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FRACTURE STRENGTH OF A NOTCHED BEAM WHEN A SMALL FATIGUE CRACK EMANATES FROM THE NOTCH ROOT

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INTRODUCTION

There are many service situations in which catastrophic failure can occur when a small fatigue crack is present ahead of sharp corners in structural parts. Cracks ahead of welded joints, keyways, fillets, undercuts, roots of turbine blades and gear teeth are a few such examples. Under dynamic service loading conditions such failures are often seen to occur due to the initiation of a small fatigue crack ahead of these stress raisers. The initiation of this type of fatigue crack can be considered in two stages. Firstly, a definite number of cycles is required to fatigue harden the notch root, and then secondly, a very small visible crack is initiated from the curved boundary of the notch root. Finally, this crack propagates under the application of stress field at the vicinity of the notch geometry until it becomes so large that it can be treated by standard linear elastic fracture mechanics. As soon as a small crack in the order of a few grains starts from a notch root surface, the stress field as well as the plasticity spread in the region alters drastically [1]. To study crack propagation laws or to calculate fracture instability load it is important to know the stress intensity factors for such cracks [2]. When the mechanism for ductile fracture initiation from a notch groove surface is studied [3] or the stress corrosion cracking behaviour of a material in a notched beam is to be tested, it becomes necessary to understand the influence of notch geometry on stress intensity factors for such short cracks. Another aspect which is of practical interest is at what stage of the crack growth stress intensity factor for the crack will be totally independent of the notch geometry, and will only be governed by the depth of the notch together with the crack length [4].

The present work is primarily concerned with the fracture behaviour of small, fatigue cracked charpy type notched beams in three point bending, and thereby demonstrating the physical meaning of a limiting crack length for a particular notch geometry. Apparent crack toughness in mode-one for three point bending load is determined on 0.33% carbon steel, 0.68% carbon steel and a 14W-3Cr-0.25V-0.60C high speed steel. The validity of plane strain crack toughness tests [5] are also studied for this geometry.

DETERMINATION OF STRESS INTENSITY FACTOR

Neuber [6] investigated the theory of notch stresses using stress functions. Williams [7] and Sih and Rice [8] derived expressions for calculating the stress field around a crack tip,

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$$\sigma_x = (K_I/\sqrt{2\pi r})\ [1 - \sin\theta/2\ \sin 3\theta/2]\ \cos\theta/2 + \ldots$$

$$\sigma_y = (K_I/\sqrt{2\pi r})\ [1 + \sin\theta/2\ \sin 3\theta/2]\ \cos\theta/2 + \ldots$$

$$\tau_{xy} = (K_I/\sqrt{2\pi r})\ [\sin\theta/2\ \cos\theta/2\ \cos 3\theta/2] + \ldots \qquad (1)$$

If a fine saw cut with a very small root radius is produced in a photoelastic sensitive material, then the strain field at any point r, θ around this slit can be found by observing iso-strain fringe patterns [9, 10]. The stress intensity factor of this fine slit, which can be regarded as a crack, is calculated from equation (1) [11, 12]. Alternatively, if a thin birefringent photoelastic sheet is properly coated on a cracked metal specimen, then it would be possible to measure elastic-plastic strain field near a crack by reflection polariscope [13, 14]. In terms of fringe constant f and fringe order N, the stress intensity factor can be expressed as

$$K_I = 2\ \sqrt{2\pi}\ Ef\ N\sqrt{r}]/[2(1+\nu)\ \sin\theta] \qquad (2)$$

For Ps-3B coating plastics, fringe order becomes

$$N = 0.03373K_I\ \sin\theta/\sqrt{r} \qquad \mathrm{MPa}.\sqrt{m}/\sqrt{m} \qquad (3)$$

Equation (3) suggests an experimental method of determining stress intensity factor.

Newman [15] developed an improved boundary collocation technique to find out theoretically the stress intensity factors for cracks ahead of cut-outs and holes under elastic situation. Recently, Mubeen [16] used birefringent photoelastic coating method, to determine the stress intensity factor of a fatigue crack ahead of machined notch in three-point bending.

Figure 1 shows a typical isochromatic fringe pattern observed during the photoelastic coating test on the surface of a fatigue cracked mild steel specimen under three point bending. Figure 2 describes the nature of stress intensity factors for different fatigue crack lengths ahead of 0.25 mm, 0.50 mm, 0.75 mm, 1.00 mm, 1.25 mm and 1.50 mm notch root radii.

