-2}, while the rates of change of internal energy and internal entropy are of the order of r^{-2}. When the quantities under consideration are confined to a small enough region close to the crack border it is apparent that internal entropy production can be disregarded. This means that the material must be *non-dissipative in the singular region*. In other words, the material must respond thermo-elastically in the singular region, or elastically if thermal effects are neglected at the crack border.

Viscoelastic constitutive equations with instantaneous elastic response automatically satisfy the condition of non-dissipation at singular crack borders, since such materials respond elastically at unbounded strain-rates which occur in the singular region during deformation whether the crack is extending or not.

ELASTIC-PLASTIC MATERIALS

For plastic or elastic-plastic materials the condition of non-dissipation in the singular region at crack borders implies restrictions on ordinary constitutive equations.

A first attempt to take the condition of non-dissipation into account may be made as follows. For the sake of simplicity the discussion is confined to a constitutive equation of the Prandtl-Reuss type. Accordingly, the strain tensor is written as the sum of the elastic and plastic strain tensors,

$$\varepsilon_{ij} = \varepsilon^e_{ij} + \varepsilon^p_{ij} \, . \qquad (3)$$

Here, ε^e_{ij} is given by Hooke's law, for instance, and ε^p_{ij} is given by

$$\dot{\varepsilon}^p_{ij} = \begin{cases} \lambda\, \partial f/\partial \sigma^{ij} & \text{during loading} \\ 0 & \text{during neutral loading and unloading ,} \end{cases} \qquad (4)$$

where the parameter λ depends on the yield function, f, and the current stress-rate.

By introduction of an effective plastic strain, ε^p_{eff}, being a functional of the plastic strain history, loading conditions satisfying fracture mechanics requirements can be specified. Besides the usual conditions for loading that the stress tensor shall be situated on the yield surface and λ be positive we impose the additional condition that the effective plastic strain shall be less than some limit value ε^p_{lim}.

Other theories for plastic behaviour may be restricted in a similar way. In this regard it is important that the constitutive equations employed allow for elastic strain superposed on the completed plastic strain.

The properties of the elastic-plastic constitutive theories necessary to render a fracture theory meaningful may be given a satisfactory physical basis in terms of crystal lattices and dislocations.

To make a rough sketch of a process of initiation of crack growth that is consistent with the theory discussed let us consider a body with a crystal structure. At low stress atom layers in the crystal lattices will respond by a small displacement which may be interpreted as elastic deformation. When the stress reaches the yield limit, the deformation is accomplished by dislocation movements on a large scale. If such deformation continues, unlimited plastic deformation would result and the material would consequently never fracture by formation of new boundary surfaces.

If, on the other hand, dislocation movements are prevented, only a further (elastic) displacement of lattice atoms remains possible until the separation of atom layers reaches such a magnitude that the material finally fractures. Thus, from the viewpoint of this kind of model it seems reasonable that plastic deformation must reach a limit before fracture can occur. The process of plastic flow must turn into a process of fracture.

CONCLUSIONS

Consideration of the energy supply through a control surface surrounding a singular crack border leads to a natural definition of the rate of change of surface energy at such a crack border. An analogous treatment of entropy makes it possible to describe the quality of the surface energy in accordance with classical thermodynamics. Then, the requirement of the uniqueness of the definitions together with the dissipation principle lead to the consequence that the material response in singular regions must be thermo-elastic, or elastic in isothermal theories.

From this viewpoint elastic-plastic materials are discussed and a simple elastic-plastic theory is proposed as a first attempt at meeting fracture mechanics requirements. Although no reference is made to any specific micro-mechanism of crack extension, it turns out that plastic flow and fracture are two distinct physical phenomena which may be interpreted within traditional dislocation theories.

REFERENCES

1. STRIFORS, H. C., to be published.
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