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ELASTIC CONTACT PROBLEMS IN FRACTURE MECHANICS

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INTRODUCTION

During a load cycle an existing crack may partially close causing the fractured surfaces to make contact. This would of course influence the stress field. The frictional forces arising at the contact surface could be expected to influence the direction of crack propagation. Contact problems in fracture mechanics have been studied by some authors. Askogan [1] presents a solution to the elastic problem of a closing Griffith crack. Paris and Tada [2] studies a closing elastic single edge crack loaded in mode I. Newman [3] studies the effects of closing cracks in fatigue crack propagation. Erdogan and Gupta [4] studies contact and crack problems of elastic wedges. All of these solutions are restricted to specific types of geometries and loadings and do not include friction. In the present paper an attempt is made to do a unified approach to elastic contact problems taking frictional effects into account.

In order to obtain a solution which takes the frictional effects into account a general slip criterion with associated slip rule is introduced. As a special case of the general one a Coulomb type of slip criterion is used in the numerical calculations. The incremental governing equations for elastostatic contact problems with friction are solved by means of the finite element method. Attention is focused on an existing crack that may partially close during a load cycle. Considering a virtual crack growth the crack extension work is derived by applying the principle of virtual work. The energy dissipation due to friction at the contacting surfaces is obtained.

A finite element computer programme for two-dimensional elastic, plane and axisymmetric, problems has been developed. Stress intensity factors are calculated and the effect of crack closure is shown. Crack extension work for different virtual propagation directions is calculated and the effects of crack closure and frictional properties are shown.

STATEMENT OF THE PROBLEM

Consider a body containing a crack which might have closed due to the loading (Figure 1). The problem is studied in the orthogonal cartesian coordinate system x_1, x_2, x_3. In order to facilitate the study of oblique or curved cracks, a local coordinate system η_1, η_2, η_3 is introduced. η_1, η_2 defines the tangent plane to the crack surface. We indicate the material on one side of the crack with A and the other with B (Figure 1). η_3 is then defined as the outward normal vector from B. Temperature effects and dynamic terms are omitted. The body is assumed to be loaded by surface loads q_i on Γ_q and volume loads X_i in V. The displacements are assumed to

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be prescribed on Γ_u. The crack surfaces are assumed to be in contact on Γ_c. On Γ_{cr} the cracked surfaces are not in contact and are unloaded.

THE CONTACT PROBLEM

Bodies generally contact each other through small irregularities in their surfaces. The applied normal load forces the summits of irregularities to flow plastically and/or to crush down until their cross sections are sufficient to support the applied load. It is easy to understand that the type of contact is different at different summits. At some there may be cold welding and micro-seizure and at other summits we have merely elastic contact. When shear forces arise the joint surfaces are displaced. The displacement ceases when the micro-seizure points within the real contact area have reached sufficient numbers to be able to offset the applied tangential load. When a stable condition is attained some of the summits are in adhesion or a welded state and others are in a state of elastic contact. It may be reasonable to assume that elastic and small plastic deformations appear at the adhesion points and that relative displacements appear at the elastic contact points. As a consequence of these assumptions it can be understood that a micro-slip appears even though the applied tangential load is smaller than the sliding force as determined by using the macroscopic coefficient of friction. Based upon these ideas a general contact constitutive relation will be introduced. The contact surface is assumed to be ideal and free from the above mentioned irregularities. The constitutive relation relating the contact stresses and the slip will however be derived in order to satisfy the real case. The derivation is analogous to the derivation of the flow rule in the theory of plasticity.

Consider the cartesian coordinate system η_1, η_2, η_3 at a contact point given on Γ_c. The contact stress increment vector is written

$$dp_i = (dp_1, dp_2, dp_3) \text{ on } \Gamma_c \, . \tag{1}$$

In the following, when the indices α, β, γ, δ occur, they are assumed to range from 1 to 2 and refer to the local coordinate system η_1, η_2, η_3. A detailed derivation of the slip rule is given by Fredriksson [5]. The derivation of the slip rule is based upon two *basic assumptions*.

