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ELASTODYNAMIC EFFECTS ON CRACK BRANCHING

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INTRODUCTION

If a homogeneous, isotropic, linearly elastic solid containing a plane crack is loaded so that the analytically computed singular parts of the near-tip stresses are symmetric relative to the plane of the crack, one might perhaps expect the crack to propagate in its own plane, when the pertinent stress intensity factor reaches a critical value. Experimental evidence often shows, however, the phenomena of skew crack propagation and crack bifurcation, especially for rapidly propagating cracks. Although it has been suggested by several authors that elastodynamic effects play an important role in crack branching, analytical investigations have only recently become available for antiplane strain, see references [1] and [2]. The computation of the elastodynamic fields has presented the principal obstacle.

The general nature of elastodynamic near-tip fields for the case that the tip of a crack propagates rapidly along a rather arbitrary but smooth trajectory in a two-dimensional geometry, was discussed by Achenbach and Bazant [3]. Let a crack be propagating in its own plane with speed v(t), and let a system of moving polar coordinates be affixed to the moving crack tip. For symmetric opening up of the crack (Mode I) we have in the vicinity of the crack tip.

$$\tau_\theta \sim \frac{1}{(2\pi)^{1/2}} \frac{1}{r^{1/2}} k_I(t,v)\, T^I_\theta(\theta,v) \qquad (1)$$

In equation (1), $T^I_\theta(0,v) = 1$, and $k_I(t,v)$ is the elastodynamic stress intensity factor. The function $T^I_\theta(\theta,v)$, which is complicated, is shown in Figure 1. It is of note that the maximum value of $T^I_\theta(\theta,v)$ bifurcates out of the plane $\theta = 0$ (the plane of crack propagation) as v(t) increases beyond a certain value.

The curves of Figure 1 could be used to suggest an explanation for crack bifurcation, if it is assumed that a crack tip follows the maximum value of the stress intensity factor. If this would happen, bifurcation should be expected at a crack tip speed somewhat higher than 0.6 c_T. One then would expect the crack branches to curve gradually out of the original plane, since the maximums gradually move out of $\theta = 0$. Experimental results do, however, not substantiate this explanation. They show that the experimentally observed pre-bifurcation speed is lower than 0.6 c_T, and that bifurcation happens with a specific half-angle, in between 10 and 20° Thus, the results of Figure 1 do not offer a direct explanation of crack bifurcation, and further study is necessary. Such further study is the topic of this paper.

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APPROACH

In the work presented here we take the view that branching of a running crack is an instability phenomenon, and that a necessary condition for branching can be determined by comparing states prior to branching and after branching has taken place. The comparison requires expressions for the elastodynamic fields near the tips of the branches. An analytical study of skew crack propagation or crack bifurcation thus consists of two parts. In the first part an expression is derived for the elastodynamic stress intensity factor for the pertinent geometry. In the second part a necessary condition for the particular type of crack propagation is established on the basis of the fracture criterion of the balance of rates of energies.

Details can best be explained by the relatively simple case of deformation in antiplane strain. Let us consider a semi-infinite crack propagating at velocity v(t). The near-tip elastodynamic stress is of the form

$$\tau_{\theta z} \sim \frac{1}{(2\pi)^{1/2}} \frac{1}{r^{1/2}} k_{III}(t,v) T^{III}_{\theta z}(\theta,v) \qquad (2)$$

where $T^{III}_{\theta z}(0,v) = 1$. For a semi-infinite crack we have

$$k_{III}(t,v) = (1 - v/c_T)^{1/2} K_{III}(t) \qquad (3)$$

where $K_{III}(t)$ is the stress intensity factor for the corresponding quasi-static problem. Equation (3) is also valid for a crack of finite length, but only for very small times after crack propagation has started. It is noted that $k_{III}(t,v) \to 0$ as $v \to c_T$, i.e., as the speed of crack propagation approaches the velocity of transverse waves.

A propagating crack tip acts as an energy sink. It is quite simple to compute the flux of energy into the crack tip. For Mode III fracture the result is

$$F = \frac{v}{2\mu\left[1 - (v/c_T)^2\right]^{1/2}} \left[k_{III}(t,v)\right]^2 \qquad (4)$$

The energy release rate G and the flux of energy into the crack tip, F, are related by F = Gv. The balance of rates of energies provides the following necessary condition for fracture

$$F = 2\,\Gamma\, v \qquad (5)$$

where Γ is the specific energy of crack extension, i.e., the energy required to produce one unit of fracture surface. Equation (5) is not only a necessary condition for fracture, but it also provides an equation for the computation of v.

At time $t = t_b$ the crack branches. This process is thought of as the arrest of the primary crack, instantaneously followed by the emanation of the branch or branches. If the branches propagate with velocities $v < c_T$, they propagate into fields that have already received signals from the arrest of the primary crack, since the latter propagate with velocity c_T.

