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GREEN'S FUNCTION FOR THRU-CRACK EMANATING FROM FASTENER HOLES

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INTRODUCTION

The application of fracture mechanics to fatigue crack growth and residual strength analyses has resulted in much progress during the last decade. Yet the presence of cracks in engineering structures still poses many serious research problems which remain to be solved. One such problem is a crack emanating from an inelastic field near a fastener hole, such as produced by an interference-fit fastener or a cold-worked hole.

Fatigue cracks usually originate in the regions of high stress concentration, which exist notwithstanding careful detail-design procedures. Hardly any assembled structure is free of geometric discontinuities, such as fastener holes and access holes. Since a hole is a source of stress concentration, and since there may be many holes involved in any one structure, it can be anticipated that fatigue cracks will start at some of these holes during its service life. A review of U.S. Air Force aircraft structural failures [1] revealed that cracks emanating from fastener holes represent the most common origin of these failures.

The stress-intensity factor, which generally depends upon crack length, remote loading and structural geometry, has been employed to characterize the severity of the crack-tip stress field. To date, there has been much useful work done on the problem of determining reliable stress-intensity factors for cracks emanating from fastener holes. Almost all of these analytical determinations are based upon modifications of a solution obtained by Bowie [2] for cracks emanating from a circular hole in an infinite elastic sheet. For cracks emanating from an inelastic field near a fastener hole, the stress intensity factors could be estimated by using the weight function approach as discussed by Bueckner [3 - 6] or the reciprocal theorem proposed by Rice [7]. Both techniques require a knowledge of the unflawed stress distribution in the region of the hole. Paris et al [8] has combined these techniques with the finite-element method to develop a weight function for the single edge cracked strip.

The closed form expressions for the weight function for edge cracks [4, 9], centre cracks [10] and collinear cracks [6] in a wide panel are available. But, the closed form weight function for cracks emanating from a fastener hole is not available. Development of such a function will be very difficult, if not impossible. Therefore, the weight function for a straight crack has sometimes been used to estimate the stress-intensity factor for radial cracks emanating from a circular hole, [11 - 13]. For a large crack, where the influence of a fastener hole on the stress intensity factor is small, such an approximation gives good results. However, for the case of a small crack, say $a/r \leqq 1.0$, such an approximation could be

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significantly in error. Grandt [14] used the reciprocal theorem due to Rice [7] to develop the following equation which estimates the Mode I stress intensity factor, K_I, for cracks emanating from any type of circular fastener hole:

$$K_I = \int_\Gamma p.h \, d\Gamma = \frac{H}{K_B} \int_0^a p(x) \frac{\partial \eta}{\partial a} dx \tag{1}$$

where p is the stress vector on the boundary; h is the weight function; η is the y-component of the crack surface displacements; K_B is the Bowie solution for the stress intensity factor; and H is an appropriate elastic modulus: it is $E/1 - \nu^2$ for plane strain and E for generalized plane stress. Since the closed form expression for η as a function of the crack length a is not available, it was determined by fitting an equation to the discrete displacements computed using the finite-element method for a series of crack lengths ranging from a/r = 0.4 to a/r = 2.8. The stresses and strains computed using the conventional finite-element method may be fairly accurate. But the differentiation of an approximate expression obtained by curve fitting finite-element results may not be warranted.

Two high order singularity elements have been developed at Lockheed-Georgia. One takes only the symmetric terms in the Williams' series and, hence, is applicable only to symmetric problems ($K_{II} = 0$); the other makes use of both symmetric and antisymmetric terms and is applicable to unsymmetric or mixed mode (K_I and K_{II}) problems. The efficiency and accuracy of these elements has been demonstrated in reference [15]. In order to obtain more accurate solutions for cracks emanating from a hole, the high order singularity element for symmetric problems was used to compute the Mode I stress intensity factor for a double-radial crack emanating from an open hole and subjected to concentrated loads on and perpendicular to the crack surface. The computed stress intensity factor was used to develop the Green's function (equivalent to the nondimensional stress-intensity factor) for a double-radial crack emanating from a circular hole. In the case of mixed mode conditions, the corresponding Green's function analogy to the symmetric case can be developed from the stress intensity factor computed using the unsymmetric crack element for the same cracked hole subjected to a pair of concentrated forces (equal and opposite in direction) on and parallel to the crack surface. However, in this paper, only the symmetric problem will be considered. Once the Green's functions are available, the Mode I stress-intensity factors for cracks emanating from any type of fastener hole can be calculated from a knowledge of the unflawed stress distribution in the region of the hole.

