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DETERMINATION OF CRITICAL STRESS-INTENSITY FACTORS UNDER MIXED MODE LOADING CONDITIONS BY SHADOW-OPTICS

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INTRODUCTION

Using the fracture mechanics approach the safety analysis of components and structures usually relies on a pure mode I consideration which is based on a single material parameter: the plane strain fracture toughness K_{Ic}. A more detailed analysis as well as experiments show, however, that this simplifying mode I approach may be insufficient in case of superimposed shear modes. In order to contribute to this problem in this paper the superposition of fracture modes I and II has been investigated on polymethylmethacrylate (PMMA) specimens. A shadow optical method which was developed by Manogg [1] and extended by Theocaris and Gdouts [2] served to determine the critical stress intensity factors K_I^c and K_{II}^c at the onset of unstable fracture. The experimental results are compared with the predictions of three fracture hypotheses [3 - 5] which try to give a physical interpretation of the fracture phenomena under mixed mode loading conditions.

PRINCIPLE OF THE SHADOW OPTICAL METHOD

Due to the stress concentration at the crack tip a parallel or slightly divergent light beam which passes a transparent plate perpendicular to its surface is locally deflected. A dark shadow spot surrounding the crack tip is formed in a reference plane behind the plate. In the case of a plane state of stress or strain the size and the shape of this shadow spot, which is determined by the stress field and the optical properties of the material, are related to the stress intensity factors K_I and K_{II}. As an example Figure 1 shows a picture of such a spot taken from a plate of PMMA - an optically isotropic material - for a ratio of $K_{II}/K_I = 1$. The shadow area is bordered by a bright and sharp caustic. The shape of the caustic can be calculated as shown in [1] for the pure opening mode and in [2] for mixed mode conditions. In Figure 2 a calculated caustic is plotted corresponding to the K_{II}/K_I ratio of Figure 1. It can be shown that this caustic optically is formed through all those light rays passing a circle around the crack tip in the plate. The radius r_o is given by

$$r_o = \lambda^{3/2} \left\{ \frac{3\, z_o B\, c}{2\sqrt{2\pi}} K_I \right\}^{2/5} \left\{ 1 + \mu^2 \right\}^{1/5} \tag{1}$$

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with the ratio $\mu = K_{II}/K_I$, the distance z_o between the plate and the reference plane, the magnification ratio $\lambda = (z_o+z_i)/z_i$ for a divergent light beam, the plate thickness B and the elasto-optical constant c. The amount of deflection of those rays forming the caustic is proportional to r_o and besides that only a function of μ. Therefore characteristic diameters D_{max}, D_{min} or D_t of the caustic as defined in Figure 2 can be normalized to r_o and are otherwise only functions of μ. These functions are plotted in Figure 3. From that the relation

$$\phi(\mu) = \frac{D_{max} - D_{min}}{D_{max}} \qquad (2)$$

can be deduced which only depends upon μ as depicted in Figure 4. Based on these analytical and numerical derivations K_I and K_{II} can experimentally be determined as follows: When the characteristic diameters are measured the ratio μ results from Figure 4, inserting μ into Figure 3 the radius r_o may be obtained which finally allows the calculation of the absolute values of K_I and K_{II}.

EXPERIMENTAL ARRANGEMENT

Specimens of PMMA (400 mm x 200 mm x 3 mm) containing oblique central cracks of the length $2a \approx 35$ mm were tested under uniaxial tensile loading σ_o. The angle between the crack plane and the loading direction was varied from 20 to 70 degrees. In order to obtain natural cracks of sufficient quality regarding a straight and sharp crack front and a smooth and plane fracture surface perpendicular to the plate surface, saw cuts have been extended on one side for a few millimeters by a dynamic loading device. The principle of the optical arrangement which is described in more detail in [6] is outlined in Figure 5. A point light source realized by a 100 μ diaphragm illuminates the crack tip zone (z_i = 1,8 m) and forms finally in the reference plane (z_o = 0,4 m) as the shadow spot which is recorded by a television system. In order to calibrate the system additional photographs were taken with a regular camera. Minimizing optical and electronical errors and measuring the shadow diameters accurately were favoured by a grating in the reference plane photographed together with the shadow spot.

