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CRACK PROPAGATION IN A TWO-PHASE MATERIAL SUCH AS CONCRETE

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INTRODUCTION

In recent papers crack propagation in porous viscoelastic materials has been studied in detail [1, 2]. Experimental results obtained by testing various porous building materials have been compared with theoretical predictions and have been published in a comprehensive report [3]. In a similar way crack propagation in materials under multiaxial state of stress has been treated [4]. In this approach the material has been assumed to be isotropic and homogeneous. Therefore the interaction of cracks with aggregates in a two-phase material such as concrete could not be treated specifically. Nevertheless it was possible to describe the behaviour of concrete under high sustained load by introducing simplifying assumptions on a rather phenomenological basis.

Several authors were able to prove that a considerable number of cracks can be observed in unloaded concrete specimens [5 - 8]. These shrinkage induced cracks are frequently located in the interface between hardening cement paste and aggregate. If an external load is applied the a-priori-cracks begin to grow. Up to about 85% of the ultimate load crack length increases to a comparatively small degree and cracks remain mostly interfacial [9]. At 95% of the ultimate load, however, cracking is no longer restricted to interfaces. Previous interfacial cracks extend into the surrounding mortar matrix. These spreading crack extensions have a tendency to direct themselves close to the direction of the externally applied load.

In this paper an attempt to simulate the experimentally determined crack growth in concrete is described by means of a computer programme (Monte Carlo-method). The theoretical background of this complex stress analysis will be described in detail elsewhere [10]. The actual structure of concrete is replaced by a two-dimensional model. Aggregates having polygonial shape are randomly distributed in a homogeneous matrix. The size distribution and the volume content of the aggregate particles can be varied in order to simulate different mix proportions. Interfacial stresses may be estimated by using formulae described in [11, 12].

BRANCHING CRACKS IN HOMOGENEOUS MATERIAL

We will start with the discussion of the simplest case of branching cracks, i.e. a randomly inclined crack in a homogeneous plate and loaded at infinity. Brace and Bombolakis [13] as well as Hoek [14] have studied the development of branching cracks. They showed that branching cracks become aligned with the axis of the major compressive stress. It is most impor-

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tant to note that branching cracks do not propagate catastrophically but they form gradually as the external load is increased. If several randomly inclined cracks are in one specimen branching cracks develop from cracks with differing inclination as the load is increased.

In Figure 1 an initial crack having a length of $2l_1$ and an inclination of α with respect to the direction of the external compressive load q is shown.

This crack might propagate as shear crack i.e. failure type II. In our example, however, at the end of the initial crack, two cracks of failure type I are created before the shear crack becomes critical. By introducing simplifying assumptions [10] the crack length l_2 can be expressed as follows:

$$\frac{P}{\sqrt{\pi l_2}} + \frac{Q}{\sqrt{\pi l_2}} \frac{\kappa-1}{\kappa+1} = K_{IC} \qquad (1)$$

In this equation P and Q are the x and y components of shear stress T (see Figure 1) created by friction which may be represented by a coefficient ρ:

$$P = T \sin\alpha + -2ql_1A(\alpha,\rho) \qquad (2)$$

and

$$Q = T \cos\alpha = -2ql_1B(\alpha,\rho) \qquad (3)$$

A and B have the following meaning:

$$A(\alpha,\rho) = \sin^2\alpha \cos\alpha - \rho\sin^3\alpha \qquad (4)$$

$$B(\alpha,\rho) = \sin\alpha \cos^2\alpha - \rho\sin^2\alpha \cos\alpha \qquad (5)$$

while T can be expressed in the following way:

$$T = 2\, l_1 \cdot \tau_{\xi\phi} = 2\, l_1\, q(\sin\alpha \cos\alpha - \rho\sin^2\alpha) \qquad (6)$$

If 0,2 is used as a characteristic value of the Poisson ratio of concrete [3], we find κ to be 7/3 and hence $(\kappa-1)/(\kappa+1)$ to be 2/5. Then equation (1) can be rewritten:

$$\left\{P + \frac{2}{5}Q\right\}/\sqrt{\pi l_2} = K_{IC} \qquad (7)$$

By inserting equations (2) to (5) in equation (7) we find:

$$q = \sqrt{\frac{l_2}{l_1}}\, \frac{K_{IC}\sqrt{\pi/l_1}}{2C_1(\alpha,\rho)} \qquad (8)$$

In equation (8) C_1 has the following meaning:

$$C_1(\alpha,\rho) = A(\alpha,\rho) + \frac{2}{5} B(\alpha,\rho) \qquad (9)$$

Equation (8) indicates the length of the branching cracks l_2 as a function of the external load q and the geometrical arrangement.