FRACTURE STRESS

Fracture tests were conducted on pre-cracked notched specimens of 0.33% carbon steel, 0.68% carbon steel and high speed steel to study the effects of notch root radius and fatigue crack lengths on instability load. During fatigue cracking the following ΔK values were maintained: (a) for 0.33% carbon steel, $\Delta K = K_{max} - K_{min} = 18.6 - 6.2 = 12.4$ MPa.√m, (b) for 0.68% carbon steel, $\Delta K = 17.1 - 11.5 = 5.6$ MPa.√m and (c) for high speed steel, $\Delta K = 16.4 - 5.6 = 10.8$ MPa.√m.

Figures 3, 4 and 5 show the variation of nominal fracture stress with crack lengths for 0.33% carbon steel, 0.68% carbon steel and a high speed steel respectively. Apparent fracture toughness K_Q was calculated from the maximum linear load using:

$$K_Q = \sigma_f\ Y_{ASTM}\sqrt{a} \qquad (4)$$

where a is the fatigue crack length $\bar{a}$ plus notch depth d_o, and Y_{ASTM} is the calibration factor recommended by A.S.T.M. [5]. Figures 6 and 7 describe the nature of apparent crack toughness variation against fatigue crack lengths for different notch root radii. Finally, K_{IC} values were measured for these three materials as per A.S.T.M. [5].

In order to check the validity of these tests on thickness effect, a large number of pre-cracked notched specimens with different thicknesses were tested in each material. Fractured surfaces were then examined for any excessive side flow around the crack tip. It was observed that for a thickness of 24 mm in 0.68% carbon steel and 12.5 mm, in high speed steel no side flow around crack tip was noticed. However, in the case of 0.33% carbon steel, even for 25 mm thickness plate, a slight contraction in thickness occurred, especially for small fatigue crack lengths.

DISCUSSION

Tests on three different materials confirm that the nominal fracture stress approaches theoretical value predicted by A.S.T.M. as the length of fatigue crack is increased. This is observed for all values of notch root radii. Apparent crack toughness K_Q also approaches a constant value beyond a certain crack length for a given notch root radius. This asymptotic constant value can be regarded as K_{IC} of the material. For 0.25 mm notch root radius specimens, the effect of notch geometry vanishes when the fatigue crack length is greater than 1.2 mm in 0.33% carbon steel, 1.0 mm in 0.68% carbon steel and 0.75 mm for high speed steel. If the notch root radius is increased from 0.25 mm to 0.50 mm, keeping all other specimen dimensions the same (including the depth of notch), then the influence of notch geometry does not exist beyond a crack length of 1.6 mm for 0.33% carbon steel, 1.3 mm for 0.68% carbon steel and 0.85 mm for high speed steel. A similar effect was noticed for other notch root radii. Hence, it may be concluded that a tougher material would always give a larger influence zone for the same notch root radius. In effect, the influence zone will not only be a function of notch root radius, but also depends on plastic zone size at crack tip.

Experimental observation shows [1, 16, 20] for short cracked specimens under three point bending, plasticity spread near a crack tip region is fairly large. Non-linearity in load-displacement diagram before unstable fracture is normally due to either a slow crack growth or excessive plastic deformation ahead of the crack tip region or plastic zone boundary touching back to the free surface [1]. In an actual situation, a combination of all these can occur. Non-linearity is more in 0.33% carbon steel compared to other materials. The degree of non-linearity increases as the crack length decreases. Thus, for a material like 0.33% carbon steel with short fatigue cracks, a simple K_{IC} approach may not be sufficient to measure the strain-energy release rate at the onset of fracture. If the plasticity spread is small compared to the crack length, then either McClintock-Irwin's method [17] of taking recorrected crack length or A.S.T.M. recommended 5% secant shift procedure [18] can be used to calculate the toughness of a material. Considering post-yielding near the tip of a short crack, the increase in specimen compliance could be appreciable at high stress levels [19 - 21]. No simple method can give an accurate result for such cases. However, Rice's [22] J-integral method of calculating the toughness of a material can be used successfully which may be determined by experiments [23 - 25]. Table 1 gives the comparative values of toughness for 0.33% carbon steel calculated by different methods.

