FRACTURE MECHANICS OF TIDAL FLEXURE CRACKS IN FLOATING ICE SHELVES

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INTRODUCTION

Estimates of the mechanical properties of floating ice shelves are required to facilitate the design of projects such as aircraft runways and scientific bases. Icebergs which calve from the margins of such ice shelves pose a threat to shipping in polar waters, while at a more esoteric level, recent well argued theories [1] suggest that the disintegration of the Antarctic ice sheet may initiate a global ice age. The detailed mechanics of the break up of floating ice are therefore of more than academic interest. This paper discusses one possible way in which failure may begin - that of flexure of the ice sheet by the rise and fall of the tide. A model is used to estimate the magnitude of the bending forces involved, then by applying the concepts of fracture mechanics to an unusual situation, the depths of flexure initiated cracks are calculated.

TIDAL BENDING STRESSES

Tidal bending (or 'strand') cracks at the land junction of a floating ice shelf have been described in some detail by Robin and Swithinbank [2]. The following analysis to estimate the bending stresses due to the rise and fall of the tide, is due to Robin [2] but has been developed independently by Holdsworth [3]. The floating ice is modelled as a long elastic cantilever strip, which in equilibrium floats at the mean level of the water. Axes are located at the neutral axis of a strip H thick, at the land based end, approximated by a clamped boundary, corresponding to the observed rapid increase in ice thickness (see Figure 1). The y axis is positive downwards, the x axis along the length of the beam. Now if the tide rises by an amount w, the restoring force per unit length on a strip of unit width is given by

$$F = \rho_w \, g \, (w + y)$$

where g is the acceleration due to gravity, ρ_w is the density of sea water and y is the deflection of the beam. This restoring force is the second derivative of the bending moment, hence, the general plane strain equation for the beam bending problem:

$$\frac{d^2y}{dx^2} = - \frac{(1-\nu^2)}{EI} M(x)$$

becomes

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$$\frac{d^4y}{dx^4} = -\frac{(1-\nu^2)}{EI}\rho_w g (w + y)$$

where ν is Poisson's ratio, E is Young's Modulus and I is the moment of inertia of the beam per unit thickness, $H^3/12$. Following the solution of beams on elastic foundations due to Hetényi [4], the general solution is

$$y = -w+e^{\lambda x} (A \cos\lambda x+B \sin\lambda x) + e^{-\lambda x} (C \cos\lambda x+D \sin\lambda x)$$

where

$$\lambda^4 = \frac{\rho_w g (1-\nu^2)}{4EI} .$$

The boundary conditions are that at the free end, the beam rises and falls with the tide, i.e.,

$$\text{as } x \to \infty,\ y = -w \qquad A = B = 0$$

while at the clamped end both the deflection and the slope are zero

$$\frac{dy}{dx} = y = 0 \quad \text{at} \quad x = 0 .$$

Therefore the deflection may be written,

$$y = -w [1-e^{-\lambda x} (\cos\lambda x+\sin\lambda x)] \tag{1}$$

from which maximum values of the modulus of the deflection occur at distances separated by π/λ and between these points the deflection curve follows a flat wave of decreasing amplitude with x.

Equation (1) can now be differentiated to yield the bending moments and, hence, the stresses at any section of the beam. Maximum values of moment and stress are found to occur at x = 0, $\pi/2\lambda$, $3\pi/2\lambda$ etc., where the dimension $1/\lambda$ controls the scale effect of the problem. By substituting typical values, $\rho_w = 1.02 \times 10^3 kg/m^3$, $g = 9.81 m/m^2$, $\nu = 0.3$, $E = 2.7 \times 10^9 N/m^2$, $1/\lambda$ is found to be in the order to 940m. The maximum bending stresses are going to occur first at the hinge, then some 1.5km out, ratios of successive maxima being given by $e^{-\lambda x}$, making the third and higher peaks negligible.

In general, therefore, once the bending moments are known for any tidal range ±w, and any ice depth, H, the stresses can be computed by assuming the linear distribution about the neutral axis associated with elastic beam theory. The maximum stress will occur at the top and bottom surfaces of the ice, alternating between tension and compression as the tide rises and falls. Using values typical of the Ross Ice Shelf in Antarctica*, at the hinge a value for the moment of 2.2×10^9Nm is obtained, leading to maximum surface stresses of $3.3 \times 10^5 N/m^2$. Robin [2] concluded that this analysis coincided reasonably well with the observed facts.

*H = 200m, w = ± 0.5m .

DEPTH OF STRAND CRACKS

The stresses in the region of the hinge can now be examined in greater detail to estimate the depth of cracking. The method employed has previously been used to calculate crevasse depths in glaciers [5]. Briefly, the hinge region is subjected to two stress systems, see Figure 2, the bend moment here shown inducing tensile stresses at the upper surface and a compressive ice overburden stress, increasing linearly with depth. Each type of loading produces a stress intensity factor at the tip of a crack of depth, a. The depth of the crack will increase until the difference between the stress intensity factors has been reduced to the fracture toughness of the material. Since ice is a brittle material, weak in tension, the fracture toughness can be taken to be zero. This is a reasonable approximation since, by equating the failure stress for ice from both the Griffith and fracture mechanics failure criteria we obtain,

$$\sigma_F = \left\{\frac{2E\gamma_s}{(1-\nu^2)a}\right\}^{1/2} = \frac{K_{IC}}{\sqrt{\pi a}}$$

where $\gamma_s \sim 0.11 J/m^2$ is the surface energy, hence, $K_{IC} \sim 26 kPa\cdot m^{1/2}$, smaller than typical values for steels by some three orders of magnitude, and may be neglected compared with the values of applied stress intensity factor used in the problem.

