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FINITE ELEMENT ANALYSIS OF CRACK PROPAGATION UNDER COMPRESSION

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INTRODUCTION

The behaviour of brittle materials under compression has been studied by, e.g., Hoek and Bieniawski [1] and Brace and Bombolakis [2]. Their experimental results can be summarized as follows:

1) Under compression, cracks propagate stably and further propagation of crack requires an increase of the applied stress.
2) Branching cracks emanate from the initial crack, deviate from the initial direction and gradually become aligned with the axis of the major compressive load.

As the stress at final catastrophic fracture is much greater than that at fracture initiation, as stated in (1), analysis of the propagation stage is quite important for the prediction of fracture under compression. Therefore, the authors paid their chief attention to the successive analysis of the change of stress states with crack propagation. Since the shape of cracks and stress states are quite complicated in the case of crack propagation under compression, they can be analysed only by finite element methods, and these must be more accurate finite elements than are conventionally used. Therefore, a 10 node, 20 degrees of freedom, triangular element, which makes it possible to adopt coarser meshes without the loss of high accuracy, was used in this analysis. A new finite element programme was developed, which can calculate the elastic contact stresses of crack surfaces at closure, in view of the fact that cracks might close under compression. The calculated results were closely comparable with the experimental results, and it was made clear that the fracture strength of brittle materials under compression cannot be evaluated without due consideration of the process of stable crack propagation.

THE 10 NODE 20 DEGREES OF FREEDOM TRIANGULAR FINITE ELEMENT

A triangular element of which the shape function is a complete cubic polynomial was used. This element possesses 10 nodes which correspond to the undetermined coefficients of the polynomial. The advantage of the use of a higher order shape function is that coarser mesh divisioning is possible without loss of high accuracy. Another advantage is that the value of stress and strain can be given by the node value.

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CONTACT PROBLEM OF CRACK SURFACES AT CLOSURE

Miyamoto and Shiratori [3] calculated contact stresses of crack surfaces at closure in order to study the opening and closure behaviour of fatigue cracks, but their analysis was limited to the case where the boundary condition of stress will be satisfied automatically due to symmetry if the boundary condition of displacement is satisfied on the contact surfaces. In the case of inclined cracks and their branching cracks which are to be studied in this analysis, the mere satisfaction of the boundary condition of displacement does not necessarily satisfy the boundary condition of stress on the closed crack surfaces.

Our newly developed finite element programme can deal with the contact problem on the closed crack surfaces where the state of contact cannot be assumed geometrically. In this finite element analysis, the boundary conditions of stress and displacement on the crack surfaces are replaced by those of the nodal force and nodal displacement on the crack surfaces. Let the node on the upper crack surface be i and the node on the lower surface be j, then the boundary conditions of closed crack surfaces are:

$$f_{Yi} + f_{Yj} = 0 \tag{1}$$

$$Y_i - Y_j + d_{Yi} - d_{Yj} = 0 \tag{2}$$

where f_{Yi}, d_{Yi} are respectively the force on and the displacement of node i in the Y direction. When there is no friction between the crack surfaces, the nodal forces in the X direction are equal to zero:

$$f_{Xi} = f_{Xj} = 0 \ . \tag{3}$$

The effect of friction is ignored in the following analysis for simplicity. As the area of contact of crack surfaces is generally not self-evident, the contact of crack surfaces is determined by the following conditons.

(1) If node i and node j contact each other, then $f_{Yi} > 0$.

(2) If node i and node j do not contact, $Y_i + d_{Yi} > Y_j + d_{Yj}$.

The correct solution satisfying the boundary condition of crack surfaces is obtained by repeating the same procedure, correcting successively the error of the boundary condition. This procedure is automatically carried out by the computer.

