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INITIAL STAGES OF CRACK EXTENSION IN TIME-DEPENDENT AND/OR DUCTILE SOLIDS

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Failure of a volume element located on the prospective path of the crack front is linked to the incremental work dissipated within the process zone just prior to the collapse of this zone. If t denotes the instant at which the control volume element breaks down, then the incremental accumulation of damage occurs within the time interval $t-\delta t<\tau<t$, where the increment δt ($= \Delta/\dot{a}$) corresponds to the time used by the crack front to traverse its own process zone. Size of such hypothetical zone, over which an intensive straining occurs before the crack may advance, is characterized by the length Δ which is assumed to be a structural constant. In metals Δ is determined by metallurgical parameters (Δ is supposed here to be much smaller than the plastic zone extent, and may for example be thought of to correspond to the distance from the tip at which strongly localized deformation sets in because of tunnelling or necking).

The rate of damage accumulation is given by the product of stress restraining separation of new surfaces, S, and the rate $\dot{u}$ at which these surfaces are being separated at a certain fixed control point P. The integral of this product taken over the time interval δt represents the damage accumulated in the material element adjacent to the crack tip while it undergoes the final stages of straining preceding failure. Thereby it is shown that the incremental criterion for crack extension.

$$\int_{t-\delta t}^{t} S\left[x_p,\tau\right] \dot{u}\left[x_p,\tau\right] d\tau = \text{material property} \tag{1}$$

can be reduced to a form containing the entities well-known in linear elastic fracture mechanics, such as COD or J-integral. From equation (1) for the small scale yielding range we obtain the following non-linear differential equations of the first order governing the slow crack growth process:

(a) in terms of the tip opening displacement,

$$\Delta \frac{d\delta}{da} + \frac{\Delta}{2} \log\left(\frac{2\beta\delta}{\Delta}\right) + \delta\psi\ (\delta) = \delta_o \tag{2}$$

(b) in terms of J-integral,

$$\Delta \frac{dJ}{da} + \frac{\Delta}{2} \sigma_{ys} \log\left(\frac{2\beta}{\Delta\sigma_{ys}} J\right) + \delta\psi\ (J) = J_o \tag{3}$$

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The material constants δ_o and J_o, appearing on the right hand sides of the equations above can be related to the experimentally determined slope of either δ-curve or J-curve as follows

$$\hat{\delta}_o = (d\delta/da)_{ini}\ \Delta + \frac{\Delta}{\beta}\{\log(2\beta\delta_{ini}/\Delta) - 1\}$$

$$\hat{J}_o = (dJ/da)_{ini}\ \Delta + \frac{\sigma_{ys}\Delta}{\beta}\{\log(2\beta J_{ini}/\sigma_{ys}\Delta) - 1\} \tag{4}$$

Symbol σ_{ys} denotes the yield stress encountered at the crack front, while the coefficient β relates the extent of plastic zone R preceding the crack tip and the tip opening δ, namely $R = (\beta/2)\delta$. For the line-plasticity model and within the small scale yielding range $\beta = \pi E/4\sigma_{ys}$. The other material property introduced here is the dimensionless compliance function

$$\psi(t) = D_{creep}(t)/D(0)$$

characterizing the time-dependent behaviour of a solid (D_{creep} denotes the creep compliance as defined in rheology). The increment $\delta\psi$ corresponds to the change of $\psi(t)$ during the time interval δt; therefore

$$\delta\psi = \dot{\psi}(0)\delta t$$

Both equations (2) and (3) have been derived on the basis of line-plasticity model (although the form of the criterion (1) is suitable for a continuum mechanical approach). One should also emphasize that the validity of the results discussed here is restricted to the small scale yielding range. Two limiting cases are investigated, namely

Case I. Inviscid ductile solid in which the material moduli are time-independent, and thus $\delta\psi = 0$. Then the equations (3) and (4) reduce correspondingly

$$\frac{d\delta}{da} = \frac{1}{\beta}\log\left(\delta_{max}/\delta\right), \qquad \delta = \delta(\Delta a) \tag{5}$$

$$\frac{dJ}{da} = \frac{\sigma_{ys}}{\beta}\log\left(J_{max}/J\right), \qquad J = J(\Delta a) \tag{6}$$

These equations allow theoretical predictions of the resistance curves (either δ or J-curves) for quasi-brittle solids. The R-curves can be obtained through a numerical integration of equations (5) and (6) if the initial values of δ and J (at the onset of failure) are known. When the maximum attainable J does not differ greatly from the initial threshold ("flat" R-curve), but the foregoing equations can be integrated in a closed form, yielding these solutions.

$$\delta(\Delta a) = \delta_{max}\left\{1 - \left(1 - \frac{\delta_{ini}}{\delta_{max}}\right)\exp\left(-\frac{\Delta a}{\beta\delta_{max}}\right)\right\} \tag{5a}$$

$$J(\Delta a) = J_{max}\left\{1 - \left(1 - \frac{J_{ini}}{J_{max}}\right)\exp\left(-\frac{\sigma_{ys}\Delta a}{\beta J_{max}}\right)\right\} \tag{6a}$$

