FRACTURE CRITERION FOR SOLID PROPELLANTS

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INTRODUCTION

Fracture of solid composite propellants is function of both time and state of stress. A method enabling the computation of fracture time for any type of loading, static or dynamic, uniaxial or multiaxial has been developed. Using judicious approximations, very simple formulas may be obtained.

TIME DEPENDENT FRACTURE

Crack Propagation Law

Recent studies [1] on solid propellants have shown that crack propagation rate $\dot{a}$ under a static load may depend on the applied stress intensity factor K only, using linear elasticity for the calculation of K. Results obtained by Francis and Jacobs [1] fit nicely with a linear plot in logarithmic coordinates. Such a dependence was found in glass by Charles [2]. Later, Evans [3] expressed this law in terms of stress intensity factor. Using time-temperature equivalence we may write

$$\dot{a}\, a_T = C\, K^p \qquad (1)$$

where C and p are materials constants and a_T is the time-temperature shift factor:

$$\log a_T = - \frac{C_1 (T - T_o)}{C_2 + T - T_o} \qquad (2)$$

Values obtained from stress relaxation measurements, for $T_o = 20°C$ are $C_1 = 9$ and $C_2 = 209°C$ [4].

Three types of tests (Table 1) on a polybutadiene propellant gave results shown on Figure 1. The crack-propagation law is:

$$\dot{a}\, a_T = 10^2\, K^6$$

The numerical value p = 6 was also found on polyurethane propellants.

This was also found by Swanson [5] using results of reference [1].

Smith's Failure Envelope

The crack-propagation law is a differential equation that can be integrated provided the initial crack length a_o and the applied stress $\sigma(t)$ are given as was done by Evans [3].

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We assume that a_o is small compared to specimen size, e.g. $2a_o$ is the size of the largest filler particle. This allows to consider the medium as infinite where

$$K = \sigma(t)\sqrt{\pi a(t)} \tag{4}$$

and a varies from a_o to infinity.

For constant stress-rate $\dot{\sigma}$, the differential equation is

$$\frac{da}{dt} = C(\dot{\sigma}t\sqrt{\pi a})^p \tag{5}$$

or

$$\int_{a_o}^{\infty} \frac{da}{a^{p/2}} = C(\dot{\sigma}\sqrt{\pi})^p \int_0^{t_f} t^p \, dt \tag{6}$$

where t_f is the time at break.

The stress at break is

$$\sigma_f = \dot{\sigma}t_f \tag{7}$$

After integration, time at break is obtained

$$t_f = \frac{p+1}{p/2 - 1} \cdot \frac{a_T}{C\, a_o^{p/2-1}\, \sigma_f^p\, \pi^{p/2}} \tag{8}$$

This formula gives a log-log plot of stress-time to break which is linear. A parallel straight line would be obtained at constant stress. This is what is usually observed in solid propellants.

Assuming a linear viscoelastic behaviour and using the Schapery formula [6] for the relaxation modulus:

$$E(t/a_T) = E_e + \frac{E_g - E_e}{\left(1 + \frac{t}{\tau_o a_T}\right)^n} \tag{9}$$

where E_e, E_g, τ_o and n are material constants, gives, for constant strain-rate $\dot{\varepsilon}$:

$$\frac{\sigma_f}{\varepsilon_f} = E_e + \frac{\tau_o a_T(E_g - E_e)}{(1-n)\, t_f}\left[\left(1 + \frac{t_f}{\tau_o a_T}\right)^{1-n} - 1\right] \tag{10}$$

Experiments show that results obtained at constant $\dot{\varepsilon}$ or $\dot{\sigma}$ do not differ appreciably and we use equation (8) in (10). It gives an equation for the Smith failure envelope [6]. Agreement between theoretical and experimental envelopes is very reasonable. Theoretical fracture strains are 10 to 30% too high, probably because of the non-linear behaviour of the material.

MULTIAXIAL FRACTURE ENVELOPE

Generalization of the crack-propagation law in terms of the strain-energy release rate G opens the way to three-dimensional loading:

$$\dot{a}\, a_T = C'\, G^{p'/2} \tag{11}$$

C' and p' are new material constants, p and p' are only slightly different, [5] owing to the time dependence of the modulus. Equation (11) is the Lake and Lindley formula [7] similar to other criterions [8, 9].

For the sake of simplicity, let us now consider only instantaneous fracture. G is then the only parameter. For a given geometry, G is proportional to the variation ΔW during fracture of the strain energy density W. When W is zero after fracture this is identical to Beltrami's criterion. If the material is incompressible and undilatable it is von Mises criterion.