FRACTURE CRITERIA

As the stress state near the tip of a propagating crack is in the mixed mode I - II condition, the strain energy density criterion proposed by Sih [4] and the maximum stress criterion proposed by Erdogan and Sih [5], which are applicable to mixed mode fracture, were adopted in this analysis. The strain energy density criterion is based on the local density of the energy field in the crack tip region. For two-dimensional problems the strain energy density factor S is given by the quadratic form:

$$S = a_{11}K_I^2 + 2a_{12}K_IK_{II} + a_{22}K_{II}^2 \ , \tag{4}$$

where K_I and K_{II} are the stress intensity factors of mode I and mode II, respectively, and a_{ij} are the coefficients which are functions of Young's modulus, Poisson's ratio and the polar angle θ. The fracture criterion can be expressed mathematically for two dimensional problems in the following simple forms:

$$\partial S/\partial\theta = 0, \quad \partial^2 S/\partial\theta^2 > 0, \quad \theta = \theta_o \tag{5}$$

$$S_{min} = S(\theta_o) = S_{cr} \tag{6}$$

where θ_o is the fracture angle.

The maximum stress criterion postulates that the crack will open up in the plane normal to the direction of maximum stress and that crack propagation will occur when the maximum $\sqrt{2\pi r}\sigma_\theta$ value reaches K_{IC}. These conditions can be expressed as

$$K_I \sin\theta_o + K_{II}\,(3\cos\theta_o - 1) = 0 \tag{7}$$

$$\frac{1}{2}\cos\frac{\theta_o}{2}\left[K_I(1 + \cos\theta_o) - 3K_{II}\sin\theta_o\right] = K_{IC} \ . \tag{8}$$

SIMULATION PROCEDURE

The procedure for simulating crack propagation under compression was as follows. At each stage of propagation, the direction of the crack and the applied stress were determined by the above fracture criteria, using the stress intensity factor values calculated by the finite element method. The increments of crack growth were taken to be about 1/10 of the initial crack length, considering the experimental results [1], [2]. The width of the propagating crack was assumed to be infinitesimally small. The initial crack analyzed was of an elliptical form, but the crack growth increment Δa was about one hundred times greater than the radius ρ of the crack tip. Thus, it was considered that there was no effect of the finiteness of radius ρ on the direction and the stress of fracture initiation. Analysis was made in plane strain and the stress intensity factors were calculated by the displacement method. The simulation was carried out until complete failure of the whole specimen.

TEST SPECIMEN ANALYZED

The geometry and the mechanical properties of the test specimen analyzed were chosen so as to be the same as those of the glass specimen used by Hoek and Bieniawski [1] (see Figure 1). The mechanical properties of the glass specimen are given in Table 1.

SIMULATION RESULTS

1. Crack Path

The branching paths obtained are shown in Figure 2. The path shown in the upper right is obtained assuming the strain energy density criterion and the path shown in the lower left is obtained assuming the maximum stress criterion. If Δa is taken to be a/10, cracks propagate without increase of the applied stress after D1 and D2. In order to find the critical point Δa is taken to be a/20 after D1 and D2, then cracks propagate stably to F1 and E2. As the branching cracks become aligned with the axis of the compressive load, the cracks propagate in a zigzag manner. On the whole the crack path thus predicted agrees quite well with the experimentally obtained crack path. Meshes near the branching crack are shown in Figures 6 and 7.

2. Change of Stress Intensity Factors with Crack Propagation

The values of K_I and K_{II} at the tip of initial crack, calculated by the finite element method are:

$$K_I/\sqrt{\pi a}\sigma = 0.228 \quad ; \quad K_{II}/\sqrt{\pi a}\sigma = 0.397 .$$

Although the finite element values are respectively, 8.8% and 8.3% smaller than the theoretical values of K_I and K_{II}, the accuracy of the prediction of the crack path is expected to be high because the error in K_{II}/K_I, which determines the direction of the crack path, is only 0.53%. The initial crack closes when the applied stress, P_c, reaches 6400 MP_a. This is about 200 times greater than the crack initiation stress, and the crack does not close at crack initiation. The relationship between the branching crack length and normalized stress intensity factors is shown in Figure 3. It should be noted that, even under compression, K_I is positive, and the extending crack does not close during the stable process of propagation. No appreciable difference between the strain energy density criterion and the maximum stress criterion is observed.

