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DEPENDENCE OF LIFETIME PREDICTIONS ON THE FORM OF THE CRACK PROPAGATION EQUATION**

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INTRODUCTION

The science of fracture mechanics has provided engineers with a new design technique for estimating the total allowable time that ceramic components can support mechanical loads [1]. Fracture mechanics are used in this application because ceramic materials fail by brittle fracture caused primarily by the growth of pre-existing flaws or cracks. When these flaws are subjected to mechanical stress they grow to a critical size, at which point abrupt failure occurs. The time required for cracks to grow from a subcritical to a critical size determines the time-to-failure.

In order to determine the time-to-failure of ceramic components it is necessary to characterize three parameters: the initial flaw size, the critical flaw size at which abrupt failure occurs, and the rate at which cracks grow from initial to critical flaw size. In fracture mechanics terms these parameters are: the critical stress intensity factor, K_{IC}: the stress intensity factor, K_{Ii}, at the most serious flaw when the component is first subjected to a load; and the functional dependence of crack velocity on the stress intensity factor, $v=v\ (K_I)$. Using these parameters the time to failure can be calculated from the following equation:

$$t = (2/\sigma_a{}^2\ Y^2) \int_{K_{Ii}}^{K_{IC}} (K_I/v)\, d\ K_I \qquad (1)$$

The parameters of equation (1) can be evaluated either by fracture mechanics techniques or by strength techniques. Fracture mechanics techniques can be used to obtain the critical stress intensity factor, and the relationship between the crack growth rate and the stress intensity factor directly. The initial stress intensity factor, K_{Ii}, however, must be determined by other techniques because the initial size, a, of the critical flaw on which K_{Ii} depends ($K_I = \sigma_a Y\sqrt{a}$) is small. For this reason, direct nondestructive techniques (at the current state of the art) cannot be used to estimate K_{Ii}. The two techniques that are used to obtain estimates of the initial stress intensity factor: are the proof testing technique which provides an estimate of the maximum flaw size [2]; and the statistical technique which characterizes the flaw size as a function of the cumulative failure probability [2,3].

Recently, Ritter and Meisel [4] have shown that equation (1) can be expressed in alternative form, in which the parameters of the equation are evaluated by strength measurement techniques. One advantage of this new

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approach to estimating the failure time is that it can be used in laboratories that do not have the more elaborate equipment required for fracture mechanics determinations.

Regardless of which technique is used to evaluate the constants of equations (1) the reliability of the predicted time-to-failure depends on the accuracy with which the parameters of equation (1) can be determined. In a recent set of papers the effect of errors of measurement on equation (1) has been determined for both fracture mechanics techniques and stress measurement techniques [5,6]. Although it was noted in these papers that the confidence limits for the time-to-failure also depend on the form of the crack propagation equation, this aspect of the failure problem has not been discussed extensively. This paper presents a discussion of this aspect of failure prediction techniques.

FORMS OF THE CRACK PROPAGATION EQUATION

Various equations have been suggested to describe the dependence of crack propagation rate on applied stress intensity factor [7-10]. While some of these were suggested on the basis of theoretical models of crack growth, others were suggested because they give good empirical fits of the experimental data. The four equations that will be examined in this paper are presented below:

$$v=v_o \exp(\beta' K_I) \qquad (2)$$

$$v=v_o \exp(\beta'' K_I^2) \qquad (3)$$

$$v=A K_I^n \qquad (4)$$

$$v=v_\infty \exp(-\alpha/K_I) \qquad (5)$$

To demonstrate the sensitivity of the estimated failure time to the form of the crack propagation equation, equation (1) was evaluated for two glass compositions: a soda-lime-silicate glass, and an ultra-low expansion glass (92.5% SiO_2; 7.5% SiO_2). Fracture mechanics techniques were used to determine the parameters in equation (1), while the initial stress intensity factor was determined by the proof-test method. The two glasses selected represent the extremes of crack propagation behaviour observed for most glasses. The data on the ultra-low expansion glass was collected in water; while the data on the soda-lime-silicate glass was collected in air, (50%) relative humidity. The doube-cantilever-beam technique was used to obtain both sets of data and replicate runs were obtained for each glass: three for the ultra-low expansion glass, and two for the soda-lime-silicate glass.