We now understand why Beltrami's criterion did not work under high pressures: this is because the strain energy due to pressure is not available for fracture if the material cannot implode. Von Mises does not work too for a dilatant material where an increase of volume occurs during straining.

The strain energy density at fracture is given by the classical formula:

$$W = \frac{\sigma_{oct}^2}{2K} + \frac{3}{4}\frac{\tau_{oct}^2}{\mu} \tag{12}$$

where $$\sigma_{oct} = 1/3\,(\sigma_1 + \sigma_2 + \sigma_3) \tag{13}$$

and $$\tau_{oct} = 1/3\sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^3 + (\sigma_3 - \sigma_1)^2} \tag{14}$$

σ_1, σ_2 and σ_3 are the principal stresses.

We shall assume a constant shear modulus μ and that the above formula may still be used with a variable bulk modulus K.

In order to plot on the same graph results obtained at different strain-rates or for different materials, reduced coordinates will be used:

$$x = \sigma_{oct} / \sigma_{oct}^T \quad \text{and} \quad y = \sigma_{oct} / \sigma_{oct}^T$$

σ_{oct}^T and τ_{oct}^T are σ_{oct} and τ_{oct} at fracture in uniaxial tension. They are related by

$$\tau_{oct}^T = \sqrt{2}\,\sigma_{oct}^T \tag{15}$$

A reduced strain energy density may be defined

$$w = \frac{4\mu W}{3(\tau^T_{oct})^2} = \frac{\mu}{3K} x^2 + y^2 \qquad (16)$$

For $\mu/K = 0$, it expresses von Mises criterion:

$$w = 1 \qquad (17)$$

Because of the granular nature of the material, shearing strains produce dewetting, porosity and volume increase, even under hydrostatic pressure [10]. The bulk modulus is then a function of σ_{oct} and τ_{oct} and may be positive or negative.

Following Reiner [11], the volume change is

$$\varepsilon_1 + \varepsilon_2 + \varepsilon_3 = \sigma_{oct}/\kappa + \tau^2_{oct}/\delta \qquad (18)$$

ε_1, ε_2 and ε_3 are the principal strains, κ is the bulk modulus of the unstrained material and δ is the dilatancy modulus. The porosity decreases when the pressure increases: δ is a function of σ_{oct}.

$$\delta = \mu\beta(\alpha - x)^2\tau^T_{oct} \qquad (19)$$

α and β are numerical constants. The compressibility is

$$\frac{1}{K} = \frac{\varepsilon_1 + \varepsilon_2 + \varepsilon_3}{\sigma_{oct}} = \frac{1}{\kappa} + \frac{y^2}{\mu\beta x(\alpha - x)^2} \qquad (20)$$

For solid propellants, $1/\kappa \simeq 0$, the reduced strain energy density is:

$$w = \frac{x\, y^2}{3\beta(\alpha - x)^2} + y^2 \qquad (21)$$

After fracture, $y = 0$, then $w = 0$ and ordinary Beltrami criterion may be used:

$$w = \text{Constant} = \frac{1}{3\beta(\alpha - 1)^2} + 1 \qquad (22)$$

The constant has been obtained by reference to uniaxial tension ($x = 1$, $y = 1$).

The equation of the multiaxial fracture envelope is:

$$y = \sqrt{\frac{1 + \dfrac{1}{3\beta(\alpha - 1)^2}}{1 + \dfrac{x}{3\beta(\alpha - x)^2}}} \qquad (23)$$

For $\alpha = 8$ and $\beta = .017$ we obtain the curve shown on Figure 2 that fits the experimental points very well except for compressive loading. The compressive strength is indeed dependent on the specimen shape.

CONCLUSION

A general criterion taking into account the whole stress tensor as well as the rheological properties of the material may be given for solid propellants: the crack propagation rate depends only on the strain energy release rate. The integration of the differential equation is a powerful method to describe the time dependency of fracture of solid propellants. Fixing the time, it has been shown that Beltrami's criterion is a particular case of the strain energy release rate criterion. It may be rehabilitated if variable elastic "constants" are used and if the elastic energy remaining in the material after fracture is subtracted from the elastic energy before fracture.

The physical reason for the increase of the strength under pressure is the work that has to be done against the pressure to increase the volume of the material. It will be measured with a Farris dilatometer [10] and related to the strength.

It should be emphasized that the phenomenon of dilatancy is common in many granular materials, for example crystals, where the volume increase is directly proportional to the dislocation density [12]. Atoms may be compared with the filler particles and dislocations with the vacuoles due to dewetting.