Symbol Δa is used to denote the amount of slow growth, $\Delta a = a - a_o$. Equations (5a) and (6a) give a fair correlation with the experimental data reviewed recently by Mai, Atkins and Caddell [3]

Case II. For a linear viscoelastic solid (with no plastic flow accounted for) one may show that the dominant terms in the governing equations (2) and (3) are the first and the last on the right hand side. They can be combined to yield equations of motion for a crack traversing a visco-elastic matrix, i.e.

$$(1 + \delta\psi)\delta = \delta_o$$

$$(1 + \delta\psi)J = J_o \tag{7}$$

or

$$\psi(\delta t)\delta = \delta_o$$

$$\psi(\delta t)J = J_o \tag{8}$$

These equations imply the following "equivalent" viscoelastic entities

$$J = J_{el}\ \psi(\Delta/\dot{a})$$

$$G = G_{el}\ \psi(\Delta/\dot{a}) \tag{9}$$

$$K = K_{el}\ \sqrt{\psi(\Delta/\dot{a})}$$

in where the subscript "el" emphasizes the fact that the quantity in question is obtained from the theory of elasticity. The last two forms above are identical to those suggested by the equations of motion for cracks in linear viscoelastic media, as derived by Knauss and Dietmann [1] and Wnuk [4]. From equations (9) it becomes evident that "effective" or "equivalent" energy release rate for a viscoelastic solid is a function of crack growth rate. The viscoelastic R-curves of the type G vs $\dot{a}$ or J vs $\dot{a}$ are, therefore, analogous to the resistance curves suggested for ductile solids by McClintock and Irwin [2]. An experimental verification of these equations is briefly discussed.

The second part of this work is concerned with the study of structure of an integral equation which describes path-dependent failure of an element adjacent to the tip of a crack propagating through an elastic-plastic or a viscoelastic-plastic isotropic solid capable of large deformation. In such a case the integrand of equation (1) has to be studied in a greater detail. Preliminary investigation indicates that the form

$$\delta\phi = S\left[x_p,\tau\right]\ \dot{u}\left[x_p,\tau\right]d\tau$$

is not an exact differential but a Pfaffian and thus the integral appearing in equation (1) is path-dependent. Therefore, failure of an element adjacent to the crack tip depends on the previous states of this element. In this way the dependence of the final state on the loading path (so called "history dependence") is accounted for in the theory developed on the basis of the incremental criterion of failure.

Effects of the rate of loading and time-sensitivity of the material response on fracture propagation are incorporated in the theory through the Crochet constitutive equations involving time-dependent yielding, i.e.

$$\sigma_{ys}(t) = A + B\exp(-C)$$

$$\chi = \left\{\left(\varepsilon^{v}_{ij} - \varepsilon^{e}_{ij}\right)\left(\varepsilon^{e}_{ij} - \varepsilon^{e}_{ij}\right)\right\}^{1/2}$$

$$\varepsilon_{ij} = e_{ij} + (1/3)e\delta_{ij}\ , \quad \sigma_{ij} = s_{ij} + (1/3)\sigma_{kk}\delta_{ij}$$

$$e_{ij} = e^{e}_{ij} + e^{v}_{ij}\ , \quad e = e^{e} + e^{v}\ , \quad e = \varepsilon_{kk}\ , \quad s = \sigma_{kk}$$

$$e_{ij}(\underline{x},t) = \int_{-\infty}^{t} J_1(t-\tau)\,\frac{\partial s_{ij}(\underline{x},\tau)}{\partial\tau}\,d\tau$$

$$e(\underline{x},t) = \int_{-\infty}^{t} J_2(t-\tau)\,\frac{\partial s(\underline{x},\tau)}{\partial\tau}\,d\tau$$

Here A, B, C are material constants (for example A = 55.2 MPa, B = 15.5 MPa, C = .0771 for polycarbonate) while χ is a function of the strain state. The last two equations shown above are valid for the "linear" regions in which the effective stress reduced from the multiaxial stress state according to Tresca criterion is below the yield point. Summation is implied by repeated indices; the superscripts v and e denote the viscoelastic and purely (short-time) elastic components of the strain. For strains increasing with time first of the foregoing equations assents that faster loading corresponds to a higher yield stress, while under constant stress it implies that yield occurs at a time which is longer the lower the stress. For initially elastic response under rapid loading $\varepsilon^{v}_{ij} = \varepsilon^{e}_{ij}$ and $\sigma_{ys}(0) = A + B$, while the minimum yield value is given by $\sigma_{ys}(\infty) = A$, provided $\varepsilon^{v}_{ij} - \varepsilon^{e}_{ij}$ is sufficiently large as may be the case for visco-elastic soft polymers.

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