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# LOW-TEMPERATURE FATIGUE CRACK PROPAGATION

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## INTRODUCTION

Recently, there has been an increasing awareness that fatigue crack propagation (fcp) is not simply related to an invariant microplasticity law that governs crack tip blunting and resharpening. Although early fcp laws often arrived at exponents of two to four for the power law relationship [1,2]

$$\frac{da}{dN} = \alpha \Delta K^{n} \tag{1}$$

later models [3-5] allowed larger variations in n. These modifications either included cyclic strain hardening exponents, $\beta$, or strain rate sensitivity exponents, $m^*$, which took into account either work hardening or dynamical dislocation behaviour. The various terms are defined as:

$$n = \frac{\partial \ln(da/dN)}{\partial \ln \Delta K} \quad ; \quad \beta = \left.\frac{\partial \ln \sigma}{\partial \ln \varepsilon_p}\right|_{cyclic} \quad ; \quad m^* = \frac{\partial \ln \dot{\varepsilon}_p}{\partial \ln \sigma^*} \tag{2}$$

Here, da/dN is crack velocity, $\sigma$ is applied stress, $\varepsilon_p$ and $\dot{\varepsilon}_p$ are plastic strain and strain rate, and $\sigma^*$ is the thermal component of the flow stress. To the continuum mechanicist, there may be little need for further refinements since the whole range of exponents may be predicted by either varying $\beta$ or $m^*$. An extreme example of this is Yokobori's kinetic model [4] in which the power law portion of the relationship gives

$$\frac{da}{dN} \propto \Delta K^{\left\{\frac{2\beta(m^* + 1) + 1}{\beta + 1}\right\}} \tag{3}$$

Thus, since $m^*$ can range from 1 to 40 and $\beta$ from 0 to 1, the whole range of exponents on $\Delta K$ is possible.

More recently, Miller [6] and Ritchie and Knott [7] have noted that extremely high exponents on $\Delta K$ are usually associated with low fracture toughness and mixed microscopic fracture modes, e.g. intergranular, cleavage and/or ductile fatigue striations. Now, one could loosely argue that mixed ductile/brittle modes merely accompany fatigue crack propagation where values of $\beta$ or $m^*$ lead to large exponents on $\Delta K$. For example, it is known that $m^*$ increases very rapidly with a decreasing test temperature in Fe and its alloys. It is also known that mixed fracture modes result in these materials during fatigue crack propagation [8]. Thus, an

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increase in the exponent on $\Delta K$ at low temperatures could be attributed to accelerations due to cleavage or accelerations due to the rate of increase in dislocation velocity. The purpose of this paper was two-fold:

i) to ascertain whether existing models are *sufficient* to describe the slope of log(da/dN) versus log($\Delta K$) curves, and

ii) to determine what is responsible for large changes of slope in n at low temperatures.

The experimental procedure was to run tensile and instrumented compact tension tests to evaluate the parameters in equation (2) on Fe and Fe-Si alloys from R.T. down to -100°C. As such, this study was part of a larger project designed to evaluate effects of strain rate sensitivity and flow parameters on the fracture characteristics of Fe-binary alloys [9].

## RESULTS AND DISCUSSION

Results for Fe-1%Si and Fe-2.5%Si at two test temperatures are shown in Figures 1 and 2. Because of the much higher yield strengths at lower temperatures, the fcp rates are displaced to lower values. The power law exponents, as determined from least squares fits, are given in the next-to-last column in Table 1. Cursory examinations by scanning electron microscopy demonstrated that the largest slopes were associated with combined striation and cleavage fracture modes. When only ductile striations were detected, the correlation coefficient, as noted by the parentheses in Figures 1 and 2, averaged 0.964. Furthermore, the slopes were surprisingly invariant with n being given by 5.11±6%. For the mixed mode results, the average correlation coefficient was only 0.71 and the slopes were as high as 11.2. How do these results compare to existing theoretical models?