The positive stress intensity factor generated by the bending moment, M, is [6]

$$K = F_B \cdot \frac{6M\sqrt{\pi a}}{H^2} \tag{2}$$

whilst for the compressive overburden pressure [7]

$$K = -F.0.683\ \rho_I\ g\ a\sqrt{\pi a} \tag{3}$$

where F is an additional correction factor due to the finite width ratio, (a/H). Equations (2) and (3) are plotted as a function of crack depth in Figure 3, using the hinge moment value, $M = 2.2 \times 10^9$Nm. The critical depth is attained at a/H = 0.55, that is, crack depth a $\sim$ 51m. The dotted line shows the values calculated for the second maximum moment (0.46×10^9N/m) some 1.5km out from the hinge. The compressive stresses are unchanged: the critical crack depth reduced to very small value of some 7m.

A check can be made on this result. The strain relief afforded by the cracking is in the order of the difference in the strain at the top surface and the crack depth in the bent, but uncracked, beam. The radius of curvature at the hinge ($\sim$ EI/M) is of the order 0.82×10^6m, so the strain relief becomes

$$\Delta\varepsilon \sim \left[\frac{H/2 - a}{R}\right] \sim 6 \times 10^{-4}.$$

Swithinbank [2] quotes examples of strand cracks 20 mm wide, spaced at 40m intervals. The corresponding strain relief would be $\sim 5 \times 10^{-4}$, in satisfying agreement with the above calculation.

DISCUSSION

In common with many geophysical problems, the mechanics of strand cracking are very complex. This paper has used a simplified model of both the geometry of the ice, the ice/land boundary conditions and the material behaviour. Future work will examine these details and include the stresses due to the forward spreading of the ice. The situation at the bottom of the ice also requires further investigation. Here, although the compressive stresses are larger than near the top surface, water can enter the crack and act as a tensile wedge. Sophisticated modern techniques have identified bottom crevasses of tidal origin [8], while Hughes [1] has estimated the rate at which water will freeze on the sides of these cracks. The important point is that both top and bottom strand cracks will act as zones of weakness when they have eventually reached the position of the ice front and contribute to the break up of the ice shelf. The methods of fracture mechanics have enabled a realistic estimate to be made of the depth of cracks subjected to complex loading conditions in a naturally occurring system.

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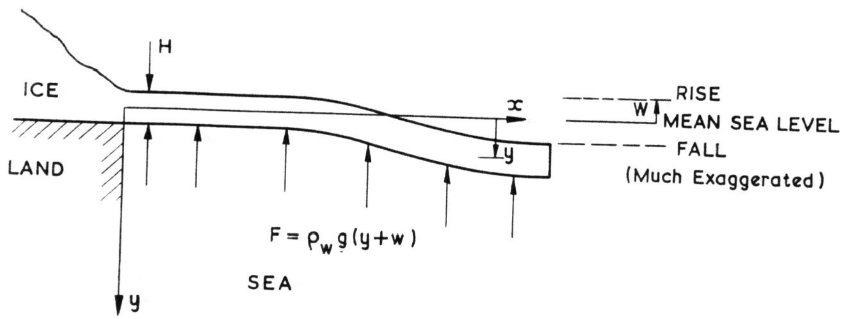

Figure 1 Restoring Force Acting on Ice Beam

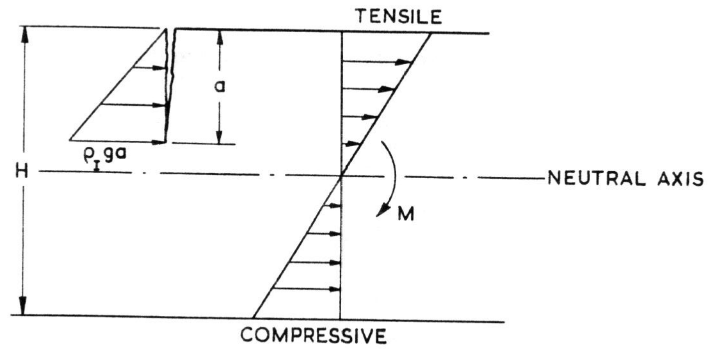

Figure 2 Details of Stresses Acting at Hinge

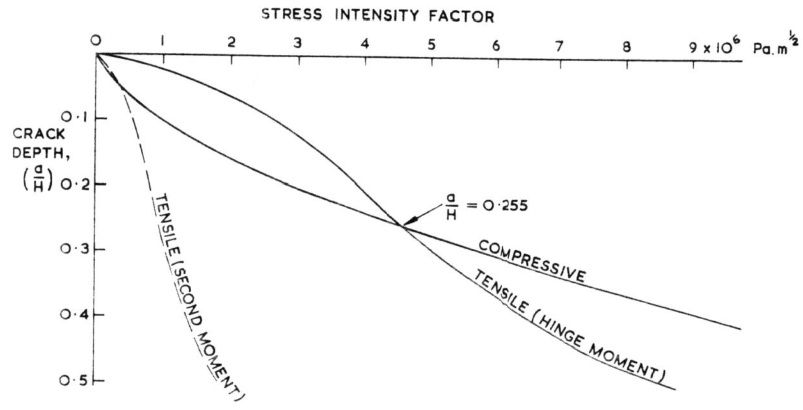

Figure 3 Stress Intensity Factors Due to Moments and Ice Compression