Fracture 1977, Volume 3, ICF4, Waterloo, Canada, June 19 - 24, 1977

# EXISTENCE OF A CRITICAL STRAIN ENERGY RELEASE RATE FOR CONCRETE

J. Mazars*

## INTRODUCTION

Concrete, a mixture of aggregates (sand, gravel) and cement, closely bonded owing to the hydratation of cement, is a heterogeneous material. The fracture surfaces, through the pulling of the concrete units, may look several different ways:

- loosened aggregates
- broken aggregates

The usual mode of fracture is a mixture of both types, in variable proportions, according to the mineralogical character, the shape and the cleanliness of the particles, their relative toughnesses compared with the cement paste.

When adequate precautions are taken, the fracture of a structure constituted by a given kind of concrete obeys laws established here through fracture mechanics.

Kaplan [1], Nauss, Lott [2], and a few others have measured, on bending tests on notched concrete beams, the critical strain energy release rate Gc, at fracture. This notion is generalized on a certain type of concrete, on fracture tests by instability on pre-cracked test specimens variously shaped and loaded.

## CRITICAL STRAIN ENERGY RELEASE RATE

Griffith [3], then Irwin [4], and Orowan [5] have introduced the energetic method to define a parameter governing the sudden failure of cracked elastic brittle materials. The generalization of that notion to three dimensional non-linear problems was recently realized by J. Lemaitre [6]. That parameter is the critical strain energy release rate:

$$Gc = \frac{\Delta Vc}{\Delta A}$$

$\Delta A$ being the increase area of the crack
$\Delta Vc$ being the energy consumed during its extension.

Let U and W be respectively the strain energy and the external work, for a mechanism of increase of the crack of an area $\Delta A$. In an elastic line problem, the fracture criterion can be thus written:

*Laboratoire de Mécanique et Technologie, E.N.S.E.T. 61, avenue du Président Wilson, 94230 Cachan, France.



MS222

$$\frac{\Delta V}{\Delta A} \geq Gc \leftrightarrow \text{sudden propagation of the crack.}$$

APPLICATION OF THE FRACTURE CRITERION TO CONCRETE

In order to show that Gc is a specific parameter of concrete, a series of fracture tests were carried out on pulled test plates, some of these with central cracks and the others with single-edge cracks, and also on beams loaded in bending.

The concrete used was made with the following ratios by weight; 1 class 325 artificial portland cement, 1.8 ratio of 0/5 sand, 2.9 ratios of 5/20 rolled aggregates (silico-calcareous), 0.5 water.

The cracks were simulated by notches made of thin steel plates 0.5 mm thick tapering in their ends and introduced into the moulds.

The system which enabled us to exert tensile stresses on the plates has been represented on Figure 1.

The effort was exerted on axles set in pipes around which highly adhesive steels transmitting the efforts to concrete had been welded (the pipes and steels being sunk in the concrete).

A system of anchorage exterior to the plate made possible a perfect distribution of the effort on the four axles transmitting the stresses.

Many measurings (strain gauges) showed that the efforts thus transmitted resulted in a tensile stress distributed in a zone neighbouring the end of the plate.

The dimensions of the plates were 600 x 340 x 80 mm, the different widths of the crack being:

- 2a = 80, 140, 200 mm for the plates with central cracks.
- a = 40, 70, 100 mm for the plates with single-edge cracks.

As far as the bending test is concerned the method used was the centre point loading.

The dimensions of the beams were 150 x 100 x 1000 mm and the different crack widths, a = 20, 35, 50 and 70 mm.

The tests took place 28 days after the concrete was cast and the specimens were tested up to sudden crack propagation.

Thus we obtained the critical stress $\sigma c$ necessary to the determination of Gc.

The study of a heterogeneous material such as concrete made a global analysis of the fracture phenomenon necessary. From the starting hypothesis according to which the global behaviour of a concrete structure was similar to that of an equivalent structure made of homogeneous material, a calculation programme was conceived in order to handle cracked plane structure problems [7].





As a calculation method we chose the finite elements method.

For a given crack A, the critical stress $\sigma c$ (provided by the test) results in concentrated efforts ($Pcx^i$, $Pcy^i$) on the exterior nodes i, the calculation programme calculates the resulting displacement of the nodes i ($ux^i$, $uy^i$);

The material being elastic linear [8], we can write:

$$Uc\ (A) = Wc\ (A) = \frac{1}{2} \sum_i \{Pcx^i\ ux^i + Pcy^i\ uy^i\}$$

For a crack A + $\Delta$A, and the same stress $\sigma c$, the calculation programme gives Uc (A + $\Delta$A), thus:

$$\frac{Uc\ (A + \Delta A) - Uc\ (A)}{\Delta A} = \frac{\Delta Uc}{\Delta A} = Gc$$

The good agreement between the results provided by this programme and those available in the literature has been studied and discussed [7], [9].

For the type of tests described in this paper, the calculation of Gc might have been established by using the analytical expression available in ASTM literature, but enlargement of this study to more complex tests (reinforced concrete, precast concrete) made the elaboration of such a calculation programme necessary.

The results given by the former calculation were verified on beams loaded in bending by means of the compliance method, which in this case, resulted in:

$$Gc = \frac{1}{2} Pc^2 \frac{\Delta C(A)}{\Delta A}$$

Pc: critical crack propagation effort
C(A): beam compliance.

The difficulty to measure the variations of compliance did not enable us to apply this method to pulled plates.