The singular part of the stress field radiating from the arresting crack is obtained from equation (2) by setting $v \equiv 0$:

$$\tau_{\theta z} \sim \frac{1}{(2\pi)^{1/2}} \frac{1}{r^{1/2}} \cos\left(\frac{\theta}{2}\right) K_{III}\left(t_b\right) H\left(t - t_b\right) \qquad (6)$$

This stress field must be removed from the surfaces of the branches, to render the branches free of shear stresses. Note that here we are interested only in very small times after branching, so that additional terms in equation (6) do not enter.

The computation of the stress-singularities at the tips of the branches is complicated. If the loading conditions are of a special type, the elastodynamic fields for skew crack propagation or crack bifurcation of a semi-infinite crack are, however, self-similar. These fields can then be analyzed in a relatively simple manner. Elastodynamic fields that are not self-similar can subsequently be obtained by approximate superposition considerations, see reference [2].

SOME RESULTS

The elastodynamic field which is generated when a branch emanates assymmetrically from the tip of a stationary semi-infinite crack, when the surfaces of the crack are subjected to shear tractions $\tau_{\theta z} = -\tau_0 H(t)$ is first investigated. The shear tractions give rise to plane waves and a cylindrical diffracted wave centred at the original crack tip. The semi-infinite crack propagates at an angle $\varkappa\pi$ and with velocity v, where $v < c_T$, at the instant that the surface tractions are applied. At time $t > 0$ the crack tip is located at point D, see Figure 2. For this problem the particle velocity is self-similar. For a similar problem, details can be found in reference [1]. Relative to the system of moving coordinates shown in Figure 2 we find near the tip

$$\tau_{\theta z} \sim \frac{1}{(2\pi)^{1/2}} \frac{1}{r^{1/2}} k_{III}(t,v,\varkappa)\; T^{III}_{\theta z}(\theta,v) \qquad (7)$$

where

$$k_{III}(t,v,\varkappa) = 2\,\pi^{1/2}\left(1 - \frac{v^2}{c_T^2}\right)^{1/4}\left(\frac{t}{v}\right)^{1/2} K(\varkappa) \qquad (8)$$

The function $K(\varkappa)$ follows from equation (3.8) of reference [1] by setting $\alpha = 0$ and $W_0 = \tau_0 c_T/\mu$.

The results obtained above can now be used to analyze the conditions for the emanation of a single branch from a running crack. After branching of the *running* crack, the shear stress near the branch tip is of the general form

$$\tau^{*}_{\theta z} \sim \frac{1}{(2\pi)^{1/2}} \frac{1}{r^{1/2}} k^{*}_{III}(\bar{t},v)\; T^{III}_{\theta z}(\theta,v) \qquad (9)$$

where $\bar{t} = t - t_b$. We have found that the near tip stress field for instantaneous skew crack propagation upon the application of equal and uniform antiplane shear tractions to the two semi-infinite surfaces of a stationary crack is given by equation (7). If for that case the crack does not branch, nor propagate in its own plane, the near tip stress field is

$$\tau_{\theta z} = \frac{B}{(2\pi)^{1/2}} \frac{t^{1/2}}{r^{1/2}} \cos\left(\frac{\theta}{2}\right) H(t), \text{ where } B = 2\left(\frac{2c_T}{\pi}\right)^{1/2} \tau_o \quad (10)$$

Clearly, the result (7) can be regarded as being the consequence of removing stresses of the form (10) from the crack branches. Thus, we now have a known stress (7) due to the removal of the known distribution of surface tractions (10), and an unknown stress (9) due to the removal of the known distribution (6). Apart from constants the difference between equations (6) and (10) is, however, only in the time dependence; equation (6) contains a step time dependence, while in equation (10) the dependence on time is as $t^{1/2}$. These results then suggest that at least for small times k^*_{III} and k_{III} are related by superposition considerations as

$$k_{III}(t) = \frac{B}{K_{III}(t_b)} \int_0^t k^*_{III}(t-s)\, d(s^{1/2}) \quad (11)$$

This equation can easily be solved for k^*_{III} as

$$k^*_{III} = \left(1 - \frac{v^2}{c_T^2}\right)^{1/4} \left(\frac{1}{2c_T v}\right)^{1/2} \frac{\pi K(\varkappa)}{\tau_o} K_{III}(t_b) \quad (12)$$

The corresponding flux of energy into the crack tip can be computed by employing equation (4). The noteworthy result is that the rate of energy flux into a propagating crack tip shows a maximum at $\varkappa = 0$ only for values of v/c_T which are smaller than approximately $v/c_T = 0.27$. Apparently the rate of energy flux into a crack tip can be higher for skew crack propagation than for a crack propagating in its own plane.

