ENGINEERING DESIGN AND FATIGUE FAILURE OF FUSED SILICA FIBRES

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INTRODUCTION

One important consideration in the design of a fibre optic communication cable is to assure the long-term, mechanical reliability of the glass fibres. Unfortunately, these glass fibres exhibit delayed failure (commonly known as static fatigue) and a wide variability in fracture strength that must be taken into account in the design of the cable. This paper will review the fracture mechanics foundation for making failure predictions for glass fibres based on fatigue strength data and will show how these predictions can be incorporated into a design diagram which gives the probability of failure in service for a given lifetime and applied stress, as well as, the proof stress necessary to assure a minimum lifetime in service. It is hoped that this paper will provide the background needed to develop better design techniques for the use of glass fibres.

FUNDAMENTALS

Wiederhorn and Evans [1 - 3] have provided a sound, fundamental background for making failure predictions and for the determination of proof test stresses and the establishment of proof test conditions. Their analysis is based on the reasonable assumption that failure of glass occurs mainly from stress-dependent growth of pre-existing flaws to dimensions critical for spontaneous crack propagation. By assuming a power functional relationship between crack velocity during the period of subcritical crack growth and the applied stress intensity factor, they derived that the time to failure (t_f) under a constant applied tensile stress (σ_a) is:

$$t_f = \left[\frac{2}{A\ Y^2\ (N-2)\ K_{IC}^{N-2}}\right] \sigma_{IC}^{N-2}\ \sigma_a^{-N} \qquad (1)$$

where K_{IC} = critical stress intensity factor, σ_{IC} = fracture strength in an inert environment, Y = geometric constant, A and N = constants. For a given glass and test environment, the term in brackets in equation (1) is constant, hence,

$$\log t_f = \log C + (N-2) \log \sigma_{IC} - N \log \sigma_a \qquad (2)$$

where C = constant. By using the average value of σ_{IC}, it can be seen from equation (2) that a log-log plot of t_f as a function of σ_a gives a straight line from which the constants C and N can be determined.

Evans [4] also derived that the fracture strength (σ_f) at a constant stressing rate ($\dot{\sigma}$) is:

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$$\sigma_f^{N+1} = \left[\frac{2}{A\, Y^2\, (N-2)\, K_{IC}^{N-2}}\right] (N+1)\, \sigma_{IC}^{N-2}\, \dot{\sigma} \tag{3}$$

where the term in brackets is the same as in equation (1). Thus, equation (3) can be rewritten as:

$$\log \sigma_f = \frac{1}{N+1}\left[\log C + \log (N+1) + (N-2) \log \sigma_{IC} + \log \dot{\sigma}\right] \tag{4}$$

Using equation (4), C and N can be determined from a log-log plot of σ_f as a function of $\dot{\sigma}$.

The probability of failure for a given t_f and σ_a (or a given σ_f and $\dot{\sigma}$) can be obtained from (equation (2)(or equation (5)) by expressing the inert strength (σ_{IC}) in terms of its failure probability distribution. By so doing, we assume that the sample with the shortest fatigue life has the lowest inert strength and that the origin of fracture is the same for both fatigue and inert failures. As will be shown, this statistical approach often leads to low design values for the allowable stress in service if low failure probabilities are required. Fortunately, the proof test method removes this disadvantage of the statistical approach to fatigue and assures that every specimen surviving the proof test will have a minimum service life since weaker specimens are eliminated. Procedures of proof testing and precautions that must be followed during proof testing are discussed fully elsewhere [3]. For our purposes we will only note that for effective proof testing the proof test should be conducted in a relatively inert environment and that moderately rapid unloading rates should be used. Also, the proof test must duplicate in the component the actual state of stress expected in service; and, after proof testing, the components must not incur damage that will negate the value of the proof test. In summary, if these precautions are followed, proof testing can be a practical method for assuring the mechanical reliability of glass [3].

From a fracture mechanics viewpoint, the value of proof testing is that it characterizes the largest effective flaw possible in the tested components since any larger flaw would have caused failure during the proof test. The minimum time to failure after proof testing is the time it takes for this maximum flaw to grow to critical dimensions for spontaneous fracture and is given by [1 - 3]:

$$t_{min} = \left[\frac{2}{A\, Y^2\, (N-2)\, K_{IC}^{N-2}}\right] \sigma_p^{N-2}\, \sigma_a^{-N} \tag{5}$$

or

$$t_{min} = C\, \sigma_p^{N-2}\, \sigma_a^{-N}$$

where σ_p = proof test stress. By comparing equation (5) with equation (2) it is seen that σ_p simply represents the minimum inert strength of the components after proof testing.