DEVELOPMENT OF THE GREEN'S FUNCTION

Figure 1 shows the scheme of the linear superposition method. The stress intensity factor of problem 1a is equivalent to the sum of that of problems 1b and 1c. Since problem 1b is crack free, the stress-intensity factor of problem 1a is equivalent to that of problem 1c. By idealizing the stress in problem 1c as N discrete loads, $P_1, \ldots, P_N$, then the stress-intensity factor, for a given crack length a, can be computed from the following equation:

$$K(a) = \sum_{i=1}^{N} K_i = \sum_{i=1}^{N} k_i\left(x_i, a\right) P_i\left(x_i\right) \tag{2}$$

where $k_i(x_i,a)$ is the normalized stress-intensity factor due to the i^{th} load, P_i, applied at location x_i. For arbitrary distributed stress, σ, instead of discrete forces, P_i, equation (20) becomes

$$K(a) = \int_0^a k(x,a) \, . \, \sigma(x) \, dx \tag{3}$$

By defining $G = k(a/\pi)^{1/2}$ and $\xi = x/a$ and substituting them into equation (3), one obtains

$$K(a) = \sigma_o \sqrt{\pi a} \int_0^1 \bar{\sigma}(\xi) \, G(a,\xi) \, . \, d\xi \tag{4}$$

where σ_o is the uniform far-field stress and $\bar{\sigma} = \sigma/\sigma_o$ is the normalized unflawed stress distribution on the prospective crack surface.

For a straight crack subjected to two pairs of concentrated forces on the crack surface as shown in Figure 2, the corresponding Green's function, G, is

$$G\left(a, \frac{b}{a}\right) = \frac{K}{P}\left(\frac{\sigma}{\pi}\right)^{1/2} = \left[\left(\frac{a+b}{a-b}\right)^{1/2} + \left(\frac{a-b}{a+b}\right)^{1/2}\right] / \pi \tag{5}$$

The Green's function G, for a double-radial crack emanating from a circular hole and subjected to two pairs of concentrated forces on the fracture surface, as shown in Figure 3a, can be obtained from the computed stress-intensity factor using finite-element analysis with inclusion of the singularity element for various crack lengths a/r and b/a ratios as follows:

$$G\left(\frac{a}{r}, \frac{b}{a}\right) = \frac{K}{P}\sqrt{\frac{a}{\pi}} \tag{6}$$

Due to the limitation of finite element methodology, when the concentrated forces were applied close to the crack tip, say b/a > 0.9, the corresponding Green's function was obtained using the central crack solution by idealizing the hole as a portion of a straight crack as shown in Figure 3b. The Green's function corresponding to this case is

$$G\left(\frac{a}{r}, \frac{b}{a}\right) = \left[\frac{4\left(1 + \frac{r}{a}\right)}{\left(1 - \frac{b}{a}\right)\left(1 + \frac{b}{a} + \frac{2r}{a}\right)}\right]^{1/2} / \pi \tag{6a}$$

The computed Green's functions were then tabulated as a function of a/r and b/a. For any a/r ratio different from those tabulated values, an interpolation or extrapolation technique was used to obtain the corresponding Green's functions.

With a knowledge of the Green's functions, G, and the stress, σ, on the prospective crack surface with the crack absent, one can compute from equation (4) the corresponding stress-intensity factor for any radial crack from a hole.

When crack face overlapping occurs or the applied force P_i is in compression, the computed K_i in equation (2) will become negative. Physically, it means the crack surfaces are closed and react against each other. Occurrences of such cases are illustrated in examples 3 and 4, where the computed negative K_i were set equal to zero.

For a case where there is only one crack emanating from a hole, instead of redeveloping the associated Green's function, it was found that the following equation will give a good estimation of the stress-intensity factor:

$$K_I \Big|_{\text{one crack}} = \frac{B_1}{B_2} K_I \Big|_{\text{two cracks}} \qquad (7)$$

where B_1 and B_2 are Bowie's factors for single and double cracks, respectively.

