

## FRACTURE MECHANICS OF VISCOELASTIC SYSTEMS

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## INTRODUCTION

Consideration is being given to viscoelastic systems of inherited type. These systems, under cyclic load, display notable dissipative warming-up and their mechanical behaviour varies with time and depends upon environmental conditions in the form of radiation flows or chemically active agents. Local fracture theory is applied to analyze the fracture processes of such systems. All the relations outlining this process have been obtained on the basis of methods of thermodynamics of irreversible processes. An experimental procedure is presented which makes it possible to appraise characteristic properties and life time prior to fracture, based on the available present level of information on the physical, chemical and mechanical states of the system.

A set of equations is under consideration for calculation of real viscoelastic constructions. This set of equations included: 1) state parameters, 2) deformation equilibrium and coincidence equations, 3) rheological relations, 4) heat equation with internal heat sources, 5) fracture criterion equations.

Deformation equilibrium and coincidence equations are adopted with allowance made for particular system geometrical shape and its loading conditions. Rheological relations are selected with regard to system mechanical behaviour. Valter operators with a Rabotnov - type (relaxation with rational) exponential nucleus have been used in this specific case.

## FRACTURE CRITERIA

Two fracture criteria have been utilized in this research:

1. An entropy criterion postulating the following: system fracture takes place at the moment when entropy increment density reaches a certain critical level  $\Delta S_f^f(T)$  i.e. material behaviour at a given temperature. After integration this criterion appears as [1]:

$$\int_{t_0}^{t_f} \dot{S}(t) dt = \Delta S^f(T) = S(t_f) - S(t_0), \quad (1)$$

where  $S(t)$  = entropy density;  $\Delta S^f(T)$  = entropy density at a critical level of material behaviour, serving as a temperature function.

Let us consider fracture in a real viscoelastic system and take as a system a prismatic rubber element loaded in accordance with the law of harmonics

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$\varepsilon = \varepsilon_0 \sin \omega t$ . We take  $\{T_E^0, T, G\}$  as a complete set of thermodynamic parameters applicable to a 'special' one which on the one hand is sufficiently small to be considered as a continuous medium point and on the other hand -- sufficiently large to exhibit all the properties of this medium. By applying the first law of thermodynamics  $U = T_\sigma \cdot \dot{T}_E + q$ , the available energy density definition  $f = U - TS$ , and the condition  $f = f(T_E^0, T, G)$  we derive the following expression for the rate of change of entropy density:

$$\dot{S} = \frac{1}{T} \left[ T_\sigma \cdot \dot{T}_E^0 - \left( S + \frac{\partial f}{\partial T} \right) \dot{T} + \left( T_\sigma - \frac{\partial f}{\partial T_E^0} \right) T_E^0 - \frac{\partial f}{\partial G} \dot{G} + \dot{q} \right] \quad (2)$$

where  $U$  = internal energy density,  $G$  = shear modulus,  $q$  = heat flow,  $T_\sigma$  = stress tensor,  $T_E^0$  = strain tensor (irreversible part),  $T_E$  - strain tensor (reversible part).

Substituting (2) into (1) and taking into account equalities [2]:

$$\frac{\partial f}{\partial T} = -S, \quad \frac{\partial f}{\partial T_E^0} = T_\sigma,$$

we derive:

$$\Delta S_f = \int_{t_0}^{t_f} \frac{1}{T} \left[ T_\sigma \dot{T}_E^0 - \frac{\partial f}{\partial G} \dot{G} + \dot{q} \right] dt \quad (3)$$

Disregarding material ageing ( $\dot{G} = 0$ ), and assuming the temperature field to be steady, this expression may be written as:

$$T \Delta S_f(T) = \int_{t_0}^{t_f} T_\sigma \dot{T}_E^0 dt - \int_{t_0}^{t_f} \dot{q} dt \quad (4)$$

A negative  $q$  indicates dissipation of heat in the system. The first integral in (7) represents strain energy density, irreversibly dispersed in  $[t_0, t_f]$  time; the second thermal energy density, released in the sample at the same time. For a closed strain path:

$$\int_{t_0}^{t_f} T_\sigma \dot{T}_E^0 dt = \int_{t_0}^{t_f} T_\sigma \dot{T}_E dt, \quad (5)$$

which results from the condition that: work done by stress in elastic deformation equals zero in this case. With regard to (5) equation (4) assumes the form:

$$T \Delta S_f = \int_{t_0}^{t_f} T_\sigma \dot{T}_E dt - \int_{t_0}^{t_f} \dot{q} dt \quad (6)$$