Measured values of strain hardening and strain-rate sensitivity exponents are given in the first two columns of Table 1. Since Fe and Fe-Si alloys are very strain-rate sensitive, we thought it most appropriate to compare these data to Yokobori's kinetic model [4] which takes into account m* effects. The calculated values of n from equation (3) as shown in the third column of Table 1, qualitatively give the right order of magnitude. However, the model must be incorrect in that the values are always low and there are some inconsistencies. For example, in both the Fe and Fe-1%Si data, large increases in n are predicted for tests at 233K compared to 296K. The fact is that increases in n were statistically insignificant. Thus, whereas dislocation models would predict large changes in n, no significant changes were observed where only ductile striations were present. Admittedly, static strain hardening exponents were used in these estimates. However, if anything, cyclic exponents would have been smaller with the resulting calculated values of n being further from those measured. A second point is that there was no significant predicted increase in n for either Fe or Fe-2.5% Si at the lowest temperatures where large increases in n were observed. Clearly, the strain-rate sensitivity models are inadequate for describing fcp under mixed fracture modes. Thus, in neither the all ductile nor the ductile/brittle fcp situations, does the kinetic model predict the appropriate trends in n.

The most interesting data in Table 1 are the large changes in n observed near the ductile-brittle transition temperatures for Fe(241K) and Fe-2.5%Si(236K). As a result, we attempted a modelling in terms of the cleavage events accompanying fcp.





## PROPOSED MODEL

There are two levels at which cleavage could be assessed. One could take large cleavage bursts with an associated increment, $(\Delta a)\sigma_f^*$, of gross crack extension and then model retardation effects on subsequent ductile growth. There was some indication that crack growth did occur by bursts in pure Fe at 173K. On the other hand, the Fe-2.5%Si test did not indicate burst phenomena at 233K. Still, some relatively "uniform cleavage" process was contributing to the slope of 11 observed. Because this relatively uniform growth process is more interesting and somewhat simpler to model, we decided to evaluate this. The schematic in Figure 3 depicts the approach. The assumptions are:

i) there is a stress intensity, under fatigue, which nucleates cleavage, $K_{nucl.}$;

ii) cleavage occurs within a few grains local to the crack tip, depending on the orientation of the cleavage planes;

iii) the ductile growth process catches up with the cleaved grains with the initial striations or microvoid growth in the process being larger than normal;

iv) the rate of ductile growth depends indirectly on cleavage through the fracture strain.

Considering models which depend on fracture strain, one can pick Weertman-McEvily (W-M)[10,11] or Tomkins' [12] models which show an $\varepsilon_f^{-1}$ dependence or a Rice-McClintock [13,14] model which demonstrates an $\varepsilon_f^{-2}$ dependence. Without going into the details, it is sufficient to write

$$(da/dN)_{R-M} \simeq \Delta K^4/(c_1 \sigma_{ys}^2 \varepsilon_f^2)$$

$$(da/dN)_{W-M} \simeq \Delta K^4/(c_2 \sigma_{ys}^3 \varepsilon_f) \qquad (4)$$

where $c_1$ and $c_2$ are constants which contain a microstructural unit such as grain size, d, or crack-tip radius. Prior to examination of the fracture strain dependence, it was important to check these models for the wholly ductile cases. First, the fourth power dependence does agree reasonably well with the observed values of n = 5.11±6%. Secondly, the yield strength dependence was approximately observed as indicated in Figure 4. By plotting $(1/\sigma_{ys})^2$ versus $\Delta K$ at constant crack velocity, it was found that equation $(4)_2$ represents the data best. Because these materials are strain rate sensitive, we also determined a dynamic yield strength associated with a strain rate appropriate to cycling at 30 Hz. For all temperature, yield strength and $\Delta K$ values of interest, a value of $\dot{\varepsilon}$ ranging from 0.17 to 0.23 $sec^{-1}$ was calculated at a distance, d, from the crack tip assuming a Rice and Hutchinson power-hardening material. From the strain rate sensitivity measurements, this allowed $(\sigma_{ys})_{T,\dot{\varepsilon}}$ to be calculated as listed in Table 1. As seen in Figure 4, this produced no significant difference in the overall trend of the data.