EXAMPLE PROBLEMS

1. Open Holes

To check the validity and accuracy of the present solution, Bowie's [2] solution for a double radial crack emanating from an open hole in an infinite plate was employed. By approximating the unflawed stress distribution as

$$\sigma = \sigma_o \left[1 + \frac{1}{2}\left(\frac{r}{r+x}\right)^2 + \frac{3}{2}\left(\frac{r}{r+x}\right)^4 \right] \qquad (8)$$

the computed non-dimensional stress-intensity factors using equation (5) for a crack emanating from an open hole subjected to uniaxial and biaxial uniform far-field loading are presented in Figure 4. The corresponding Bowie's solutions are also included in the figure. As can be seen, the current results are within 2 percent of Bowie.

2. Neat-Fit Hole with Fastener Load Transfer

The normalized unflawed tangential stress distributions along the plane perpendicular to the load-line in the hole of a 7075-T6 aluminum plate fitted with a Ti-6Aℓ-4V titanium fastener are given in Figure 5 for various percentages of fastener load transfer. The non-dimensional stress-intensity factors computed using those unflawed stresses and equation (4) for a double crack emanating from the neat-fit hole are presented in Figure 6. From this figure, one can see that the stress-intensity factor for a neat-fit hole without fastener load transfer is lower than that of an open hole (shown as dotted line). However, when the amount of fastener load transfer increases the stress-intensity factor increases rapidly, especially for short crack lengths. The computed non-dimensional stress-intensity factor at the edge of the hole is approximately equal to 1.12 times the normalized unflawed stress at that location. This agrees very well with the edge crack solution. When the crack length is larger than 2½ times the hole radius, the effect of fastener load transfer becomes negligible.

3. Interference-Fit Fastener Holes

The Green's function approach is also used to compute the stress-intensity factor for a double radial crack emanating from an interference-fit fastener hole. Figure 7 shows the unflawed stress distributions for an aluminum plate with a steel interference-fit fastener caused by 0.010 centimeters interference and subsequent edge loading and unloading [16]. The computed stress-intensity factors are shown in Figure 8. From this figure, one sees that for $a/r < 0.5$, when the far-field loading (172 MN/m^2) is removed, the computed K is less than zero. Physically, it means that the fracture surfaces are completely closed and compress each other. The effective stress-intensity factor range equals the difference between curves 2 and 3. For a similar plate with an open hole subjected to 172 MN/m^2 far-field loading, the corresponding K (also ΔK) is plotted in the same figure as dotted lines for comparison purposes. For small crack lengths, the computed ΔK is much smaller for an interference-fit fastener hole than an open hole. This explains why the crack emanating from an interference-fit fastener hole grows much slower than the corresponding crack in an open hole when the crack length is small. When the crack length is longer than 3 times the radius of fastener hole, the growth rates are about the same, since the effective stress-intensity factor ranges are about the same. This indicates that the influence of the interference-fit fastener is negligible when $a/r > 3$.

4. Cold-Worked Holes

Figure 9 shows the unflawed stress distributions in the region of a 4.4% cold-worked hole in a 7075-T6 plate caused by 110 MN/m^2 edge loading and subsequent unloading [17]. After the edge loading is removed, a residual compressive tangential stress remains at the edge of the hole ($a/r < 1$). The computed stress intensity factors using the current approach is presented in Figure 10 as dotted lines. Curve A is the computed K_{max} corresponding to 110 MN/m^2 edge loading while Curve B is the stress-intensity factor range $K_{max} - K_{min}$. K_{min} was computed using the unflawed stress corresponding to 5.5 MN/m^2 edge loading. For the same level of cold-working (4.4%) and edge loadings ($\sigma_{max} = 110\ MN/m^2$, $\sigma_{min} = 5.5\ MN/m^2$), the stress-intensity factors obtained from crack growth tests reported in references [18] and [19] and the one predicted using the linear superposition method [18] are also included in the figure. As can be seen, the current analysis gives an excellent correlation with the experimental data.

CONCLUSIONS

The Green's function for a double-radial crack emanating from a fastener hole was developed from the computed stress-intensity factor for such a crack loaded with concentrated forces by using the finite element method with the inclusion of a high-order singularity element. The stress-intensity factors for cracks emanating from any type of fastener hole were able to be computed from a knowledge of the unflawed stress distributions in the region of the hole and the developed Green's function. Stress-intensity factors computed using this approach agree very well with known solutions for open holes and neat-fit holes and correlate excellently with data generated using cold-worked hole specimens. The approach can also be used to estimate the stress-intensity factors for cracks emanating from interference-fit fastener holes.