EXPERIMENTAL RESULTS

The crack propagation curves obtained in this study are presented in Figures 1 and 2. In each of these figures the crack propagation rate is plotted as a function of the stress intensity factor in a form suggested by equations (2) - (5). A straight line was fitted to the data by the method of least squares, minimizing the error along the axis representing

the stress intensity factor. In addition a plot of the residuals was obtained as a function of the crack velocity in order to accentuate the curvature of the fit. For the sake of brevity, these plots of the residuals are not presented in this paper.

For the ultra-low expansion glass (Figure 1) the best fit was given by equation (4). Scatter of the data points about the least fit line was random over the entire range of data points. The next best fit was given by equation (2), for which a slight curvature was observed. The data showed a negative deviation from the best fit line over the middle range of crack velocities and a positive deviation at both low and high velocities. However, the curvature of the data for equation (2) was not as great as that exhibited for equations (3) or (5). The most severe curvature of the data was obtained for equation (3), which showed the largest deviations from the best fit line over the entire range of crack velocities.

At the very low velocities, less than 10^{-10} m/s, the data (Figure 2) for the soda-lime-silicate glass exhibited a rapid decrease in crack velocity that was very suggestive of an approach to the static fatigue limit. Because of this decrease, data at velocities less than 10^{-10} m/s were not used in the least squares fit. In all, five data points of the set of 60 were eliminated from the least squares fit for the equations. The data for the soda-lime-silicate glass was curved for all four equations under discussion. The best fit and the least curvature was obtained for equation (2). Here the deviations were greatest at the lowest and highest velocities. From 10^{-9} m/s to approximately 3×10^{-5} m/s the straight line fit indicated no perceptible curvature of the data. By contrast the logarithmic fit, equation (4), did exhibit slight curvature over the entire range of data, exhibiting a positive deviation from the best fit line in the mid-range of the data and a negative deviation both at the high and the low crack velocities. The worst fits were obtained for equations (3) and (5), for which severe curvature occurred over the entire range of data. In both cases the deviations were negative in the mid-range of the data and were positive for the higher and lower crack velocities.

DISCUSSION

From the data presented in this paper, we conclude that the best fit of the crack propagation data for glass is obtained by using either a logarithmic representation (equation (4)) or a semi-log represented (equation (2)) of crack velocity data. This conclusion is supported by studies conducted on other glasses not presented in this paper. Equation (3) gives by far the worst fit to the experimental data, while equation (5) gives a fit that is somewhat intermediate between equations (2) and (4) on the one hand and equation (3) on the other hand. However, considering the slight curvature that is exhibited by all of these fits within the data range, it can be concluded that any of the representations will adequately predict time-to-failure over the range in which the data were collected. It is only when the data must be extrapolated beyond the data range that the type of fit becomes important.

The sensitivity of equation (1) to the form of the crack propagation equation is illustrated in Figure 3, which is a logarithmic plot of $\sigma_a^2 t_{min}$ versus the proof test ratio σ_p/σ_a. The three curves on this plot were calculated for equations (2), (4), and (5). Equation (3) is not presented because it gave the poorest representation of the crack velocity data within the data range. On each curve the vertical line marks the upper proof test value, which is determined by the lowest measured crack velocity. Time-to-failure predictions at higher proof test ratios represent extrapolations to low crack propagation rates. It is observed that within the data range (portions of the curves in Figure 3 that lie to the left of the vertical line) all of the three fits give a good prediction of time-to-failure. However, as the proof test ratio is increased beyond the limits of the data we see that the three lines deviate from one another. The deviations can be quite large as the proof test ratio is increased beyond the limits set by the crack propagation data, causing a substantial uncertainty in the predicted time-to-failure. Thus, for a proof test ratio of 3 (which is not unusually high for glass), the predicted times-to-failure for equations (4) and (5) are approximately 4 and 50 times that predicted from equation (2) for soda-lime-silicate glass. Similarly, for the ultra-low expansion glass, the predicted times-to-failure are approximately 60 and 10,000 times that predicted from equation (2). Uncertainties in lifetime predictions can be reduced substantially by extending the crack propagation data to the lowest possible crack velocities (as was done in the present paper). Conversely, a high value for the low velocity limit of the crack propagation data increases the uncertainty lifetime prediction. For the soda-lime-silicate data, for example, the predicted times-to-failure would be approximately 10 and 1600 times larger for equations (4) and (5) than for equation (2) if the lower limit of the crack velocity range were only 10^{-9} m/s.