In summary, equations (2) and (5) give lifetime predictions before and after proof testing and are dependent on the inert strength distribution and the crack propagation constants C and N. These constants must be measured under the service conditions and can be obtained from either of two types of fatigue strength experiments. In one, the time to failure

under constant stress is determined as a function of applied stress and equation (2) is used to calculate C and N. This type of experiment is commonly termed static fatigue. In the other, fracture strength is measured as a function of stressing rate and equation (4) is used to calculate the required constants. This type of experiment is known as dynamic fatigue.

It is important to note that there is an uncertainty in the failure predictions discussed above due to the experimental scatter associated with determining the crack growth parameters and the inert strength. Statistical techniques have been derived to estimate this uncertainty in the failure predictions [5] and the statistical analysis of typical fatigue strength data for glasses show that this uncertainty is quite sensitive to the experimental error in the crack growth parameters [5]. Fortunately, much of this uncertainty in the failure predictions can be eliminated by increasing the required proof stress or by decreasing the allowable stress in service by a small increment [5].

DESIGN DIAGRAMS

The failure prediction techniques discussed in the preceding section are most easily understood by expressing the fatigue strength data in terms of a design diagram [1 - 3]. Design diagrams for a material in a given environment give the probability of failure for a given lifetime and applied stress and also the proof stress necessary to insure a minimum lifetime at a given stress. To illustrate the construction and use of these diagrams, static and dynamic fatigue data [6] for fused silica fibres in air at room temperature was analyzed.

A least square analysis of the static fatigue data [6] gives:

$$\log t_f \text{ (sec)} = 72.02 - 19.46 \log \sigma_a \text{ (MPa)} \tag{6}$$

Using the average inert strength as measured in liquid nitrogen, 1.38 x 10^4 MPa (2 x 10^6 psi), the constants in equation (2) can be determined as follows:

$$\log t_f = 0.26 + 17.4 \log \sigma_{IC} - 19.46 \log \sigma_a \tag{7}$$

The inert strength distribution of these fibres can be reasonably assumed to be given by the Weibull relationship:

$$\ln \ln \frac{1}{1-F} = 7.0 \ln \sigma_{IC}/1.45 \times 10^4 \tag{8}$$

Figure 1 gives the design diagram based on the static fatigue data where the curves for failure probability without proof testing are obtained from equations (7) and (8) and the proof test ratio curves from equation (5) where log C = -0.26 and N = 19.46.

A least square analysis of the dynamic fatigue data [6] gives:

$$\log \sigma_f \text{ (MPa)} = 3.616 + 0.04554 \log \dot{\sigma} \text{ (MPa/sec)} \tag{9}$$

From this data and the average inert strength, the constants in equation (2) can be determined as follows:

$$\log t_f = -0.43 + 18.96 \log \sigma_{IC} - 20.96 \log \sigma_a \quad (10)$$

Figure 2 gives the design diagram based on the dynamic fatigue data where the curves for failure probability without proof testing are obtained from equations (8) and (10) and the proof test ratio curves from equation (5) where log C = -0.43 and N = 20.96.

Design diagrams are most useful to the designer in deciding if a proof test is necessary and, if necessary, in determining the proof test required to insure a given lifetime. For example, from Figure 1 one can see that the point of intersection for a lifetime of 10 years at a constant applied stress of 689 MPa (10^5 psi) gives a failure probability of 3×10^{-4}. In other words, without proof testing 0.03% of the samples will fail within 10 years at a stress level of 689 MPa. By comparison, a proof stress ratio of 6.7 is necessary to insure no failures for these same service requirements.

The important thing to note on comparing the two design diagrams based on static and dynamic fatigue data (Figures 1 and 2) is that they give similar failure predictions. For example, for a lifetime of 10 years at a stress level of 689 MPa, the predicted failure probability without proof testing based on the static fatigue data is 0.03% and based on dynamic fatigue data is 0.01%. To assure no failure under these same service requirements, a proof test ratio of 6.7 is estimated from the static fatigue data and 5.8 based on the dynamic fatigue data. This difference in the predicted proof test ratios necessary for no failure is about 15% and is probably within the experimental uncertainty in the predictions.

It should also be noted that the failure probability curves in Figures 1 and 2 are based on laboratory specimens and have not been scaled for the fact that these samples are much smaller than will be the actual fibres in the cables. Recent data [7] on silica fibres 1000 m long show that this size effect can reduce the strength to about 1/10 that of the laboratory specimens. Thus, this strength reduction limits the use of the fibres in service for a 10 year lifetime at the 10^{-4} probability level to a design applied stress of about 68.9 instead of the 689 MPa obtained from Figures 1 and 2.