BRANCHING CRACKS FROM INTERFACIAL CRACKS

Now we consider a homogeneous matrix with one polygonial inclusions, representing an aggregate. An initial interfacial crack with length $2\,l_1$ is assumed to be located along one side AB (see Figure 2). This problem can be treated in a similar way as was shown with the example of the behaviour of an inclined crack in a homogeneous material in the previous section. In the interface stress concentrations perpendicular and parallel to the crack surface have to be taken into consideration. This can be done by introducing coefficients of stress concentration k_σ and k_τ. It can be shown that the initial crack spreads (Type II) as soon as the critical load q_{II}^{IF} is reached

$$q_{II}^{IF} = - \frac{K_{IIC}^{IF}}{\sqrt{\pi l_1}\cdot D_{IF}(\alpha,\rho)} \qquad (9)$$

Index IF denotes that critical values of the interface have to be used and D_{IF} has the following meaning:

$$D_{IF}(\alpha,\rho) = k_\tau \sin\alpha \cos\alpha - k_\sigma\rho \sin^2\alpha \qquad (10)$$

This shear crack develops in an unstable manner until it reaches the length $2L_1$ (see Figure 2b). In this situation further crack growth is stopped. If the external load is increased another critical value q_I^M is reached:

$$q_I^M\Big|_{l_1=L_1} = - \frac{K_{IC}^M \sqrt{3/2}}{\sqrt{\pi l_1}\; D_{IF}(\alpha,\rho)} \qquad (11)$$

Here index M denotes that under these conditions cracks run through the matrix. The actual crack length in the matrix can be given as a function of load in analogy to equation (8):

$$q = - \frac{\sqrt{\pi l_2}\; K_{IC}^M}{2L_1\; C_{1IF}(\alpha,\rho)} \qquad (12)$$

where

$$C_{1IF}(\alpha,\rho) = D_{IF}(\alpha,\rho) \sin\alpha + \frac{2}{5} D_{IF}(\alpha,\rho) \cos\alpha \qquad (13)$$

According to equation (12) crack length is steadily increased as the load increases. l_2 in equation (12) corresponds to the distance AA' as shown in Figure 2c.

CRACKS INTERFERING WITH AGGREGATES

In Figure 3a the situation shown in Figure 2c is repeated. But in the present problem it is assumed that branching crack AA' meets a second inclusion as it propagates (Figure 3b). Further crack growth will follow the interface MN. In principle this interfacial crack growth may occur according to type I or to type II. The respective critical loads are given by:

$$q_I = - \frac{2\, K_{IC}^{IF} \sqrt{\pi l_2} \,/\, L_1}{C_{1IF}(\alpha,\rho)\left[3\cos\frac{\beta}{2} + \cos\frac{3\beta}{2}\right] - 3C_{2IF}(\alpha,\rho)\left[\sin\frac{\beta}{2} + \sin\frac{3\beta}{2}\right]} \qquad (14)$$

$$q_{II} = - \frac{2\, K_{IIC}^{IF} \sqrt{\pi l_2} \,/\, L_1}{C_{1IF}(\alpha,\rho)\left[\sin\frac{\beta}{2} + \sin\frac{3\beta}{2}\right] + C_{2IF}(\alpha,\rho)\left[\cos\frac{\beta}{2} + 3\cos\frac{3\beta}{2}\right]} \qquad (15)$$

In equations (14) and (15) C_{2IF} has the following meaning:

$$C_{2IF}(\alpha,\rho) = \frac{2}{5}\, D_{IF}(\alpha,\rho)\, \sin\alpha + D_{IF}(\alpha,\rho)\, \cos\alpha \qquad (16)$$

Once a crack has reached a second inclusion further crack growth depends both on the geometry of the crack path A'ABB' and the inclination of the interface MN. Whether crack propagation according to type I or to type II is to become critical depends on the sign of β and is indicated by equations (14) and (15). Shear cracks (type II) are facilitated by an external load. In contrast the component of q perpendicular to the plane MN makes the formation of opening cracks (type I) less likely. As a consequence in a material with randomly distributed inclusions cracks will propagate with high probability as indicated in Figure 3c.

SIMULATION OF CRACK PROPAGATION IN CONCRETE

In the preceding sections all essential elements to describe crack propagation in a two-phase material have been described. Now we can simulate the structure of concrete using Monte Carlo-method. A typical example of one computer realization is shown in Figure 4a. 30 polygonial inclusions have been randomly distributed in the matrix. Each aggregate particle is supposed to have one initial interfacial crack (marked with a thick bar in Figure 4). As the load is increased up to a certain level some cracks become critical and propagate in an unstable way within the interface and then as the load is further increased in a stable manner through the matrix. Three different levels of loading are shown in Figures 4b - d. A gradual increase of the mean crack length is observed. Finally a composite crack runs with slight overall inclination through the specimen and causes macroscopic failure. Inclined cracks have been experimentally determined in compressed concrete specimens (see f.e. [15]). As far as we know the theoretical background of this characteristics behaviour has been outlined for the first time in this contribution. Conditions for the formation of inclined cracks are formulated by equations (14) and (15).