Regardless of the range of the crack propagation data, safe design practices dictate a conservative approach to the use of predicted lifetimes. To assure reliability the shortest predicted lifetime should be used for purposes of design. Based on this criterion, it can be concluded that for glass, equation (2) gives the most conservative prediction of lifetime under load, and is the appropriate one to use for design applications*.

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*It is worth noting here that for the soda-lime-silicate glass all three fits will give conservative estimates of the failure time at high proof test ratios if in fact the crack propagation data approaches a static fatigue limit as suggested by the data at low velocities.

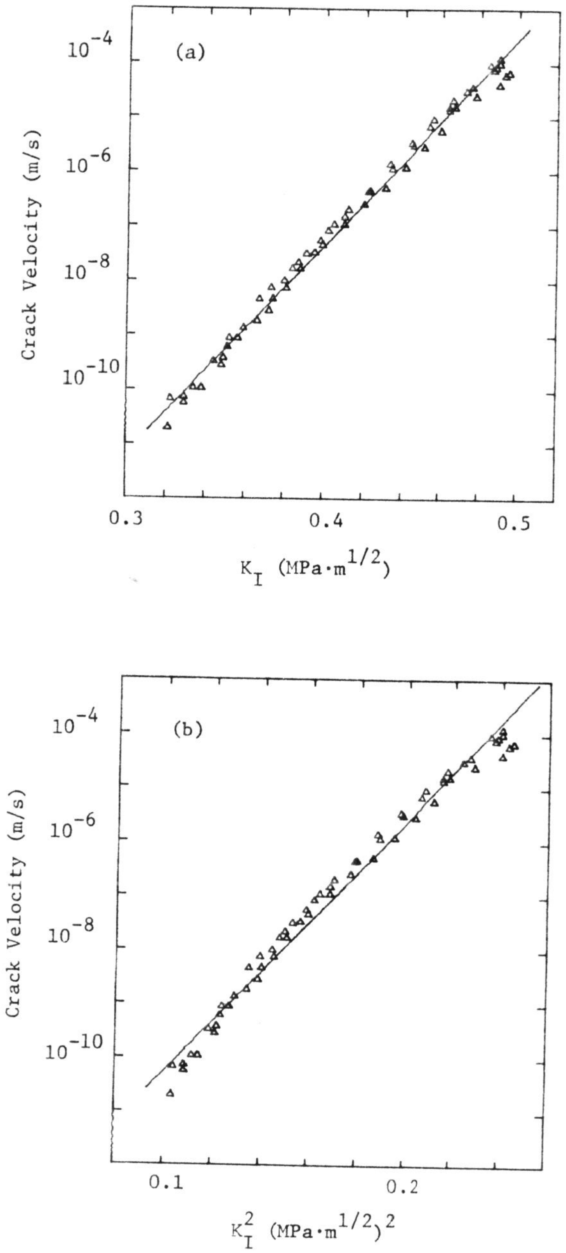
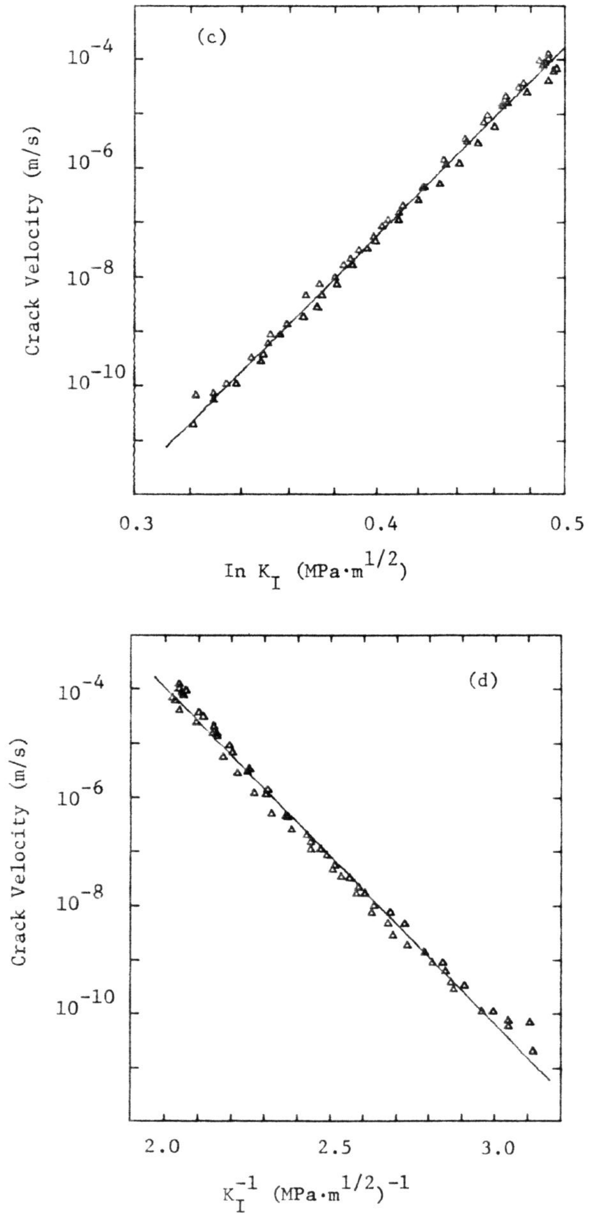