The tendency towards skew crack propagation can be examined on the basis of the balance of rates of energies. This fracture criterion is stated by equation (5). For essentially brittle fracture Γ is the specific surface energy, which is independent of $\varkappa$. In a plot of F vs. $\varkappa$ and $2\Gamma v$ vs. $\varkappa$ for specific v/c_T, the term $2\Gamma v$ is then represented by a horizontal line. In accordance with the balance of rates of energies, the values of v and $\varkappa$ are determined by a point of intersection of the curves for F and $2\Gamma v$. Since both v and $\varkappa$ are as yet unknown an additional condition is required. Such an additional condition is that only an intersection where $2\Gamma v$ is tangent to F (i.e., F is a maximum with respect to $\varkappa$) defines a case of stable crack propagation relative to variations of $\varkappa$. Thus, in Figure 3, the maximums of F with respect to $\varkappa$ have been replotted versus v/c_T, and values of $\varkappa$ at which the maximums of F are reached have been indicated. In this figure $2\Gamma v$ is a straight line through the origin. The intersection of $2\Gamma v$ and F defines a case of crack propagation and the pertinent values of v and $\varkappa$ follow from the point of intersection in Figure 3. The foregoing discussion defines Γ as the principal quantity controlling skew crack propagation. For small enough Γ, $2\Gamma v$ is tangential to F at $\varkappa = 0$, and thus v/c_T will be relatively small and the crack will propagate in its own plane. For large values of Γ the relevant intersection is at $\varkappa > 0$, i.e. skew crack propagation can be expected.

An analogous analysis for crack bifurcation in antiplane strain was presented in reference [2].

THE IN-PLANE PROBLEM

Computations of deformations in antiplane strain (Mode III), though important for geophysical situations, are of minor practical significance for engineering problems. Solutions of antiplane problems do, however, frequently suggest the proper steps for the attack on in-plane problems. There are, however, some important differences in the basic fracture mechanics of the antiplane and inplane cases, and these should be kept in mind. For example, for inplane deformation the branches of a primary crack are subjected to both Mode-I and Mode-II fracture conditions. Mixed fracture conditions do not occur for crack bifurcation in antiplane strain. Thus it is necessary to analyze the inplane problem separately.

The case of inplane strain is, of course, must more complicated. In the physical plane the region in which the stress field must be analyzed consists of wedge-shaped segments which are connected ahead of the propagating crack tip(s). For inplane deformations there are two wave equations governing the displacement potentials. By taking advantage of self-similarity, these wave equations can be reduced to Laplace's equations in half-planes. The solutions to these equations are, however, coupled along the real axes by conditions which stem from the conditions along the crack surfaces and along the wavefronts. The coupling conditions give rise to singular integral equations for the displacement potentials. A numerical scheme based on series expansions in terms of Chebyshev polynomials has been developed to obtain numerical solutions. Results are forthcoming.

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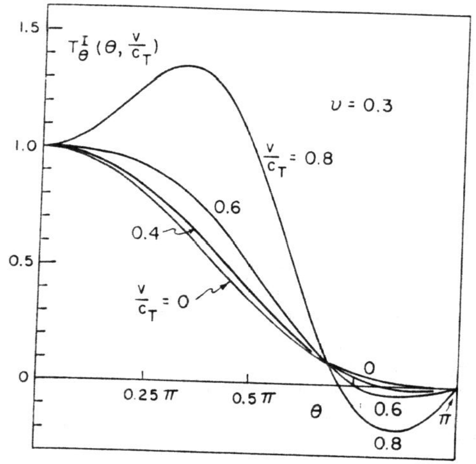

Figure 1 Function T_θ^I (θ,v) versus θ for Various Values of v/c_T.

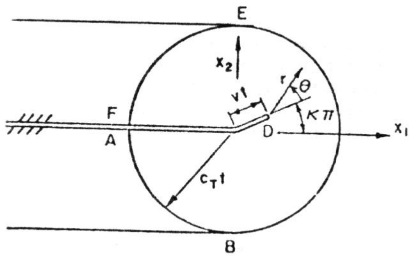

Figure 2 Pattern of Wavefronts and Position of Crack Tip for Crack Branching Under the Influence of a Suddenly Applied Antiplane Shear Traction

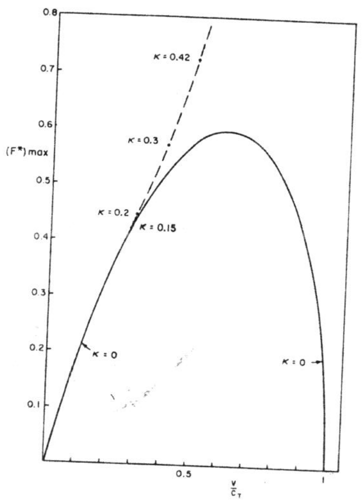

Figure 3 Maximums of F with Respect to $\varkappa$, Plotted vs. v/c_T;
$F^* = 4\mu F/c_T[K_{III}(t_b)]^2$