A new and basic approach to simulate crack propagation in concrete has been described. In Figure 4 normal concrete has been chosen as an example. In lightweight concrete and in high strength concrete cracks may penetrate through aggregate particles [16]. The behaviour of concrete under high sustained load as well as under impact load can also be studied in this way. Simulated crack patterns agree reasonably well with experimental findings. Therefore it seems that a solid basis for further investigations has been provided.

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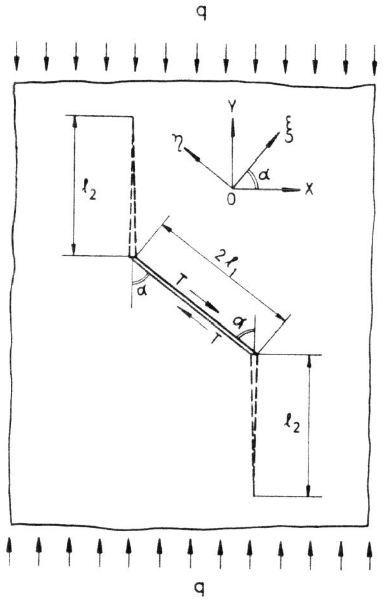

Figure 1 Schematic Representation of the Development of Branching Cracks and Definitions of Symbols Used in Equations (1) to (9)

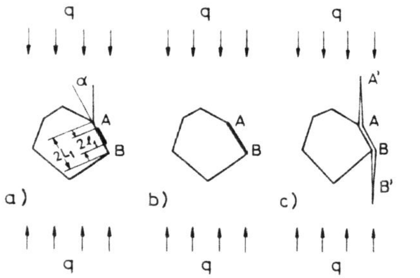

Figure 2 An Initial Crack with Length $2l_1$(a) Grows in an Unstable Manner Along an Interface AB (b) and Finally Stable Branching Cracks AA' and BB' are Created as the Load is Increased

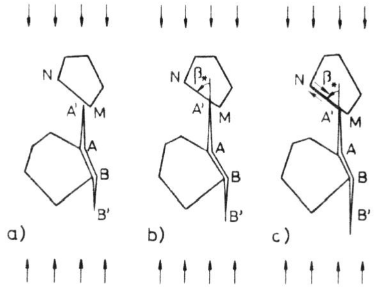

Figure 3 (a) A Crack Path as Shown in Figure 2c, (b) Meets a Second Inclusion, (c) Finally a Shear Crack in the Interface MN will Occur

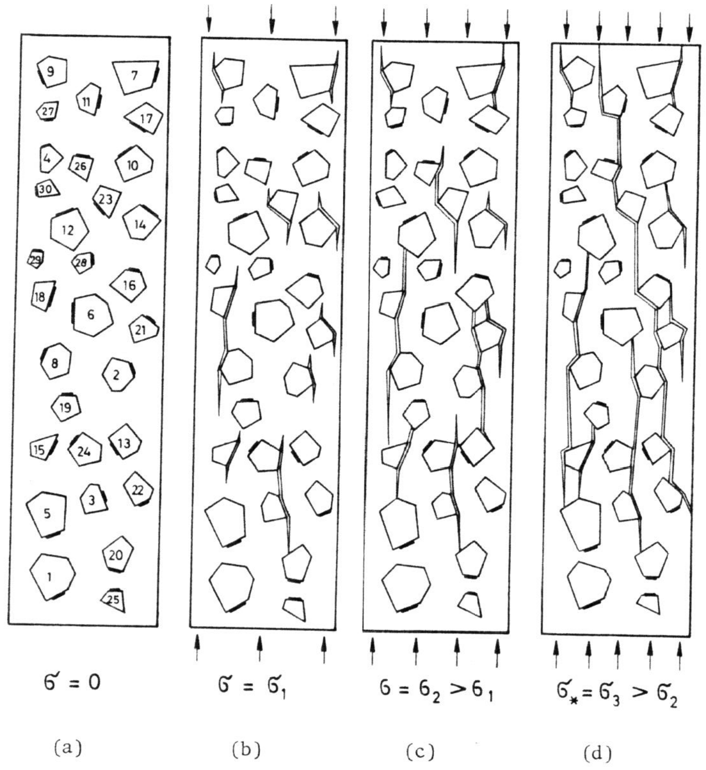

Figure 4 (a) Computer Simulation of the Structure of Concrete
(b) and (c) Crack Growth of Initial Interfacial Cracks as the Load is Increased
(d) Finally Failure Occurs as a Slightly Inclined Crack Runs Through the Specimen