1) The slip increment dv_α is linearly dependent on the contact stress increment. That is,

$$dv_\alpha = du_\alpha^A - du_\alpha^B = h_{\alpha\beta}\, dp_\beta \tag{2}$$

2) There exists a *slip surface* $g(p_i) = 0$ in the contact stress space on which slip will occur. At each state of the slip no further slip will occur unless

$$\frac{\partial g}{\partial p_i} dp_i > 0 \, . \tag{3}$$

The first assumption implies that the slip has the same direction as the outward normal vector to the locus generated from the intersection between the surface and the plane p_3 = constant.

For *slip hardening* we obtain

$$dv_\alpha = \frac{1}{L} \frac{(\partial g/\partial p_\alpha)(\partial g/\partial p_\beta)}{(\partial g/\partial p_\delta)(\partial g/\partial p_\delta)} dp_\beta \text{ on } \Gamma_c^s \tag{4}$$

when

$$g = 0, \ \frac{\partial g}{\partial p_i} dp_i > 0, \ p_3 < 0 \, .$$

Γ_c^s is the part of Γ_c, where the slip criterion is satisfied. In the part Γ_c^a of Γ_c where the slip criterion is not satisfied there is no slip increment and the displacement increment must satisfy

$$dv_\alpha = du_\alpha^A - du_\alpha^B = 0 \text{ on } \Gamma_c^a \, . \tag{5}$$

If we assume *ideal slip* (4) is replaced by the slip rule

$$dv_\alpha = \lambda \frac{\partial g}{\partial p_\alpha} \text{ on } \Gamma_c^s \tag{6}$$

$$\lambda \geq 0 \text{ when } g = 0 \text{ and } \frac{\partial g}{\partial p_i} dp_i = 0, \ p_3 < 0$$

$$\lambda = 0 \text{ when } g < 0 \text{ or, } \frac{\partial g}{\partial p_i} dp_i < 0, \ p_3 < 0$$

$\frac{\partial g}{\partial p_i} dp_i > 0$ does not exist in ideal slip. In ideal slip the slip surface is fixed.

The functions g and L depend on properties of the contact surface, for instance type of material and surface roughness. In the case of ideal slip the parameter λ is indeterminate.

Assuming *Coulomb isotropic slip criterion* [5] we obtain

$$g(p_i) = \frac{1}{\mu} (p_\alpha p_\alpha)^{1/2} + p_3, \ p_3 < 0 \tag{7}$$

and the associated slip rule for hardening slip

$$dv_\alpha = \frac{1}{L} \frac{p_\alpha p_\beta}{p_\delta p_\delta} dp_\beta \tag{8}$$

μ is the coefficient of friction.

For ideal slip we obtain

$$dv_\alpha = \lambda \frac{p_\alpha dp_\alpha}{\mu (p_\delta p_\delta)^{1/2}} \, . \tag{9}$$

Introducing the *effective contact stress*

$$p_e = (p_\alpha p_\alpha)^{1/2} \tag{10}$$

and the *effective slip*

$$v_e = \int dv_e; \quad dv_e = (dv_\alpha \, dv_\alpha)^{1/2} \tag{11}$$

it can be shown [5] that for slip hardening

$$L = -p_e \frac{d\mu(v_e)}{dv_e} . \tag{12}$$

Thus, the single curve $\mu = \mu(v_e)$ yields both the shear stress necessary to obtain slip and the function L, which might be called the *slip modulus*. For ideal slip L is zero and equation (9) has to be used. The parameter λ is indeterminate and the stiffness properties of the contacting bodies must be used to obtain the slip increment.

