

## FRACTURE MIRROR FORMATION IN SINGLE CRYSTAL ALUMINA

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## INTRODUCTION

Interest in fracture mirror formation, or crack branching in brittle materials originates from practical applications. Mirrors have been used to locate fracture origins [1], predict flaw sizes [2-4], verify failure stresses [5,6], estimate the magnitudes of residual stresses such as the compressive surface stresses from tempering [6,7], to interpret impact failures [8], etc. They have received particularly intensive study in the fracture of glass. More recent studies have emphasized the general applicability of this particular fracture surface analysis to all brittle ceramic materials, particularly fully dense polycrystalline ceramics [9].

The general empirical relationship that has emerged is:

$$\sigma_f r_m^{1/2} = A = K_b, \quad (1)$$

where  $\sigma_f$  is the fracture stress,  $r_m$  is the fracture mirror radius and  $A$  is referred to as the mirror constant. It has the units of fracture toughness  $\text{MPa}\cdot\text{m}^{1/2}$ ; therefore, it is sometimes referred to as the stress intensity at crack branching,  $K_b$ . However, as pointed out by McKinney [10], the mirror boundary forms only after the stress intensity at the crack tip substantially exceeds  $K_{IC}$ . In fact, all reported  $K_b$ 's exceed  $K_{IC}$  for the various brittle materials which have been tested. Numerous authors have treated the experimentally measured  $K_b$ 's as material constants, in spite of contradictions in values [1] and the highly tenuous association with  $K_{IC}$ .

Recently Abdel-Latif, *et al.* [11] have shown that for a soda lime silica glass the mirror constant is dependent on the macroscopic stress state of the test specimen at fracture and have presented energy balance analyses which yield quantitative agreement between the experimentally determined mirror constants and calculated values. It also explains the tensile and flexural differences in  $K_b$ . In this paper, the energy balance criterion is extended to theoretically predict the mirror constants for  $\text{Al}_2\text{O}_3$  single crystals demonstrating anisotropy of fracture surface energy and elastic constants.

## THEORETICAL BACKGROUND

The criteria for fracture mirror boundary formation which have been presented in the literature all condense to three basic approaches: (1) a critical velocity criterion, which argues that the propagating planar crack branches at some characteristic or critical velocity for a given

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material [12]; (2) a critical stress, or strain intensity criterion which indicates that the crack branches at some characteristic critical stress, or strain intensity [13-15]; and (3) the energy balance criterion which assumes that the crack branches when the rate of release of strain energy exceeds the demand of the propagating planar crack such that four surfaces can be created instead of two [16-20]. The failure of the first two to explain the effect of the macrostress state of the specimen on the observed mirror radii in glass has been discussed by Abdel-Latif, *et al.* [11]. The energy balance approach outlined briefly here has previously been discussed, but has only recently been applied in detail to explain differences in mirror constants for specimens fractured in flexure and in tension.

The energy balance approach to crack branching as first suggested by Mott [16] applies the classical Griffith treatment for the propagation of a pre-existing flaw, but includes a kinetic energy term since the kinetic energy of the crack surfaces is substantial at crack branching velocities. For a semicircular, planar crack propagating from the surface in a cylindrical rod in uniform tension, the total energy of the system can be evaluated. One must establish an affected volume of the rod, or a domain of the crack from which most of the released strain energy for the whole system is obtained to accelerate the crack, and to be included in the kinetic energy term. In the present work this domain has been chosen as a volume of semicircular cross section in the plane of the crack,  $A = 1/2 \pi a^2$ , and extending  $2a$  in both directions perpendicular to the crack plane in which  $a$  is the crack radius [11]. Using this approach, the following terms for the total energy of the cylindrical rod in tension containing a semicircular surface crack can be evaluated:

$$U_t = U_o - U_e + U_s + U_k, \quad (2)$$

where  $U_t$  is the total energy,  $U_o$  is the energy of the specimen without the crack,  $U_e$  is the elastic strain energy,  $U_s$  is surface energy, and  $U_k$  is the kinetic energy term. Substituting for the respective energy terms at crack branching yields:

$$U_t = U_o = \left( \frac{\pi \sigma_f^2 a^3}{E} \right) + \left( 2\pi \gamma_f a^2 \right) + \left( \frac{k\rho V^2 \pi \sigma_f^2 a^3}{E^2} \right), \quad (3)$$

where  $\sigma_f$  and  $a$  have been defined,  $E$  is the elastic modulus,  $\gamma_f$  is the fracture surface energy,  $k$  is a proportionality constant,  $\rho$  is the density, and  $V$  is the crack velocity. Note that  $4\gamma_f$  has been substituted for  $2\gamma_f$  in the surface energy term since branching is being modelled as the formation of four surfaces instead of two for the normal fracture process. When the strain energy release rate equals the energy demands to propagate two cracks instead of one, the mirror is bounded. Differentiating the above relationship with respect to the crack radius, and then equating to zero, yields the crack branching condition:

$$\frac{\partial U_t}{\partial a} = - \left( \frac{3\pi \sigma_f^2 a^2}{E} \right) + \left( 4\pi \gamma_f a \right) + \left( \frac{3\pi k\rho V^2 \sigma_f^2 a^2}{E^2} \right) = 0, \quad (4)$$

then, setting  $a = r_m$  and  $V = V_b$  and rearranging:

$$\sigma_f^2 r_m \left( \frac{3}{E} - \frac{3k\rho V_b^2}{E^2} \right) = 4\gamma_f. \quad (5)$$

The other solution to equation (4)  $V = 0$ , is not applicable to the branching phenomenon.

In terms of the mirror constant form, the condition becomes:

$$\sigma_f r_m^{1/2} = \left\{ \left( \frac{4E\gamma_f}{1 - \frac{k\rho V_b^2}{E}} \right) (1/3) \right\}^{1/2} \quad (6)$$

Thus, only for a very specific set of test conditions will the product  $\sigma_f r_m^{1/2}$  be constant for tensile fractures. With respect to the fracture of single crystal alumina, which has a highly anisotropic  $\gamma_f$  [21], as well as considerable anisotropy of  $E$  [22], even if branching occurs at constant velocity (which is not correct in detail for sapphire, but nearly so [19]), there should be a significant orientation dependence of  $\sigma_f r_m^{1/2}$ .

Expressions having the same form as equation (6) can be derived for different stress states in a test specimen. In the case of the four point flexural test, the stress terms in equation (3) are not simply  $\sigma_f$ , but decrease from  $\sigma_f$  at the surface of the specimen to zero at the neutral axis. By substituting a linear relationship for  $\sigma_f \{ \sigma_f (R-a/R) \}$ , where  $R$  is the radius of the test rod, into equation (3) and performing the same mathematical manipulations a similar expression for  $\sigma_f r_m^{1/2}$  can be derived. In Appendix A the relationship for four point flexure is given. Based on these relationships, it is obvious that the "mirror constant" cannot be a constant for a given material since the sample dimensions, and the test geometry all enter into the expression when the kinetic energy term is included in the energy balance criterion for crack branching. Note that for large specimens and large test spans relative to the fracture mirror size, the geometry dependent term for the flexural case reduces to 1/3, the tensile case.

#### EXPERIMENTAL RESULTS AND DISCUSSION

In the case of an isotropic material the previously described theoretical analysis has been shown to quantitatively predict the mirror "constants" for the three test geometries considered [11]. The following experimental results and calculations illustrate that the approach correctly predicts the "mirror constant" relationships for a highly anisotropic single crystal material alumina.

Sapphire filaments\*,  $\sim 0.025$  cm in diameter, with the C-axis parallel to the filament axis were tested in tension and four point flexure at room temperature in air using a constant strain rate (1.27 cm/min cross-head speed) testing machine. The gauge length in tension was 2.54 cm and in four point bending the outer test span was 1.90 cm and the inner 0.95 cm inches. A very small area,  $\sim 0.1$  cm in length, of the gauge length was lightly abraded with 15  $\mu$ m diamond by touching the fibres with a polishing cloth containing the diamond paste. Flame polished ruby\*\* rods with the C-axis  $\approx 60^\circ$  from the rod axis and 0.043 cm in diameter were also tested in four point flexure to assess the effects of the anisotropy of fracture surface energy on the mirror constant by comparing the results to those for C-axis sapphire. The fracture surfaces were examined and photographed

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in the scanning electron microscope. Detailed measurements of the mirrors were taken from the micrographs.