In conclusion, from design diagrams one can make long-term failure predictions for glass fibres with and without proof testing. These diagrams are based on crack growth parameters that can be measured from fracture mechanics experiments or from static and dynamic fatigue strength data. Being able to base failure calculations on fatigue strength measurements is most important since it cannot be assumed that data from large, preformed cracks (as present in fracture mechanics samples) are relevent to the propagation of submicroscopic cracks present in glass fibres. Also, failure predictions for fused silica fibres based on dynamic fatigue data approximates that based on static fatigue data which gives support to the use of the short-term dynamic fatigue tests in lieu of the longer-term static test in determining the crack growth parameters.

ACKNOWLEDGEMENT

This work was supported in part by the National Science Foundation.

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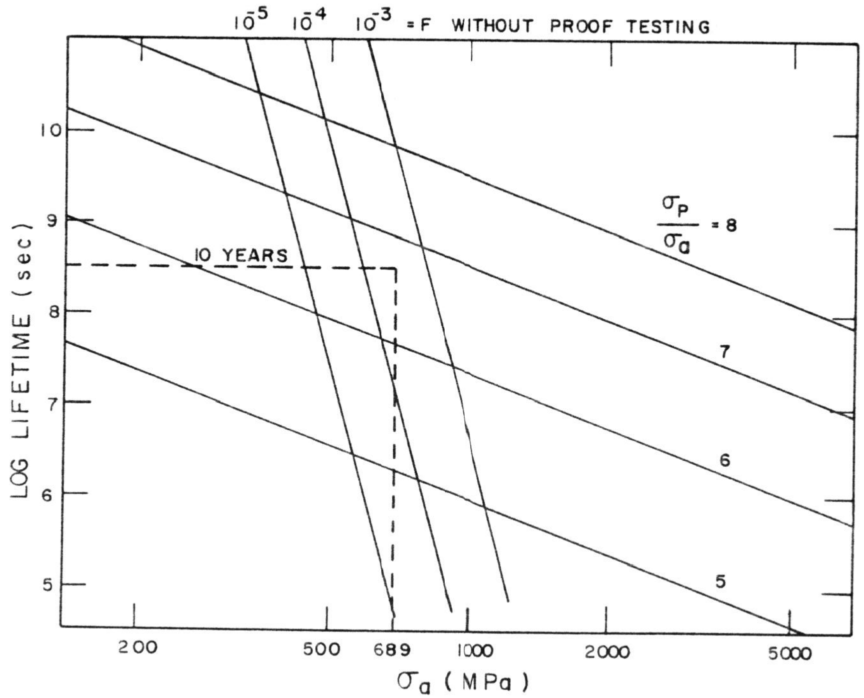

Figure 1 Design Diagram for Fused Silica Fibres Based on Static Fatigue Data in Air at Room Temperature [6]

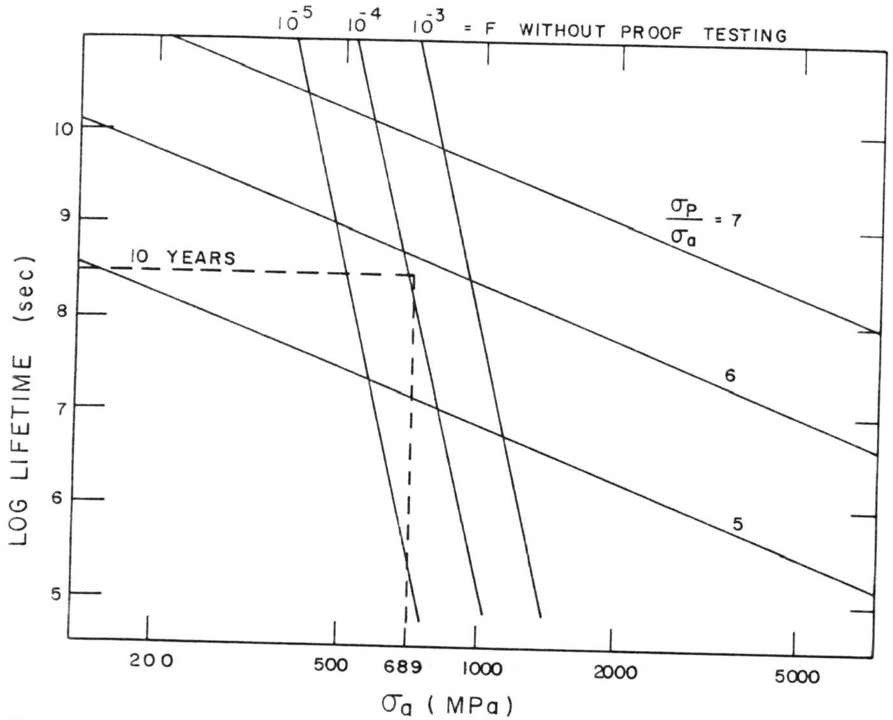

Figure 2 Design Diagram for Fused Silica Fibres Based on Dynamic Fatigue Data in Air at Room Temperature [6]