It was next suitable to evaluate the effects of cleavage on the fracture strain. The concept is an extension of Gell and Smith's [15] analysis of cleavage initiation on twist or tilt planes oriented away from the crack plane which is normal to the applied stress. It may be shown that twist cracks are more difficult to nucleate and so we have assumed that this orientation factor is controlling. In terms of the local stresses required for cleavage, one can show that the stress intensity required for cleavage on a new plane, twisted an angle $\psi$ away, is given by [15]

$$K^{Req.}_{I(\psi)} = K_I \sec^2\psi \tag{5}$$

If all grains were in perfect orientation for cleavage, then $K_I = K_{nucl.}$ would produce catastrophic fracture. As $\psi \neq 0$ for nearly all grains, most grains would not cleave even at $K_I = 1.1\ K_{nucl.}$ and non-propagating cleavage microcracks would result. We can use this result to estimate the fracture of grains that have cleaved for a given stress intensity, $K_{max} > K_{nucl.}$. If $K_{max}$ is large enough, then all twist orientations ±45° could cleave. Furthermore, as there are orthogonal sets of cleavage planes in the bcc system, this ±45° range would be sufficient to assure that all grains along the crack front could cleave, representing a cleavage fraction of one. It follows that the fraction cleaved is

$$f = \frac{4}{\pi} \sec^{-1}(K_{max}/K_{nucl.})^{1/2} \tag{6}$$

It is next possible to treat this "void fraction" effect on the fracture strain associated with a partially cleaved crack front. If one assumes a dislocation model for planar hole growth, the volume fraction and hole size are

$$f = \frac{d}{\lambda'} \; ; \; D = d(1+\varepsilon_p) \tag{7}$$

referring to Figure 5. Considering D grows to $\lambda'$ when $\varepsilon_p = \varepsilon_f$, that the plane strain value is 1/3 the tensile value and that there is some $f_o$ inherent in the material and not associated with grains cleaving, one finds

$$\varepsilon_{f_{ps}} \simeq \frac{1}{3}\left[\frac{1}{f_o + \alpha f} - 1\right] \tag{8}$$

The value of $\alpha$ is less than one indicating the greater difficulty in connecting up between tilts and twists. For convenience, we will pick $f_o$ to be 1/3 and $\alpha$ to be 2/3 since these give $\varepsilon_{f_{ps}}$ ranging from 0.67 to 0 for a fraction, f, of cleaved grains ranging from 0 to 1. Additional justification for this reduced fracture strain concept comes from El-Soudoni and Pelloux [16] who demonstrated that growth rates in 7075-T6 aluminium were improved if the void fraction of inclusions was reduced. Now this only applied to relatively large stress intensities where some void growth around inclusions could be contributing. Since our values of $\Delta K/(\sigma_{ys})T,\dot{\varepsilon}$ were actually higher than those reported by El-Soudoni and Pelloux [16], hole growth could be an important consideration. In fact, on several of the fracture surfaces, we did find evidence of microvoid coalescence mixed in with fatigue striations.

As a final step, the model is completed by combining equations (4),(6) and (8) to give





$$\left(\frac{da}{dN}\right)_{R-M} = \frac{\Delta K^4}{c_1\sigma^2_{ys}\left\{\frac{1}{1+\frac{8}{\pi}\sec^{-1}(K_{max}/K_{nucl})^{1/2}} - \frac{1}{3}\right\}^2}$$

$$\left(\frac{da}{dN}\right)_{W-M} = \frac{\Delta K^4}{c_2\sigma^3_{ys}\left\{\frac{1}{1+\frac{8}{\pi}\sec^{-1}(K_{max}/K_{nucl})^{1/2}} - \frac{1}{3}\right\}} \tag{9}$$

For comparison to the Fe-2.5%Si data at 233°K, we chose $K_{nucl.} = 23\text{MPa-m}^{1/2}$ for the initiation of cleavage and calculated $c_1$ and $c_2$ from the first data point. For the zero to max cycling used, $K_{max}$ was $\Delta K$ and so da/dN could be determined directly. The resulting agreement is reasonable as shown in Figure 6. The slope of four prior to nucleation is indicated at the left while slopes afterwards would average out to be ten or more. Two further comments are that $K_{nucl.} = 23\text{MPa-m}^{1/2}$ does produce stresses equal to $\sigma_f^*$ at a distance of 2d from the crack tip and R ratio effects can be at least qualitatively predicted.