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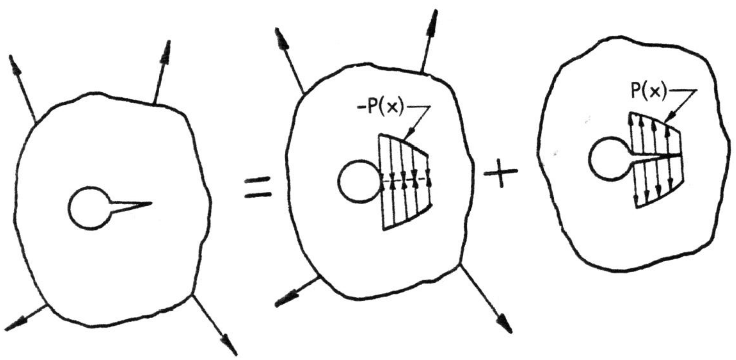


(a) (b) (c)

Figure 1 Schematic of Linear Superposition Method

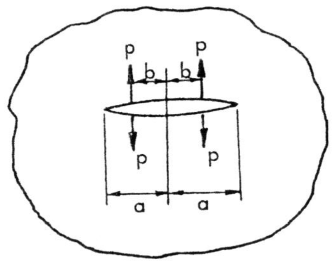
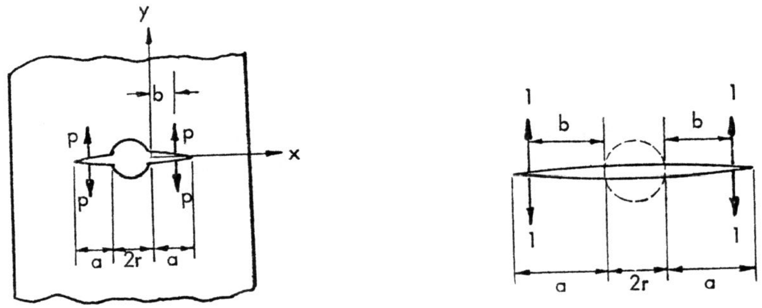


Figure 2



(a) (b)

Figure 3

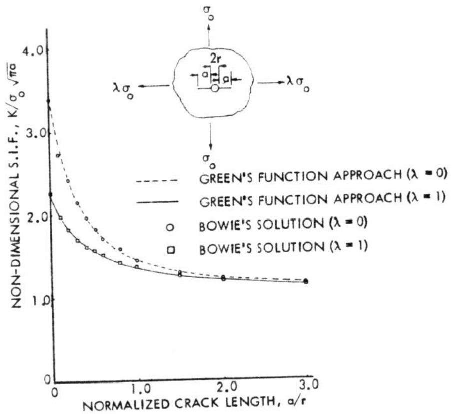


Figure 4 Normalized Stress-Intensity Factors for a Double-Crack Emanating from an Open Hole

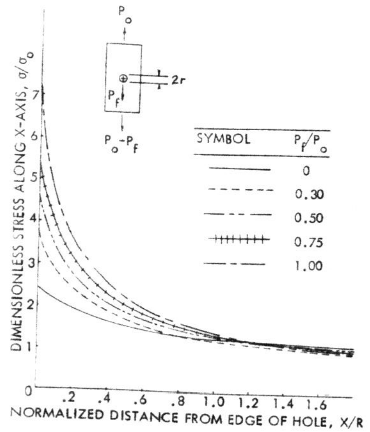


Figure 5 Normalized Unflawed Stress Distributions Along x-Axis in the Plate with Various Amounts of Fastener Load Transfer

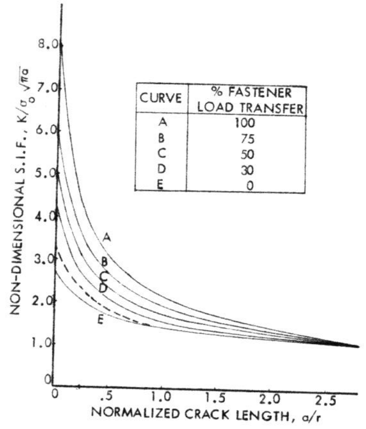


Figure 6 Normalized Stress Intensity Factors for a Double-Crack Emanating from Neat-Fit Hole with Fraction of Fastener Load Transfer