CONCLUSIONS

Stress intensity factor for a fatigue crack emanating from a notch groove surface under three point bending has been measured by photoelastic coating on mild steel. For a charpy type notch in mild steel under three point bending load, stress intensity factor asymptotically approaches 93% of the A.S.T.M. value for an ordinary long crack emanating from top flat free surface. Based on the maximum linear load during fracture, an apparent crack toughness is measured for this kind of geometry. Apparent toughnesses in 0.33% carbon steel, 0.68% carbon steel and high speed steel show a good agreement with predicted values by photo-elastic tests. Fracture tests on carbon steel and tool steel materials show, there exists a limiting crack length for a given notch root radius beyond which apparent crack toughness becomes equal to K_{IC} of the material. The instability load for a crack larger than this limiting crack length can be predicted from standard A.S.T.M. method, by assuming the total crack length to be equal to notch depth plus fatigue crack length. The limiting crack length is a function of notch geometry, more particularly of notch root radius. This limiting crack length depends on the size of plastic zone at crack tip as well. A non-linearity in load displacement diagram is noticed when the fatigue crack is small. This is more pronounced in 0.33% carbon steel than 0.68% carbon steel. For such cases the toughness of a material is better determined by Rice's J-integral energy method rather than standard K_{IC} approach.

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Table 1 Comparative Values of Toughness Calculated by Different Methods (Material: 0.33% Carbon Steel)

Fatigue crack length ahead of notch $\bar{a}$ mm	Total crack length from top flat free surface a mm	Maximum linear load kN	Differential area under P/B vs u record for crack lengths a and (a + Δa) under constant displacement control N	Total area under the graph of P/B^2 vs C.O.D./B for a crack length, a MN/m²	Area under linear portion of P/B^2 vs C.O.D./B diagram for a crack length, a MN/m²	Apparent crack toughness based on maximum linear load, K_Q MPa.$\sqrt{m}$	Toughness calculated by equi-valent energy method, (Ref. [24]) MPa.$\sqrt{m}$	"J" integral energy value for crack length a kJ/m²
0.524	4.724	15.8	-	0.1765	0.0627	40.9	68.7	-
0.575	4.775	15.5	7.26	0.1959	0.0618	40.5	72.6	142
0.625	4.826	15.1	7.35	0.1961	0.0597	40.0	72.2	147
0.670	4.870	14.8	6.13	0.2197	0.0608	39.8	75.4	136
0.820	5.020	14.1	18.61	0.2285	0.0557	38.7	78.7	124
1.080	5.280	13.7	30.40	0.2158	0.0538	38.0	76.0	117

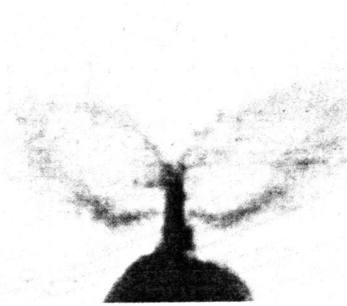

Figure 1 A typical fringe pattern in mild steel around a crack of 0.9 mm length ahead of 1.5 mm root radius notch. Applied load = σ_{Nom}/σ_y = 0.74. (Fatigue pre-cracking at constant $\Delta K = K_{max} - K_{min}$ = 21.7 - 7.8 = 13.9 MPa.$\sqrt{m}$).