In summary, we have shown that existing kinetic models do not correlate to strain rate sensitivity data nor do they predict the appropriate trends in $\partial \ell n(da/dN)/\partial \ell n \Delta K$, the fatigue law exponent. Exponents were found to be relatively independent of alloy content and test temperature as long as growth was in a ductile mode. Combined ductile/brittle modes gave exponents as high as 11 and a model which assesses the effects of a cleavage produced void fraction on reduced ductility and enhanced growth rate is seen to predict the trends in the data.

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Table 1 Flow and fatigue parameters

| Materials† and test Temperature | β†† | m* | $\sigma_{ys}$ (MPa) | $(\sigma_{ys})_{T,\dot{\varepsilon}}$ (MPa) | $\frac{2\beta(m^*+1)+1}{\beta+1}$ | Observed n | Fatigue mode |
|---|---|---|---|---|---|---|---|
| Fe-296K | 0.19 | 4 | 120 | 177 | 2.44 | 5.2††† | Ductile |
| Fe-233K | 0.2 | 11.5 | 235 | 300 | 4.25 | 5.43 | Ductile |
| Fe-173K | 0.105 | 18 | 356 | 440 | 4.50 | 6.0 | Ductile/ cleavage |
| Fe-1%Si-296K | 0.23 | 6 | 150 | 184 | 3.43 | 4.96 | Ductile |
| Fe-1%Si-233K | 0.256 | 8 | 183 | 229 | 4.45 | 5.27 | Ductile |
| Fe-1%Si-173K | 0.16 | 11.5 | 293 | 374 | 4.30 | 4.97 | Ductile |
| Fe-2.5%Si-296K | 0.26 | 7 | 200 | 222 | 4.10 | 4.82 | Ductile |
| Fe-2.5%Si-233K | 0.245 | 8.5 | 222 | 255 | 4.54 | 11.2 | Ductile/ Cleavage |

†alloys in at.% ††values of β are monotonic †††Ref. [8]