EXPERIMENTAL PROCEDURE

For the investigated specimens and the given loading conditions the K-values can be calculated as deduced in [7] by

$$\begin{aligned} K_I &= \sigma_o \sqrt{\pi a}\,\sin^2\beta \\ K_{II} &= \sigma_o \sqrt{\pi a}\,\sin\beta\,\cos\beta \end{aligned} \qquad (3)$$

(for the symbols see Figure 6). In a first series of tests with non-critical loads these K-values have been determined from the experimental data and compared with those measured by the shadow optical technique. For this comparison the shadow spots have been evaluated by two methods distinguished one against the other by the way of determining μ. The first way is to compute μ from the specimen geometry using $\mu = \text{ctg}\beta$ (which follows from equation (3)); the second way is to determine μ by means of Figure 4 using the measured diameters D_{max} and D_{min}. Then both methods continue by determining r_o in Figure 3, μ and r_o deliver K_I and K_{II} according to equation (1). In both cases for the whole range of loading the comparison shows a good agreement within the accuracy of measurements. To get critical K-values the specimens were loaded up to rupture at a constant displacement rate of 0.1 mm/min. In the case of mixed mode conditions the crack started rapidly in a velocity range of m/s and showed in general no slow propagation before. In the last picture of the television record, showing the shadow spot up to at most 20 ms (given by a picture frequency of 50 s^{-1}) before the unstable crack growth, the diameters D_{max} and D_{min} can be measured. During this time interval the increase in σ_o and thereby in K_I and K_{II} is neglectible.

To normalize the critical K-values for mixed mode conditions the fracture toughness K_{Ic} was measured too ($K_{Ic} = (1.46 \pm 0.11)$ MPa$\cdot$m$^{1/2}$). It was determined in the usual manner from the external critical load and the critical crack length a_c (using equation (3) with $\beta = 90°$) because under pure mode I conditions the crack began to propagate slowly and left the picture area with a velocity in the range of mm/s so that the value K_{Ic} defined for unstable crack propagation could not be measured shadow-optically. According to Döll [8] in PMMA a_c is given by the sharp line marked on the crack surface at the transition to the unstable crack growth with velocities in the range of m/s.

In addition to the critical K-values the angle of crack extension θ_o (see the insert in Figure 6) has been measured directly at the crack tip by a microscope in different planes parallel to the specimen surface and finally averaged.

RESULTS AND DISCUSSION

The experimental results for the angle of crack extension θ_o and for the critical stress intensity factors K_I^c and K_{II}^c normalized to the fracture toughness K_{Ic} are plotted in Figure 6 and Figure 7, together with the corresponding predictions of three fracture hypotheses which assume three different physical quantities to be responsible for the crack behaviour: the maximum stress (Erdogan and Sih [3]), the energy release rate (Hussain, Pu and Underwood [4]) and the strain energy density (Sih [5]).

The results for the angles θ_o (Figure 6) show, in spite of the difficulties in principle to measure θ_o, a small scatter, and agree with the fracture criteria, stated above, in the range of $40° < \beta < 60°$, whereas a modified maximum stress criterion, proposed by Williams and Ewing [9] and Finnie and Saith [10], is in accordance with the results over the whole investigated range of β. The measured critical K-values scatter more (Figure 7) than the angles θ_o and are partly smaller than the corresponding values of the criteria, particularly in the case of dominating shear loading.

These results differ from those in [3], also obtained on PMMA (K_{Ic} = 0.52 MPa$\cdot$m$^{1/2}$) with the same specimen geometry, but because of the unknown detailed testing conditions and the big difference in K_{Ic} cannot be compared directly. In practice the behaviour of real cracks is influenced by the curvature of the crack front and crack surface, the crack tip radius, the strain rate depending of the material and the structure of the plastic zone. These things are not considered by fracture criteria assuming linear elastic material properties and ideal cracks. Whereas the crack extension direction seems not to be affected seriously by these effects, an influence on the critical crack behaviour should be taken into consideration. This influence could lead to smaller critical values since our results show that the cited fracture criteria for combined loading do not represent a conservative estimation for the cases studied.

A second result is that the shadow optical method is applicable to mixed mode problems and can be used to determine critical values for any plane problems without knowing the external loading conditions. This is because the whole information about K_I and K_{II} is obtained from the small circle around the crack tip where the stress distribution creates the caustic.

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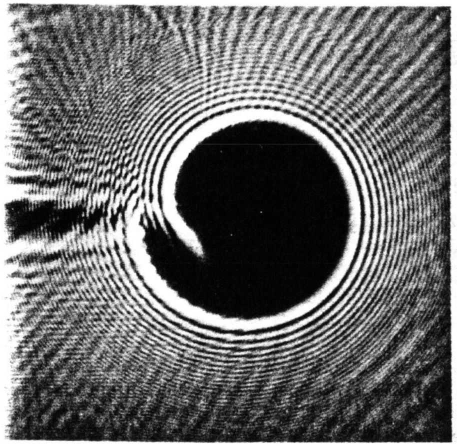