Figure 1 Ultra-Low Expansion Glass



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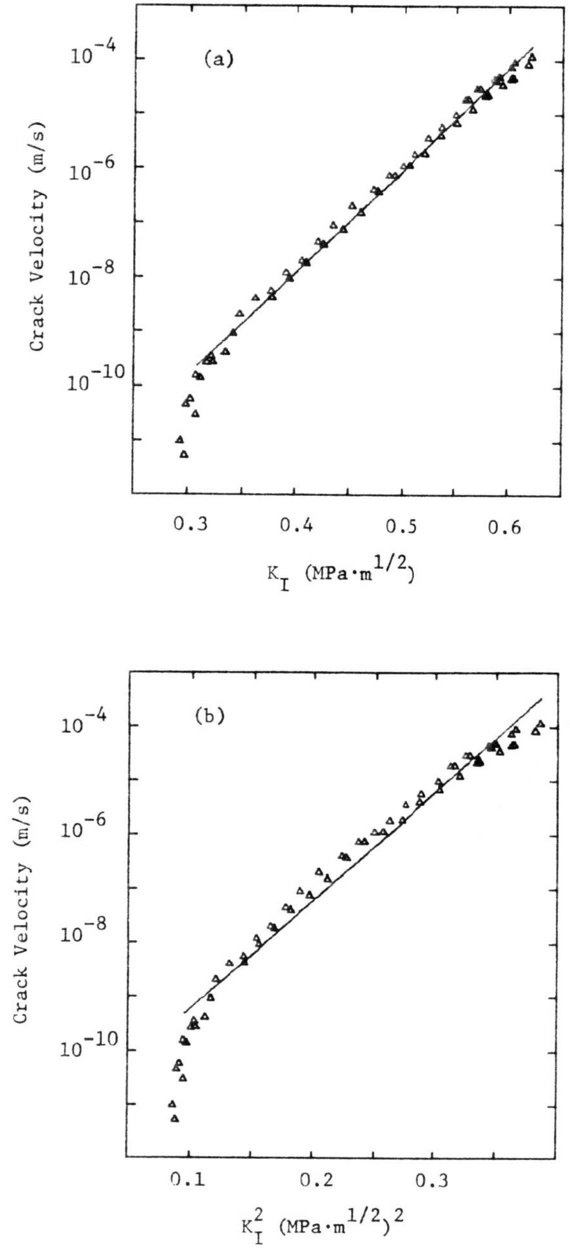
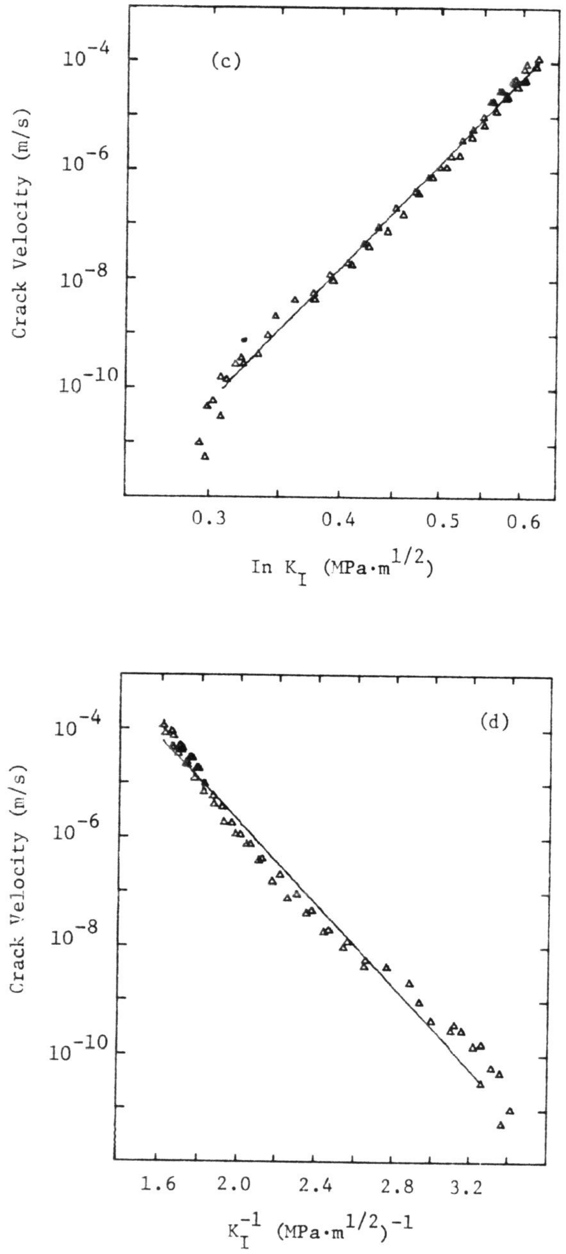


Figure 2 Soda-Lime-Silicate Glass