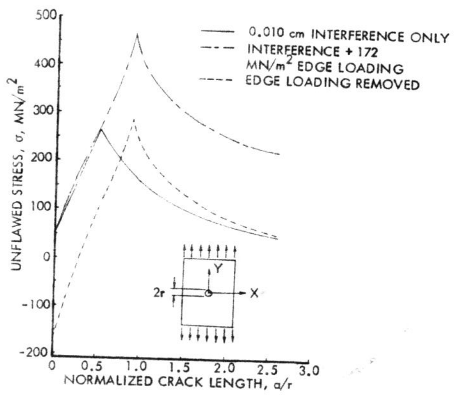


Figure 7 Stress σ_Y at Y = 0 in an Aluminum Plate with a Steel Interface Fit Fastener Caused by Interference, Edge Loading and Unloading

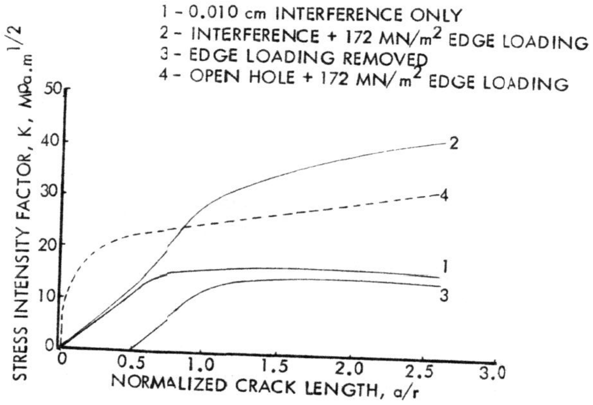


Figure 8 Stress-Intensity Factors for a Double-Crack Emanating from an Interference-Fit Fastener Hole

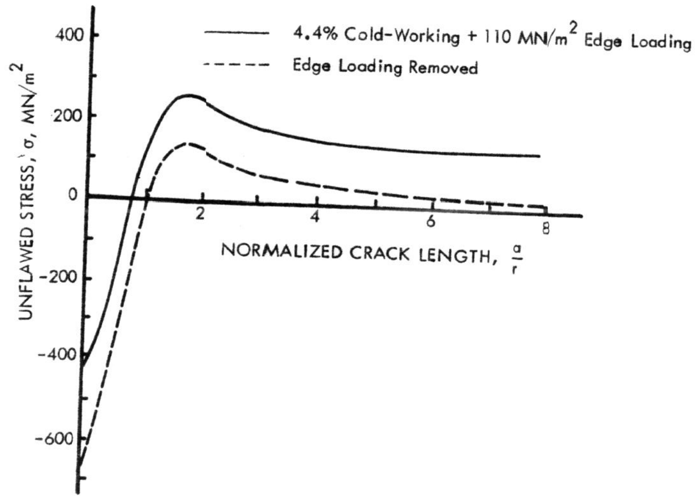


Figure 9 Stress at the Region of 4.4% Cold-Worked Hole in 7075-T6 Aluminum Plate Caused by 110 MN/m^2 Edge Loading and Subsequent Unloading

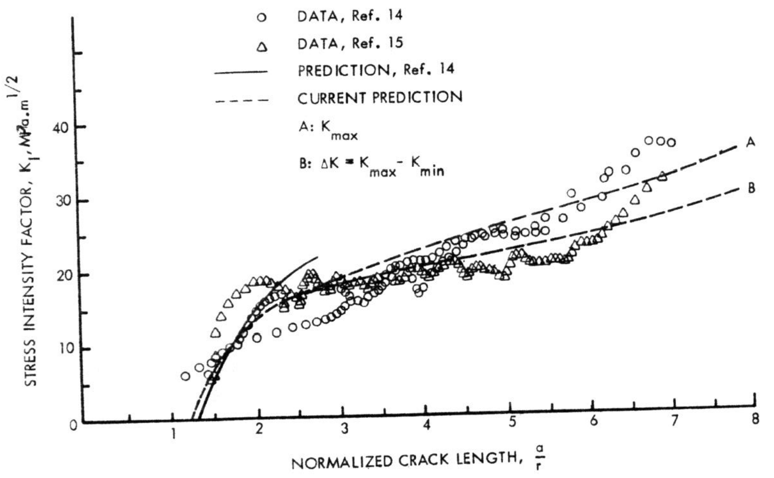


Figure 10 Stress-Intensity Factors for a Single-Crack Emanating from Cold-Worked Hole