Figure 1 Shadow-Spot for $K_{II}/K_I = 1$

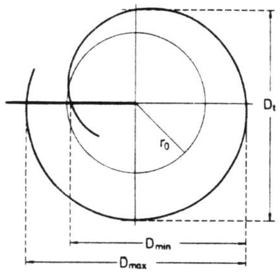

Figure 2 Calculated Caustic for $K_{II}/K_I = 1$

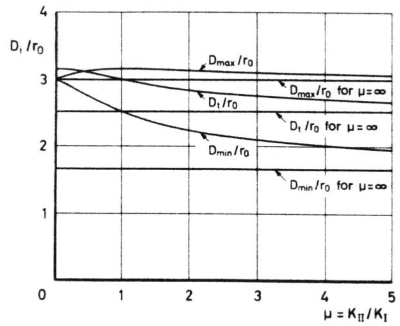

Figure 3 Normalized Shadow-Diameters as Functions of μ

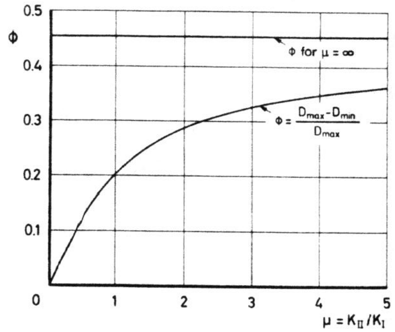

Figure 4 Ratio Ø as a Function of μ

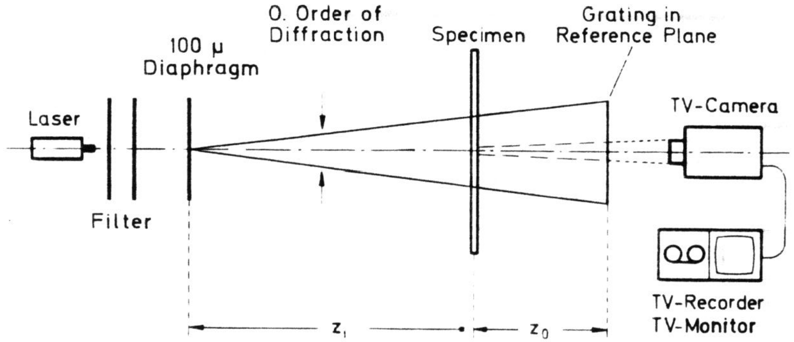

Figure 5 Schematic of the Experimental Arrangement

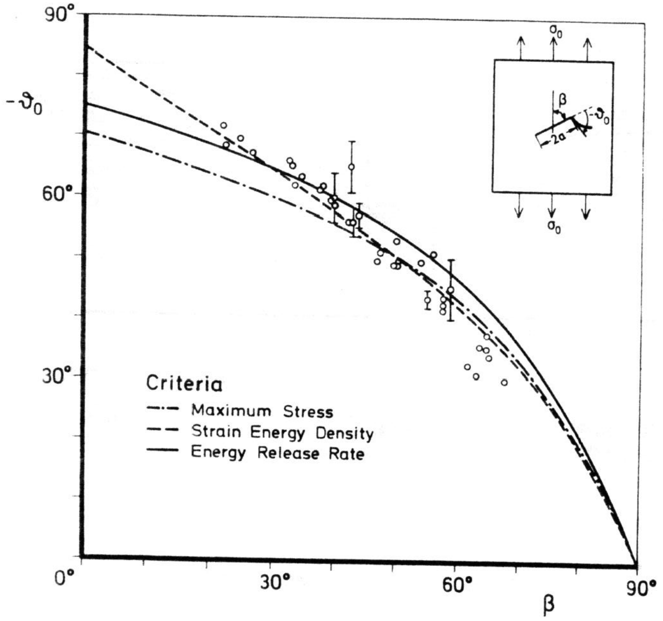

Figure 6 Crack Extension Angle θ_o

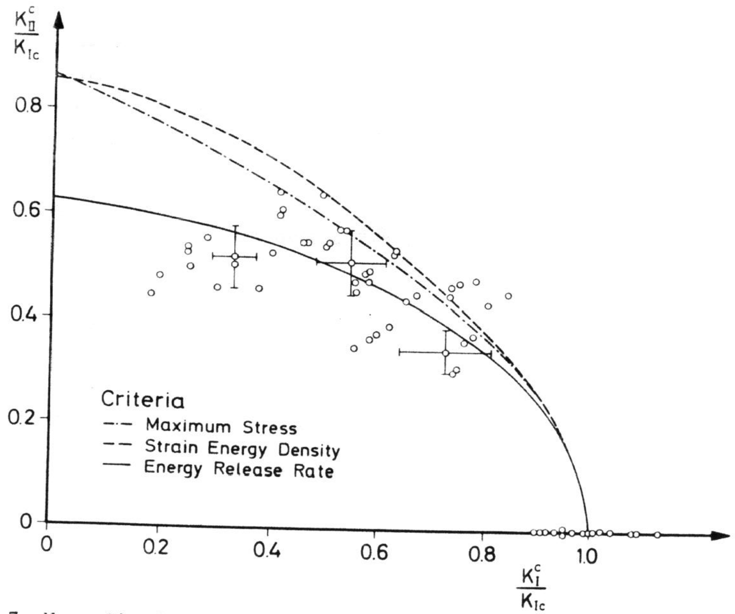

Figure 7 Normalized Critical Stress Intensity Factors for Combined Loading