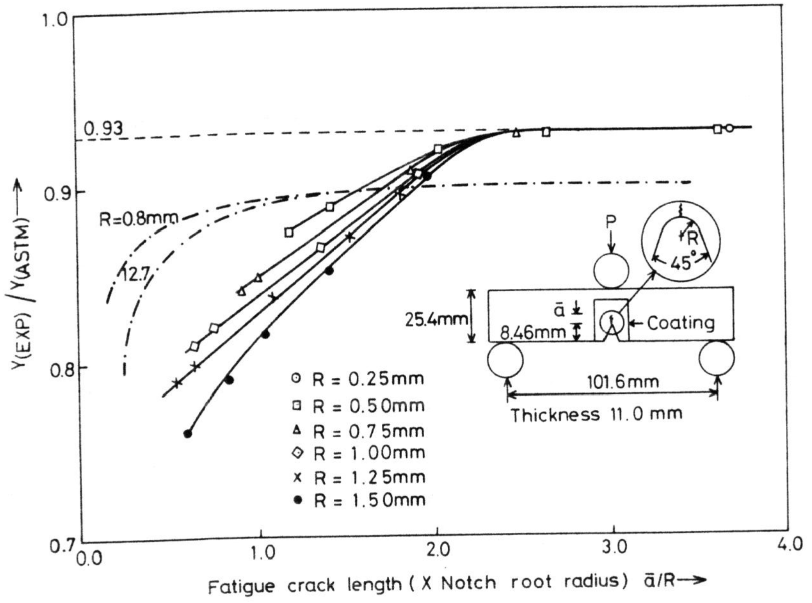

Figure 2 The nature of variation for Y_{EXP}/Y_{ASTM} against the ratio of fatigue crack length to notch root-radius. Specimen material: mild steel (0.20C, 0.7 Mn, 0.20 Si, 0.10 S and 0.04 P) σ_y = 294 MPa, σ_{ult} = 530 MPa and 39% maximum elongation in 20 mm gauge length. Fatigue pre-cracking at constant $\Delta K = K_{max} - K_{min}$ = 21.7 - 7.8 = 13.9 MPa.$\sqrt{m}$ ——— shows experimental results from photoelastic coating tests on mild steel. — · — · — gives a comparison with the results obtained by FEM solution in Reference [2].

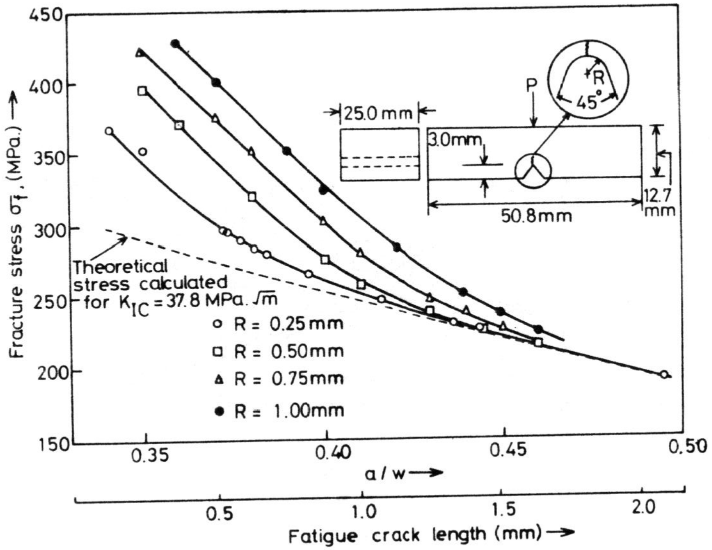

Figure 3 Nominal fracture stress in 0.33% carbon steel as a function of fatigue crack length ahead of machined notch. Composition: C 0.33, Si 0.29, Mn 0.6, S 0.04, P 0.03, σ_y = 328 MPa and σ_{ult}= 573 MPa, A = B.W

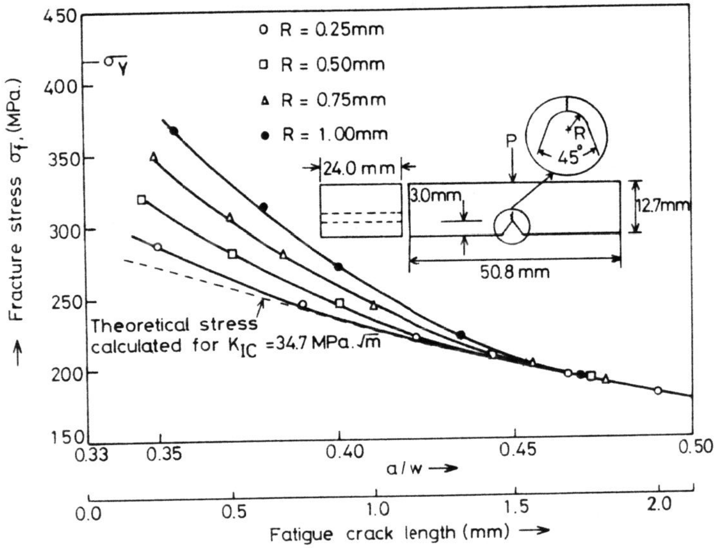

Figure 4 Nominal fracture stress in 0.68% carbon steel as a function of fatigue crack length ahead of machined notch. Composition: C 0.68, Si 0.33, Mn 0.65, S 0.04, P 0.04 - σ_y = 442 MPa and σ_{ult} = 765 MPa, A = B.W