A typical fracture mirror is illustrated in Figure 1 for a "60°" ruby flexure test. Note that there is no mist region as in glass. From Wiederhorn's [21] fracture surface morphology, it is clear that, energetically, the morphological rhombohedral planes are the "easy" cleavage planes. In the case of the "60°" ruby fracture surface the original crack plane in the sample is nearly parallel to the morphological rhombohedral plane. Thus, one observes rather broad cleavage surfaces after the crack branches from the original plane. The steps on those surfaces, nearly parallel to the filament axis, are probably other morphological rhombohedral planes of the same zone. In the case of the C-axis filament the mirror is coplanar with the basal plane and the branched crack produces hackle at steeper angles to the original crack plane and the C-axis, again they are the characteristic morphological rhombohedral cleavage planes. The fracture mirror pattern is also well defined in other published micrographs of single crystal alumina fracture surfaces, including those of Wachtman and Maxwell [23], Kotchick and Tressler [24], Firestone and Heuer [25], and Haggerty [26].

The results are presented in the classical fracture mirror plot in Figure 2 along with literature data for three types of polycrystalline alumina with significantly lower strengths; a hot pressed variety ( $A = 10.3$ ), AlSiMag #614 ( $A = 8.3$ ), and an undesignated variety ( $A = 5.4$ ). The first two polycrystalline aluminas were fractured in flexure; the last one in uniaxial tension. Analyzing first the observed differences in the mirror constants for the single crystal materials, qualitatively the trend is the same as observed for tension vs. flexure in glass specimens; the mirror constant being larger for testing in flexure than for tension. In addition, the mirror constant for the "60°" material in flexure is somewhat smaller than the C-axis value in tension.

Because of the complex form of the expressions for the flexure tests one cannot solve unequivocally for the  $\sigma_{fr}^{-1/2}$  value unless a characteristic relationship between  $r_m$  and  $V$ , the velocity, is known, which is not the case. Therefore, only the expression for the tensile case can be evaluated numerically and compared to the experimental value. Inserting numerical values of the parameters in equation (1), a value for  $\sigma_{fr}^{-1/2}$  can be calculated for the C-axis tensile test. Using  $\gamma_f = 40 \text{ J/m}^2$  [21],  $E = 4.65 \times 10^{11} \text{ N/m}^2$  [23],  $k = 22$  [11],  $\rho = 3.96 \times 10^3 \text{ kg/m}^3$ , and  $V = 1.50 \times 10^3 \text{ m/sec}$  [8] yields a mirror constant of  $6.4 \text{ MPa}\cdot\text{m}^{1/2}$ , considering the uncertainty in the value of  $V$  and the approximation in using the  $E$  value parallel to the C-axis and the  $\gamma_f$  value for a planar crack perpendicular to the C-axis. If the analogous calculation is done for the "60°" crystals using  $\gamma_f$  for (1012) cleavage of  $6.0 \text{ J/m}^2$  [21], and  $E$  value of  $3.45 \times 10^{11} \text{ N/m}^2$  for the direction perpendicular to the "60°" plane [22] and the same velocity, a mirror constant of  $2.50 \text{ MPa}\cdot\text{m}^{1/2}$  results. The calculated values illustrate the significant anisotropy that one should find in mirror constants from tensile tests in these different orientations of single crystal alumina.

From Figure 2 it is apparent that the "mirror constant" for C-axis sapphire in flexure is nearly twice that in tension. Referring to the equation in Appendix A, for small rod radii (as in this case) the mirror constant at a given set of conditions must be larger for the flexural case. Likewise the flexural "mirror constant" for the "60°" crystals must be larger than the tensile mirror constant; which it is, comparing

the experimental value to the calculated value. It must be emphasized that in the case of the flexural tests the apparent mirror constant that one observes experimentally, is very much dependent on the test geometry, and, therefore, is not a constant for a particular material.

For the polycrystalline materials, it is noteworthy that the published "mirror constants" from small test bars tested in flexure are on the order of twice the mirror constant reported for polycrystalline alumina tested in uniaxial tension, thus substantiating the validity of the energy balance approach and the test geometry predictions thereof.