Furthermore, the displacement increment perpendicular to the contact surface must satisfy the kinematical condition

$$du_3^A - du_3^B = 0 \text{ on } \Gamma_c . \tag{13}$$

CRACK EXTENSION WORK IN CRACK CLOSURE PROBLEMS

Consider a virtual quasistatic crack growth. It is assumed that macroscopic (continuum mechanics) theory is applicable [6]. The virtual quasistatic crack growth generates a new crack surface Γ_{cr}^+ with a corresponding fracture area ΔS. The work done on the fracture process zone per unit of fracture area [7] during this virtual growth from state 1 to state 2 is the *crack extension work* G [7]

$$G = - \lim_{\Delta S \to 0} \frac{1}{\Delta S} \int_{\Gamma_{cr}^+} \left(\int_1^2 q_i \, du_i \right) d\Omega . \tag{14}$$

$-q_i$ is, by definition, the stress vector acting from the continuum on the fracture process zone.

By applying the principle of virtual work the crack extension work can alternatively be expressed in global terms. Assume an infinitesimal virtual crack growth with corresponding displacements du_i. Applying the principle of virtual work to the total stress field we obtain

$$\int_V \sigma_{ij} \, d\varepsilon_{ij} \, dV + \int_{\Gamma_c} p_\alpha \, dv_\alpha \, d\Gamma = \int_V X_i \, du_i \, dV + \int_{\Gamma_q} q_i \, du_i \, d\Gamma + \int_{\Gamma_{cr}^+} q_i \, du_i \, d\Gamma \tag{15}$$

where it is assumed that $du_i = 0$ on Γ_u. The strain increment $d\varepsilon_{ij}$ and the slip increment dv_α are both compatible with the displacement increment.

Integrating equation (15) from state 1 to state 2 and introducing the total potential energy increment

$$\Delta\Pi = \int_V \left(\int_1^2 \sigma_{ij} \, d\varepsilon_{ij} - \int_1^2 X_i \, du_i \right) dV - \int_{\Gamma_q} \left(\int_1^2 q_i \, du_i \right) d\Gamma \tag{16}$$

we obtain

$$- \Delta\Pi - \int_{\Gamma_c} \left(\int_1^2 p_\alpha \, dv_\alpha \right) d = - \int_{\Gamma_{cr}^+} \left(\int_1^2 q_i \, du_i \right) d\Gamma . \tag{17}$$

Assume that the process is described using the fracture area S as a parameter [7]. Dividing equation (17) by the finite increment ΔS we obtain in the limit

$$- \Pi' - C' = G \tag{18}$$

where

$$\Pi' = \lim_{\Delta S \to 0} \frac{\Delta\Pi}{\Delta S}$$

is the change in total potential energy per unit of fracture area and

$$C' = \lim_{\Delta S \to 0} \frac{1}{\Delta S} \int_{\Gamma^c} \left(\int_1^2 p_\alpha \, dv_\alpha \right) d\Gamma \tag{19}$$

is the dissipated energy due to friction at the contact surface. Thus the sum of G and C' expresses the total energy dissipation.

From equation (18) it can immediately be concluded that the frictional properties influence the crack extension work G. When there is no friction present C vanishes. Although C vanishes the closure of the crack still influences G since the stress field is influenced and thereby the potential Π.

APPLICATIONS

The incremental governing equations for the contact problem are solved by means of the finite element method [8]. In the computer programme Coulomb slip criterion with associated ideal and hardening slip is included. The surfaces of the existing crack (or cracks) are defined as contact surfaces and the nodes in the finite element model are defined as contact nodes. The contact nodes at the crack tip may be allowed for cohesional forces. The external nodes are next applied and the contact nodes are checked for closure. When closure occurs iterations are performed until the slip rule is satisfied and convergence is achieved. By releasing the pair of contact nodes at the crack tip the crack extension work for a finite crack growth may be calculated. Relaxation must generally be performed incrementally because of the nonlinearity at the contact surface. This method of relaxation was first suggested by Andersson [9] and is also used by Hellan [7].