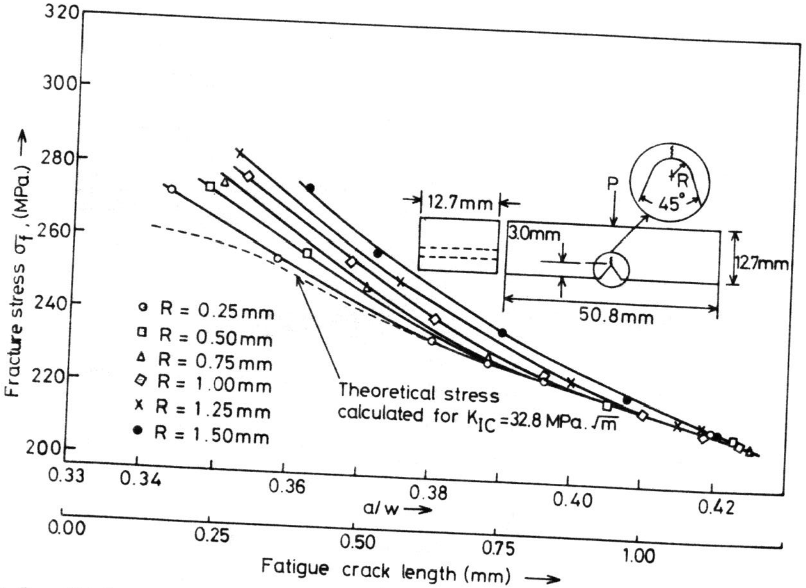

Figure 5 Nominal fracture stress in a high speed steel as a function of fatigue crack length ahead of machined notch. Composition: C 0.60, Si 0.25, Mn 0.20, W 14, Cr 3.2 and V 0.25. σ_y (0.2% proof stress) = 637 MPa and σ_{ult} = 814 MPa, A = B.W

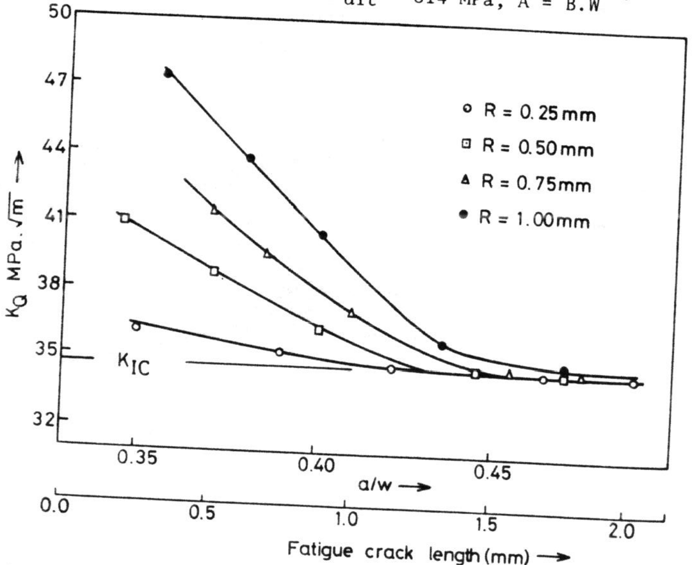

Figure 6 Variation of apparent crack toughness against fatigue crack length in 0.68% carbon steel for different notch root radii

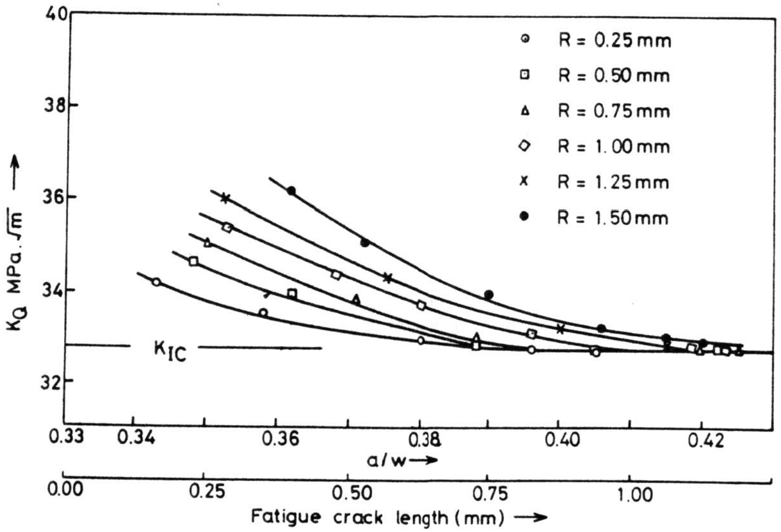

Figure 7 Variation of apparent crack toughness against fatigue crack length in 14 W - 3 Cr - 0.25 V - 0.60 C high speed steel for different notch root radii