The fact that the  $\sigma_{fr}^{-1/2}$  value for C-axis sapphire tested in tension is similar to the polycrystalline alumina value for tensile tests is not readily resolvable, since the fracture surface energies for polycrystalline  $\text{Al}_2\text{O}_3$  are generally lower than the  $40.0 \text{ J/m}^2$  used in this analysis [27]. There is not sufficient data reported concerning the particular material used in the tensile tests to speculate any further.

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## APPENDIX A

THE "MIRROR CONSTANT": FOUR-POINT FLEXURAL TEST [R = Specimen Radius]

$$\sigma_f r_m^{-1/2} = \left\{ \left( \frac{4EY_f}{1 - \frac{K\rho V_b^2}{E}} \right) \left( \frac{R^2}{3R^2 - 8Rr_m + 5r_m^2} \right) \right\}^{1/2}$$

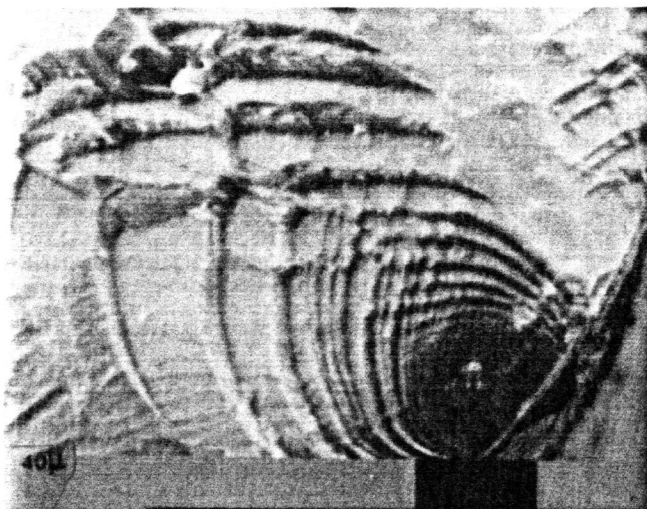


Figure 1 SEM Fractograph of a "60°" Ruby Specimen Fractured in Four Point Flexure Illustrating a Well-Defined Mirror with the Associated Hackle

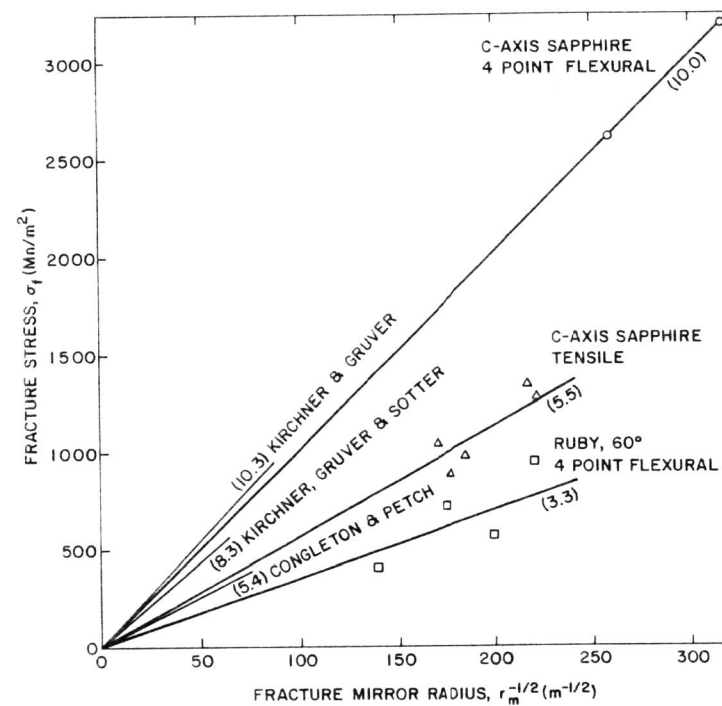


Figure 2 The "Mirror Constant" Graph of (Fracture Stress) versus (Mirror Radius)<sup>-1/2</sup> for Single Crystal Alumina Specimens (this study) and Literature Data for Polycrystalline Alumina. The Numbers in Parentheses are the "Mirror Constants" Calculated from the Slopes of the Lines