THE RESULTS

The results are summed up in Figure 2. There were five tests on plates with central cracks, three tests on single-edge cracked ones, five tests on beams in bending.

In spite of the restricted number of the tests carried out, we can reasonably estimate that the value of Gc = 10.2 J/m$^2$ is characteristic of the fracture of that type of concrete.

From fracture tests on beams in bending realized by Nauss and Lott [2], we derived value of Gc equal to 7.2 J/m$^2$ for another type of concrete (the exact ratios have not been given). Gc having been derived from the values of $K_{IC}$ given by the analytical expressions of literature.

In our tests the good agreement between the values given by the tension and bending experiments corroborates the existence of the intrinsic number Gc as far as concrete is concerned.

REFERENCES


1. KAPLAN, M. F., "Crack Propagation and the Fracture of Concrete", J. of ACI, 58, 1962, 591.
2. NAUS, D. J. and LOTT, J. L., "Fracture Toughness of Portland Cement Concretes", J. of ACI, 66, 1969, 481.
3. GRIFFITH, A. A., "The Phenomena of Rupture and Flow in Solids", Phil. Trans. of Roy. Soc. of Longon, Series A 221, 1920, 163.
4. IRWIN, "Analysis of Stresses and Strains Near the End of a Crack Traversing a Plate", J. Appl. Mec., September 1967, 254.
5. OROWAN, J., "Energy Criteria of Fracture", Welding J. 34, March 1955.
6. LEMAITRE, J., "Extension de la notion de taux d'énergie de fissuration aux problèmes tridimensionnels et non linéaires", Compte-rendu de l'Académie des Sciences, 282 Série B, 1976, 157.

CHABOCHE, J. L. and MONTHULET, A., "Calcul du facteur d'intensité des contraintes pour la prévision de la progression des fissures", Recherche Aérospatiale 1974-4.

8. MASO, J. C., "Comportement mécanique du béton", Cours de 3éme Cycle, (Mécanique des structures déformables), Institut Henri Poincaré, Paris 1973.
9. MAZARS, J., "Prévision de la rupture des structures en béton par la mécanique de la rupture", Thèse de 3éme Cycle, Paris 1976.





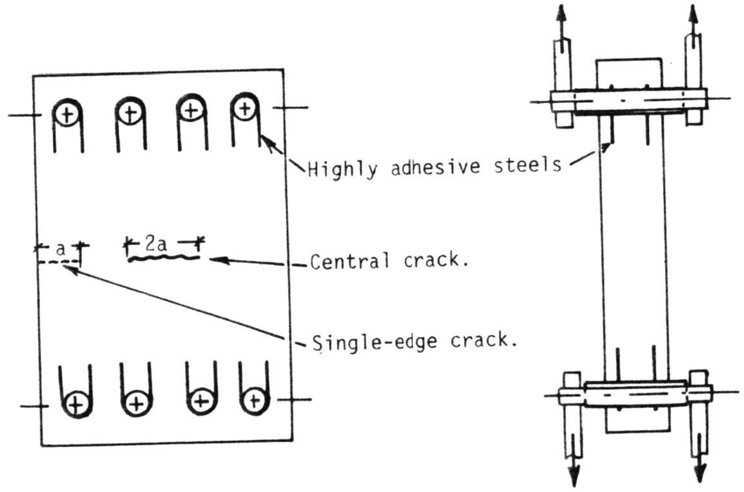
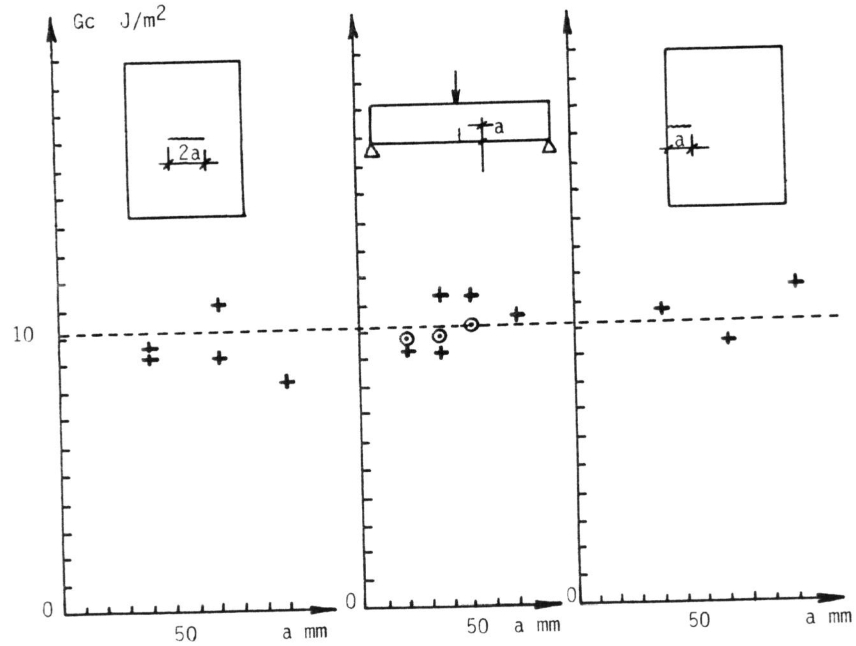




Figure 1 System used to exert tensile stresses on the plates




+ Calculation by the finite elements method.

⊙ Compliance method.

Figure 2 Results