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Table 1

Specimen type	Single edge notched	Sheets, central notch	Sheets, central notch
Loading rate	8.3 10^{-5}m/s	8.3 10^{-6}m/s	F = Constant
Temperatures	-40, -20, +20, +60°C	-20, +20°C +40°C	20°C
Thickness	5 mm	5 mm	6 mm
Width	10 mm	230 mm	230 mm
Gage length	50 mm	40 mm	40 mm
Initial crack length	2 mm (= a_o)	40 mm (= $2a_o$)	40 mm (=$2a_o$)

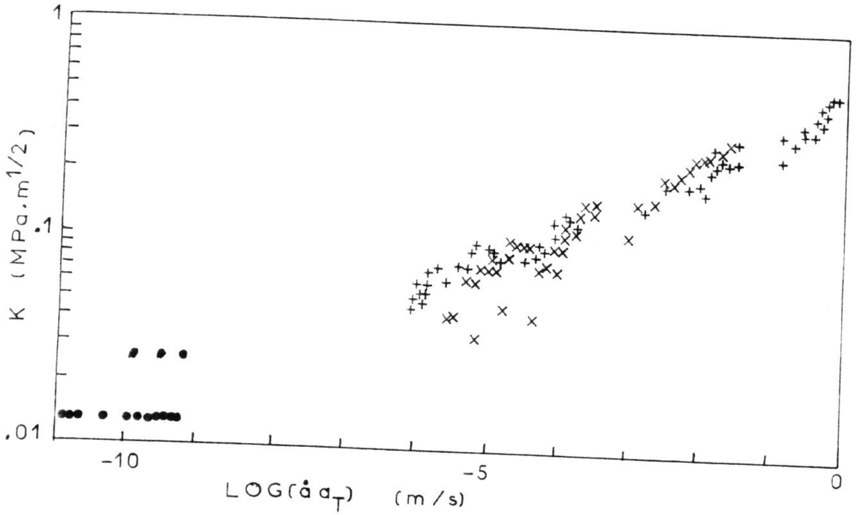

Figure 1 Stress-Intensity Factor versus Reduced Rate of Crack Propagation

(+) Single Edge-Notched and (x) Sheet Specimens at Constant Crosshead Speed and Various Temperatures
(•) Sheet Specimens at Constant Load and 20° C

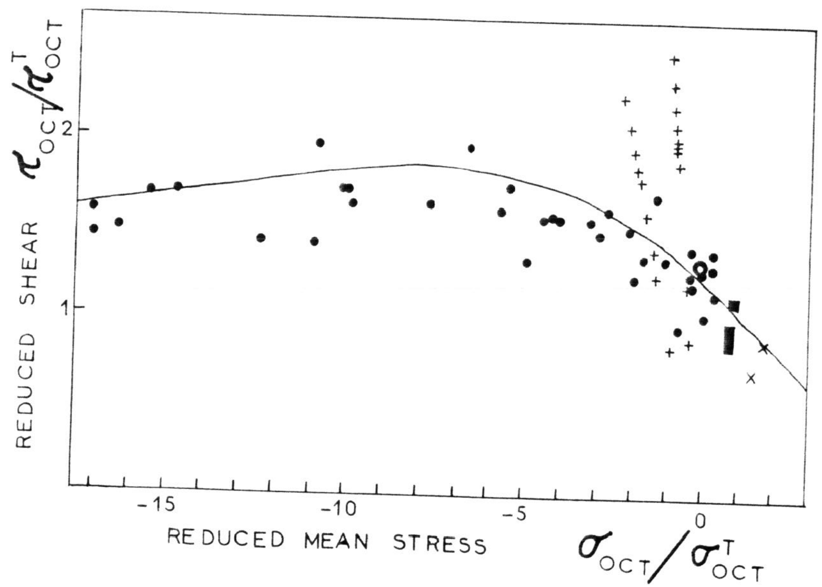

Figure 2 Multiaxial Fracture Criterion. Reduced Octahedral Shearing Stress versus Reduced Octahedral Normal Stress. Coordinates of Uniaxial Tension are x = y = 1. Most Results are Expressed in True Stresses. Atmospheric Pressure has been Neglected.

(•) Tension Under Various Pressures, all on the Same Polyurethane Propellant. Similar Results have been Obtained on Polybutadiene Propellants.
(+) Axial and Diametral Compression of Cylinders of Various Shape Ratios.
(▮) Tension of Hollow Cylinders with Varying Internal Pressure
(o) Torsion of Hollow Cylinder
(x) Equal Biaxial Tension (Membrane)