3. Change of the Compressive Stress P_{cr} with Crack Propagation

The change in the compressive stress P_{cr} required for further crack extension with crack propagation is shown in Figure 4, together with the experimental data of Hoek and Bieniawski [1]. In the case of brittle materials, cracks, once initiated, will immediately lead to catastrophic failure if the applied stress is tensile. Therefore, the criterion for initiation can be regarded as the fracture criterion for total failure under tensile loading. Under compression cracks propagate stably and the "fracture hardening" phenomenon, which means that the applied stress must be increased in proportion to further propagate cracks, is observed; in the case of the strain energy density criterion, cracks do not propagate catastrophically until the L/2a value reaches 0.4. Therefore, quantitatively good agreement between experiment and simulation is found in the relation of P_{cr} to normalized crack length. Moreover, there is no appreciable difference between the maximum stress criterion and the strain energy density criterion. A great difference is observed, however, between simulation and experiment in terms of the fracture stress, but if we note the ratio of fracture stress/crack initiation stress, good agreement is found between simulation and experiment.

DISCUSSION

In this paper, the effect of friction was ignored for simplicity. This assumption can be regarded as appropriate, at least for this simulation, since the cracks did not close during the stable process of propagation. When the roughness of the crack surfaces becomes greater than the opening width of the crack, the crack surfaces are expected to contact. As the frictional force decreases the K_{II} value, the fracture stress will be increased if friction exists, and it is thought that this is one reason why the agreement between predicted and observed ultimate failure loads is so far off.

Several fracture criteria have been proposed for mixed mode problems, but they are originally derived for crack initiation and not for crack propagation. As cracks propagate stably under compression, a fracture criterion for crack propagation is required for analysis. In this paper, the strain energy density criterion and the maximum stress criterion are extended to crack propagation. From a continuum mechanics point of view, the ideal crack path can be mathematically expressed as a smooth differentiable curve. Because, when a finite change of angle occurs with an infinitesimal increment of crack growth, it is probable that the crack path depends on the crack growth increment Δa. Therefore, the fracture criterion for crack propagation should be compatible with the differentiability of the crack path curve. Many experimental results [4], [5] show that cracks change their direction in a zigzag manner if K_{II} is not zero. It follows from this that the fracture criterion for crack propagation must satisfy the condition that the K_{II} value be zero. Otherwise the differentiability of the crack path curve is not satisfied. Kitagawa and Yuuki [8] also suggested a "$K_{II} = 0$" criterion.

Nuismer [9] derived the following stress intensity factors for deflecting cracks when the crack growth increment converges to zero.

$$\overline{K}_I = \frac{1}{2} \cos \frac{\theta}{2} \left[K_I(1+\cos\theta) - 3K_{II}\sin\theta\right] \tag{9}$$

$$\overline{K}_{II} = \frac{1}{2} \cos \frac{\theta}{2} \left[K_I\sin\theta + K_{II}(3\cos\theta - 1)\right] \tag{10}$$

where K_I and K_{II} are the stress intensity factors at point O, and $\overline{K}_I$ and $\overline{K}_{II}$ are those at point $\overline{O}$ respectively. Figure 5 shows the comparison of Nuismer's results and Kitagawa and Yuuki's results. Here, β = arc tan (K_I/K_{II}), and θ is taken positive in the clockwise direction. Good agreement between the two results can be observed. If Nuismer's solution is correct, then the "$K_{II} = 0$" criterion is no other than the maximum stress criterion. But Kitagawa and Yuuki's results show that the effect of the crack growth increment being finite is not negligible, and that, in the case of a finite crack growth increment the "$K_{II} = 0$" criterion and the maximum stress criterion agree only approximately. Oscillating solutions of K_{II}, as seen in Figure 3, can be explained by this K_{II} decreasing effect, and the zigzag crack path in Figure 2 could be made smooth if we let the crack growth increment converge to zero.