Stress Intensity Factor Calculations

In terms of the stress intensity factors the crack extension work for coplanar extension is written

$$G = \frac{1+\nu}{E}\left[\frac{\kappa+1}{4}\left(K_I^2 + K_{II}^2\right) + K_{III}^2\right] . \qquad (16)$$

K_I, K_{II} and K_{III} are the stress intensity factors in mode I, II and III respectively [7]. For plane strain $\kappa = 3 - 4\nu$ and for plane stress $\kappa = (3-\nu)/(1+\nu)$.

The present method has been tested on a plate of unit thickness in a state of plane strain with a single edge crack subjected to tension and moment loads in mode I. In Figure 2a the stress intensity factor in pure tension is shown as function of the crack length and compared with the solution by Gross [10] for an infinite strip with a single edge crack. Next a moment load was applied and the stress intensity factor calculated. The result is shown in Figure 2b. As the moment is applied the stress intensity factor decreases linearly until the crack starts to close. The linear decrease in the stress intensity factor then ceases. When the crack is closing the problem becomes nonlinear. The stress intensity factor is almost constant after crack closure. This is in agreement with the result presented by Paris and Tada (a/W = 0.55). Due to conditions of symmetry the frictional properties do not influence the result.

Crack Extension Work in Crack Closure Problems

The crack extension work for the single edge crack previously studied was computed for different types of loading (Figure 3a). Different virtual crack propagation directions ϕ were studied and the crack extension work was calculated. Applying the criterion of maximum crack extension work [7] the crack propagation direction may be predicted.

The plate was first assumed to be loaded in pure tension σ_0 and in pure tension plus antisymmetric shear F, M. The normalized crack extension work as a function of ϕ is plotted in Figure 3a. At this loading no contact forces arise. Computations were done for nine virtual propagation angles ϕ from -90° to +90°. G_{max} is the maximum crack extension work in tension plus antisymmetric shear loading. From the pure tension curve it is seen that G has a maximum at $\phi = 0$ and that coplanar extension is predicted. This result is in agreement with previous findings. Some discretization errors are observed. In the case of tension plus antisymmetric shear the maximum G appears at $\phi \simeq -40°$. If the critical G was reached the angle of propagation is predicted to -40°, that is, the crack tends to propagate downwards in a combined mode.

The plate was next simultaneously loaded in tension, in antisymmetric shear and in compression. The crack then partially closes. The influence of the frictional properties on the crack extension work G was studied. G was calculated for five different directions of virtual crack extension, from 0 to -90°. In view of the first example it is evident that the G has maximum for ϕ between 0° and -90°. In Figure 3b the normalized crack extension work for the frictionless case is compared with the case of friction. An ideal Coulomb model with $\mu = 1$ was assumed. G_{max} is the maximum crack extension work for frictionless case. From these results it could be concluded that the crack extension force is decreasing when

taking frictional effects into account. Energy is dissipating at the surface of contact. This effect of course decreases the risk for crack propagation and gives a less dangerous situation. As the present loading conditions the crack surfaces starts to make contact at the left and the contact developes inwards. When the antisymmetric shear is applied slip takes place over the whole surface of contact. When the crack is virtually extended downwards the contact continues to develop inwards and the slip is increasing.

CONCLUDING REMARKS

A method of taking contact and frictional effects in crack closure problems into account was presented. Stress intensity factors was calculated from the crack extension work and compared with known solutions. It was also shown how the stress intensity factor is affected by a partial crack closure. By studying the crack extension work for virtual crack extensions at different angles the crack propagation direction was predicted. When studying cracks in combined modes it was shown how the present method could be used to study effects of frictional properties when the crack is closing.