Supposing that rubber adheres to viscoelasticity linear inherited theory and that its properties do not vary in the process of loading, we may specify stress-strain bond as:

$$T_\sigma(t) = 3G_0 \left[ T_E(t) - x \int_{-\infty}^{t-t'} k(t-t') T_E(t') dt' \right] \quad (7)$$

where  $k(t-t')$  is a nucleus of relaxation expressed as a Rabotnov rational - exponential function,  $x$  = rheological parameter,  $G_0$  = instantaneous shear modulus. With regard to (7) expression (6) assumes a final form:

$$T \Delta S_f = \left( \frac{3\omega}{8\pi^2} \psi G_0 \varepsilon_0^2 - q \right) t_f \quad (8)$$

Here  $\psi$  = material dissipation coefficient;  $\Delta S_f$  occurred in the course of experimental studies of sample endurance, the samples being in the form of double-sided blades and was independent when investigating endurance of full-scale rubber structures in the form of a prismatic shear element. The heat flow  $q$  for the sample's most loaded point has been determined when solving a heat conduction equation of thermal conduction with an internal heat source.

For the sandwich-type rubber-metallic element with the following parameters  $G_0 = 1,76$  MPa,  $\varepsilon_0 = 0,1$ ,  $T = 320$  K,  $\omega = 12$  Hz,  $\psi = 0,31$ ,  $\Delta S_f = 2,03 \cdot 10^3$  J/m<sup>2</sup> deg,  $q = 5,31 \cdot 10^3$  J/m<sup>3</sup> sec estimated with regard to (8) time  $t_f = 4800$  hours, time resulting from the experiment for the lot of 36 full-scale articles constitutes (4 to 5)  $\cdot 10^3$  hours. Agreement, as seen, is quite satisfactory.

2. The range under consideration as the second criterion of fracture is the process of achieving by the system a critical level of damage, when the system converts into labile state. A criterion equation for the shear element loaded by the law of harmonics has been obtained as follows:

$$P_{kp} = 1 - \sqrt{\frac{0,4}{\gamma G}} \tau, \quad (9)$$

and time-period prior to 'special' volume fracture:

$$t_f = \frac{P_{kp} \exp(Q/kT)}{C k_0 I_0 (\gamma \tau / kT)} \quad (10)$$

where  $P_{kp}$  = critical concentration of stored defects;  $k_0$  constant of action;  $c$  = material behaviour, depending upon initial components concentration and forms of the elementary reactions;  $\gamma$  = specific energy of defect formation;  $I_0$  = Bessel function.

Expressions (9) and (10) are obtained for the case where thermo-conductancy equation is represented as:

$$\frac{\partial T}{\partial t} = a \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial x_2^2} + \frac{\partial^2 T}{\partial x_3^2} \right) + \frac{W}{C_1 p}$$

and the revolution equation of material damage has been recorded due to the fact that damage concentration rate change is related to the intensity of physical-chemical processes as:

$$\dot{p} = C k_0 \exp \left[ - \frac{1}{kT} \left( Q - N^0 T - N^1 \frac{\partial T}{\partial x} - N^2 : T_E \right) \right]$$

Here  $a$  = thermo-conductance coefficients;  $N^0, N^1, N^2$  - tensors of the null, first and second valency correspondingly;  $c_1$  = specific heat capacity;  $p$  = density.

A number of problems have been solved estimating a period of time prior to local fracture of viscoelastic systems in the form of silent-blocks and sandwich-type shear elements under cyclic load. If material constants have been determined correctly then (10) shows a satisfactory agreement with practice.

Material damage in the process of continuous loading may also be disclosed in the course of direct physical experiments. In the general case for rubber, concentration of damage obeys the relation:

$$\Delta p(t) = P_{kp} [1 - \exp(-nt)], \quad (11)$$

where  $\Delta P_{kp}$  = damage storage from the moment of system loading till its failure;  $h$  = constant. For CKH - 3 based rubber  $P_{kp} = 5,3$  (in terms of per unit value) and  $n = 0,0016$ . This concept has been confirmed in independent experimental studies wherein material damage (concentration of rupture with time up to system failure) has been determined on models by infrared spectroscopy.

We may derive pre-fracture time from (11) if  $\Delta p(t)$  is known over the wide range of stress and temperature variations. Promising results have been obtained for thin films, as well as good agreement between theory and experiment.

#### REFERENCES

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