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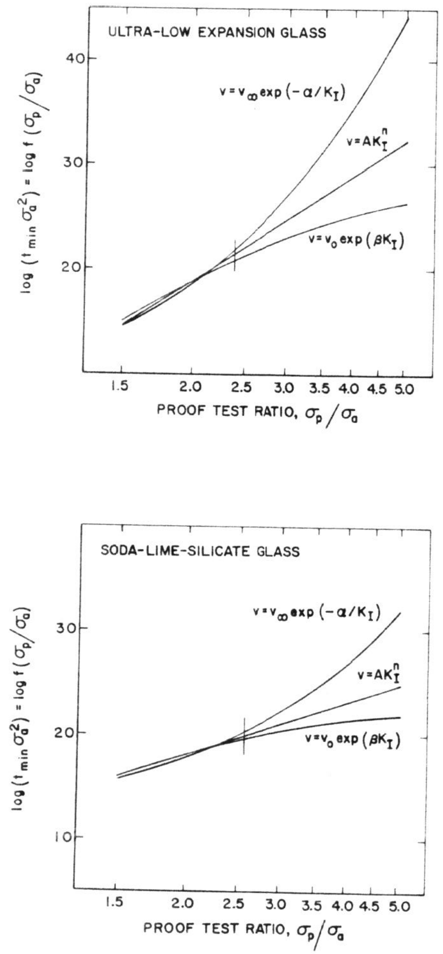

Figure 3 Proof Test Diagram Comparing Three Representations of the Crack Propagation Data