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REFERENCES

1. ASKOGAN, O., Int. J. of Frac. Mech., 11, 1975, 659.
2. PARIS, P. C. and TADA, H., Int. J. of Frac., 11, 1975, 1070.
3. NEWMAN, J. C., Eighth Nat. Symposium on Fracture Mechanics, Providence, Rhode Island, August 26 - 28, 1974.
4. ERDOGAN, J., Elasticity, 6, 1976, 57.
5. FREDRIKSSON, B., Linköping Studies in Science and Technology, Diss. No. 6, Linköping, Sweden, 1976.
6. CARLSSON, J., Fracture Mechanics, Div. of Solid Mech., Royal Inst. of Technology, Stockholm, Sweden, 1974.
7. HELLAN, K., Publ. No. 73:6, Norwegian Inst. of Technology, Norway, 1973.
8. FREDRIKSSON, B., An Int. J. of Com. and Struc., 6, 1976, 281.
9. ANDERSSON, H., J. Mech. Phys. Solids, 21, 1973, 337.
10. GROSS, B. et al, NASA TN D-2603, 1965.
11. FREDRIKSSON, B., Report LiTH-IKP-R-034, Linköping Inst. of Technology, Linköping, Sweden, 1975.

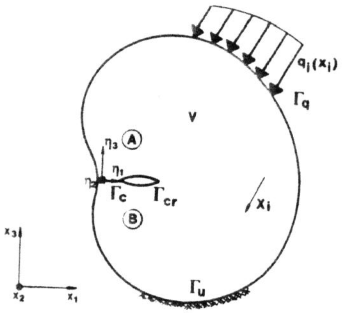

Figure 1 An Elastic Body Containing a Partially Closed Crack

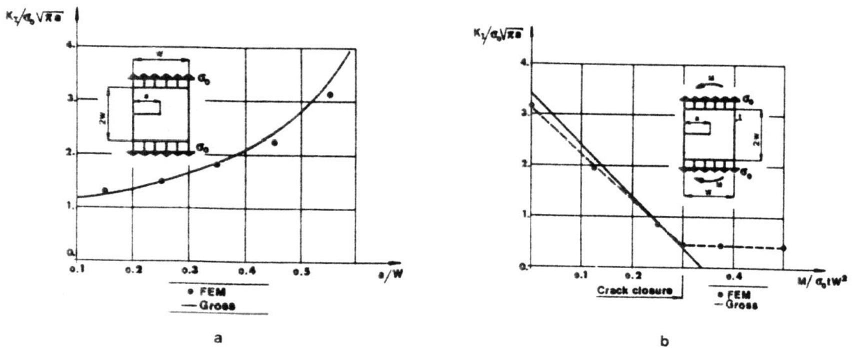

Figure 2 Single Edge Crack. FEM-Model: 178 Constant Strain Elements, 268 Degrees of Freedom. Stress Intensity Factors in

a) Pure Tension
b) Tension and Bending

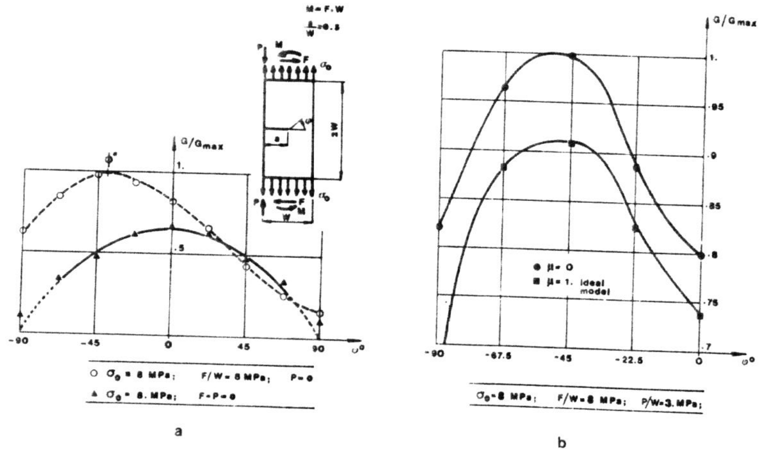

Figure 3 Single Edge Crack. Normalized Crack Extension Work in

a) Tension and Antisymmetric Shear
b) Tension, Compression and Antisymmetric Shear