CONCLUSIONS

(1) Fracture behaviour in compression can be well explained by a finite element simulation in which special attention is paid to crack propagation.

(2) Even under compression, K_I was positive, and cracks propagated in the opening mode in this simulation after the first crack growth increment.

(3) It was made clear that the fracture criterion for crack propagation must contain a "K_{II} = 0" condition, from the continuum mechanics point of view.

ACKNOWLEDGEMENT

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Table 1 The Mechanical Properties of the Glass Specimens

E (MPa)	ν	Kc (MPa·m$^{1/2}$)	Scr (MPa·m)
73550	0.25	1.47	2.91

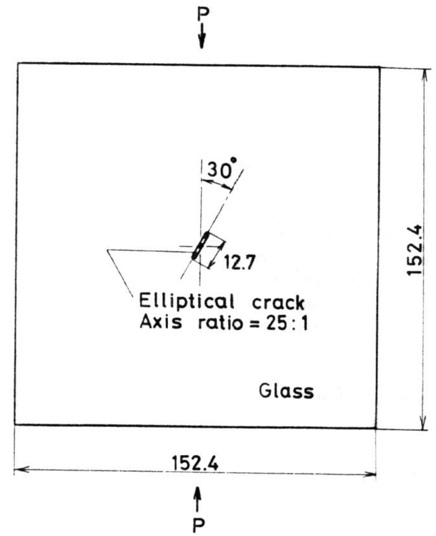

Figure 1 Test Specimen Analyzed

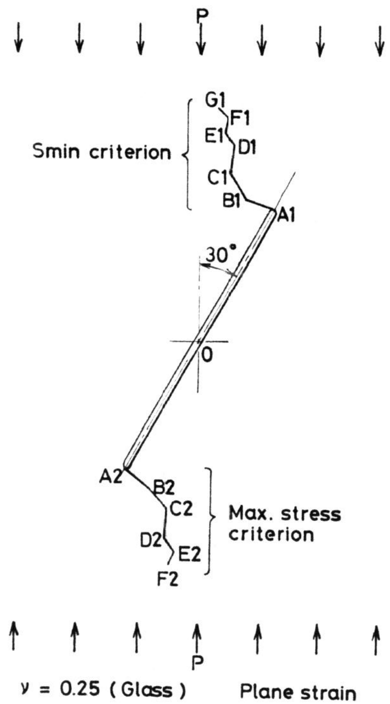

Figure 2 Crack Path

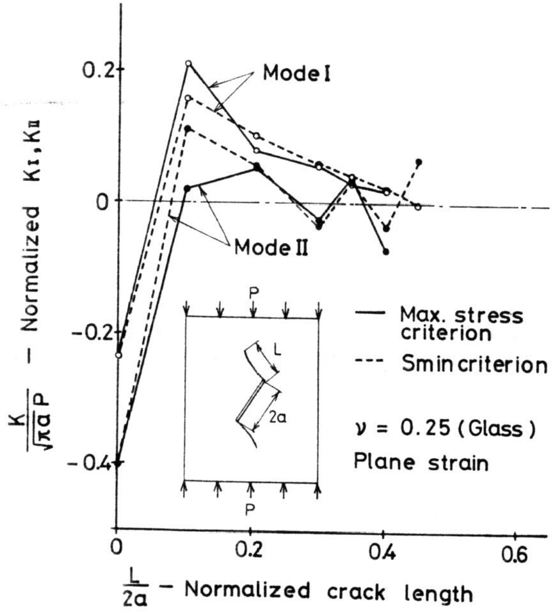

Figure 3 Change of Stress Intensity Factors with Crack Propagation

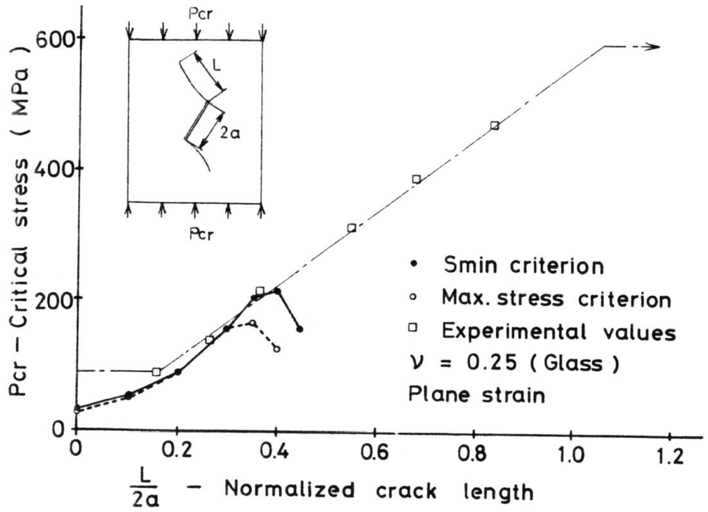

Figure 4 Change of the Compressive Stress with Crack Propagation

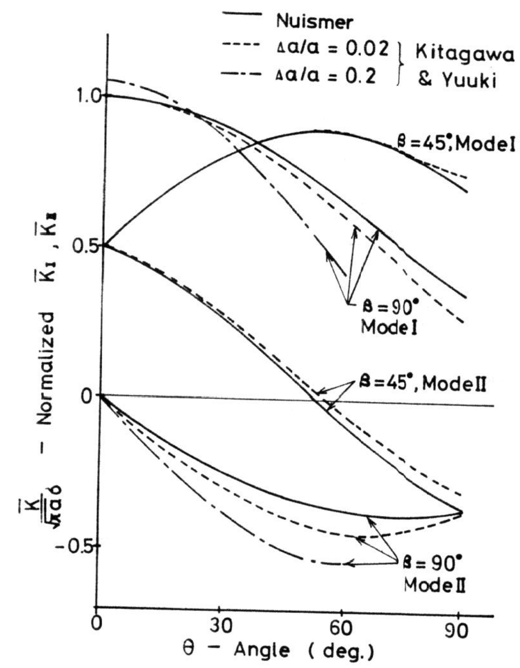

Figure 5 Stress Intensity Factors for Deflecting Crack

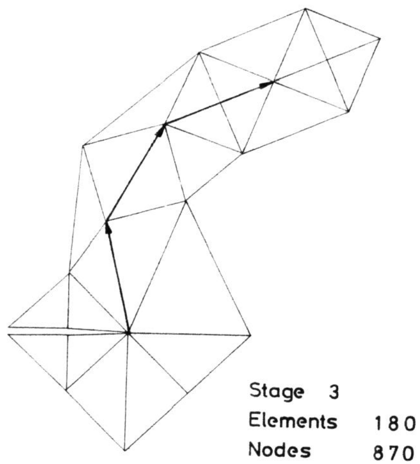

Figure 6 Breakdown of Branching Crack

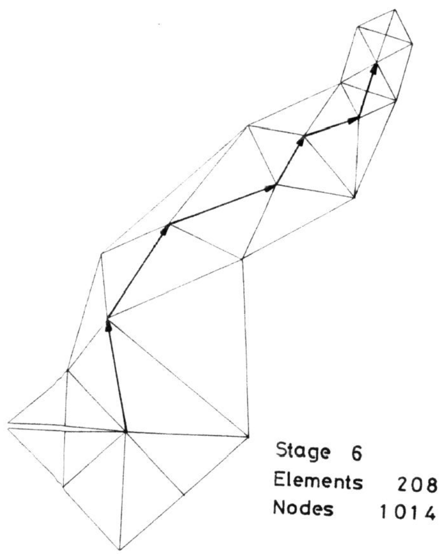

Figure 7 Breakdown of Branching Crack