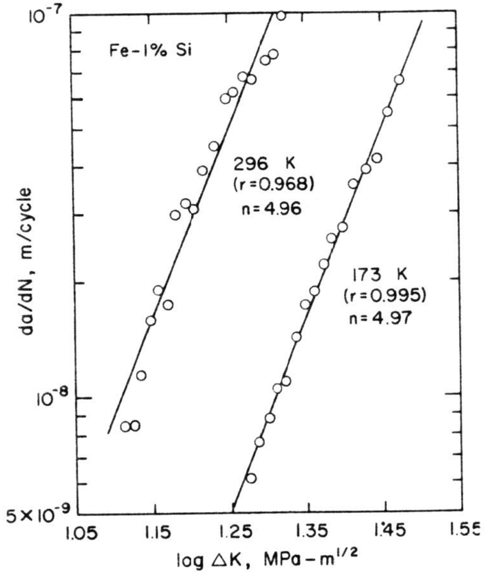


Figure 1 Fatigue crack propagation data for Fe-1%Si

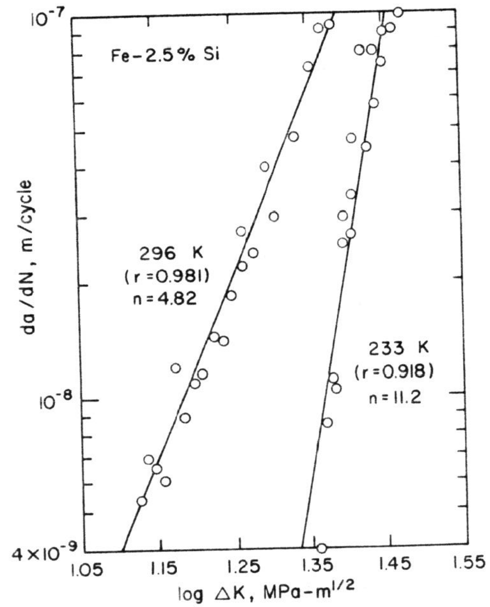


Figure 2 Fatigue crack propagation data for Fe-2.5%Si

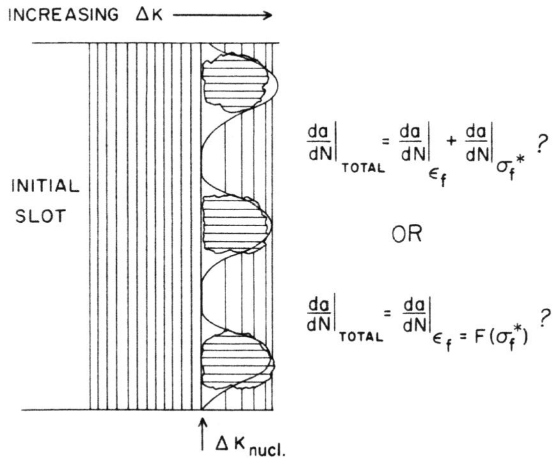


Figure 3  Schematic of ductile/cleavage fatigue crack propagation model

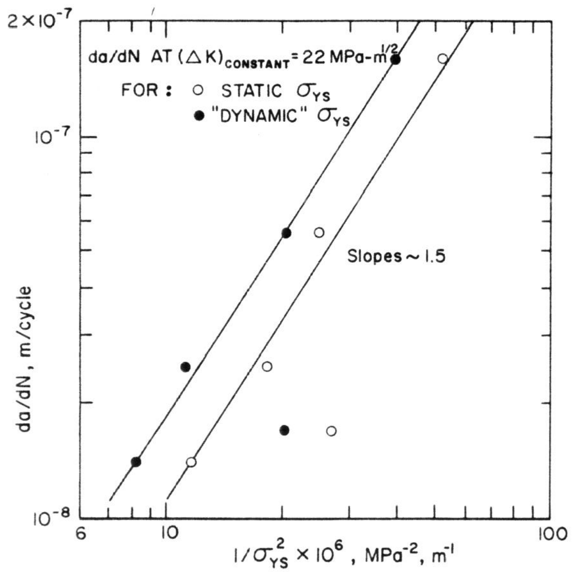


Figure 4  Yield strength dependence of crack propagation rate





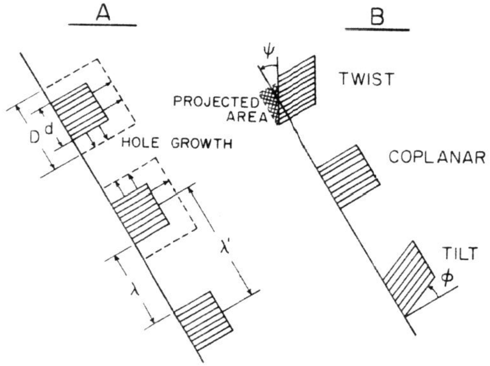


Figure 5  Fracture strain model for (A) coplanar cleavage holes (B) tilt and twist cleavage holes

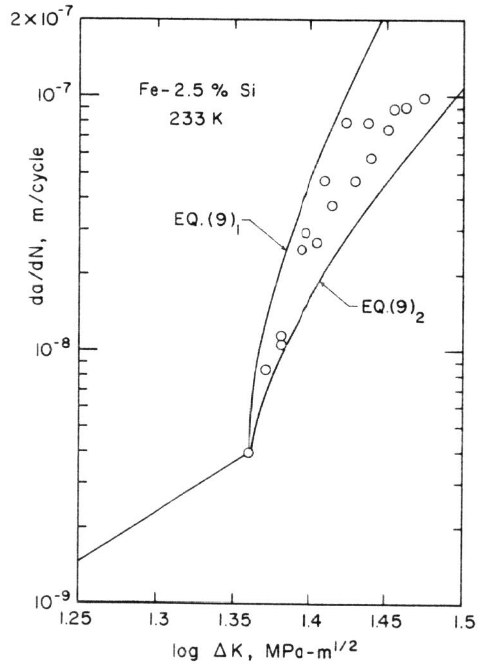


Figure 6  Comparison of ductile/cleavage fatigue model to data for Fe-2.5%Si