Fracture 1977, Volume 3, ICF4, Waterloo, Canada, June 19 - 24, 1977

BRITTLE FRACTURE UNDER BIAXIAL NORMAL STRESS

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INTRODUCTION

A great deal of literature exists concerning the fracture of brittle materials under uniaxial tension. However, relatively little work has been done on situations involving biaxial states of normal stress. While a nonuniaxial state of stress is more complex, there are many practical applications that involve such states of stress.

The most commonly applied strength criterion for brittle materials is the *maximum tensile stress criterion* in which the material is assumed to fracture when the principal tensile stress reaches a critical value. This criterion appears to hold reasonably well as long as the state of stress is uniaxial (simple tensile test, thin ring test involving simple hoop tension or bending tests leading to a modulus of rupture). However, when the state of stress is biaxial the maximum tensile stress criterion gives poor results. Examples of this may be found in the literature. For example, when several investigators [1 to 5] have attempted to correlate fracture stress in the disc test with results obtained in uniaxial tension a discrepancy of from 200 to 250% has been observed, the principal fracture tensile stress in the disc test being too low.

DISC TEST

The disc test (Figure 1) involves a simple penny shaped specimen that is loaded diametrically at the centre of the disc. The principal stresses at the centre of the disc are:

$$\sigma_1 = \frac{2P}{\pi dt} \tag{1}$$

$$\sigma_2 = -3\,\sigma_x \tag{2}$$

$$\sigma_3 = 0 \text{ (perpendicular to paper)} \tag{3}$$

where t is the axial thickness of the disc.

The source of the discrepancy between uniaxial and biaxial test results has been discussed in a recent paper [6] in which it is shown that a maximum tensile strain criterion offers a much better correlation than a maximum tensile stress criterion. According to this criterion, fracture occurs when the tensile strain reaches a critical value. For the general triaxial state of elastic stress

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$$\varepsilon_1 = \frac{1}{E}(\sigma_1 - \nu[\sigma_2 + \sigma_3]) \tag{4}$$

For the disc test this becomes:

$$E\,\varepsilon_1 = (1 + 3\nu)\,\sigma_1 = \sigma_{ef} \tag{5}$$

where $E\,\varepsilon_1$ is designated the equivalent stress (σ_{ef}).

For a uniaxial tensile test

$$\sigma_2 = 0 = \sigma_3 \text{ and} \qquad \sigma_{ef} = E\varepsilon_1 = \sigma_1 \tag{6}$$

Thus, according to the maximum tensile strain criterion the fracture value of σ_1 for the disc test should be smaller than the fracture value for the uniaxial test by the factor $(1 + 3\nu)$. This factor will vary from 1.75 to 2.5 as Poisson's ratio (ν) goes from 0.25 to 0.50.

Bridgman [7] has shown that the fracture stress for a brittle material increases with an increase in the hydrostatic compressive stress (σ_H) acting on the specimen. Since the mean principal stress approximates a hydrostatic state of stress we may assume

$$\sigma_H = \frac{\sigma_1 + \sigma_2 + \sigma_3}{3} \tag{7}$$

In reference [6] the strengths of tungsten carbide specimens containing 6 and 12 percent cobalt are compared. Both of these compositions are relatively brittle. Three types of specimens were tested:

1. Ring tests. Thin (1.25 mm) rings of 50.8 mm diameter by 12.5 mm width loaded by hydraulic pressure transmitted uniformly to the inner diameter through a rubber bag. This test, described in detail in reference [8], provides a uniaxial state of stress

$$\sigma_1 = \frac{pd}{2t} \qquad \sigma_2 = 0 = \sigma_3 \tag{8}$$

where p = hydrostatic pressure
d = mean diameter of ring
t = thickness of ring

2. Disc tests. Discs 12.5 mm diameter by 3.17 mm thick loaded across a diameter (Figure 1).

3. Sphere tests. Spheres 12.5 mm in diameter loaded across a diameter, for which the stresses at the centre of the sphere are as follows (8) in the transverse σ_1 and σ_2 and in the direction of the load σ_3:

$$\sigma_1 = \sigma_2 = \frac{4P}{\pi d^2}\left(\frac{21}{28 + 20\nu}\right) \tag{9}$$

$$\sigma_3 = -\frac{4P}{\pi d^2}\left(\frac{42 + 15\nu}{14 + 10\nu}\right) \tag{10}$$

where P is the load on a sphere of diameter d at fracture, and hence from equation (1)

$$\sigma_{ef} = E\varepsilon_1 = \frac{4P}{\pi d^2}\left[\frac{21}{28 + 20\nu}(1 - \nu) + \left(\frac{42 + 15\nu}{14 + 10\nu}\right)\nu\right] \tag{11}$$

The values of σ_H for these three types of tests are

Ring test:

$$\sigma_H = \frac{\sigma_1}{3} = \frac{pd}{6t} \tag{12}$$

Disc test:

$$\sigma_H = -\frac{2}{3}\sigma_1 = -\frac{4P}{3\pi dt} \tag{13}$$

Sphere test:

$$\sigma_H = \frac{\sigma_1+\sigma_2+\sigma_3}{3} = \frac{1}{3}\left[\frac{2(21)}{28+20\nu} - \frac{42+15\nu}{14+10\nu}\right]\frac{4P}{\pi d^2} = -\left(\frac{14+10\nu}{7+5\nu}\right)\frac{P}{\pi d^2} \tag{14}$$

If ν is 0.5 (plastic at rupture which is the case for the carbide specimens in the disc and sphere tests), then for the disc

$$\sigma_{ef} = 5.00\,\frac{P}{\pi dt} \tag{15}$$

and for the sphere test

$$\sigma_{ef} = 6.312\,\frac{P}{\pi d^2} \tag{16}$$

$$\sigma_H = -2.0\,\frac{P}{\pi d^2} \tag{17}$$

Median values of σ_{ef} at fracture are shown plotted in Figure 2 against σ_H for the three types of tests performed on two sintered tungsten carbide compositions containing 6 and 12 percent cobalt.

The influence that hydrostatic stress σ_H has on fracture is seen to be significant and a function of the ductility of the material. The more ductile composition (12 percent Co) undergoes a smaller increase in strength with hydrostatic compression than does the more brittle material (6 percent Co).

In designing a structure to be made from a brittle material it is important to know the complete state of stress at the critical point where fracture occurs. Otherwise the wrong material may be specified. For example, if the mean stress at the critical point is tensile, then a 12 percent Co will be stronger than a material containing 6 percent Co, while the reverse is true if the mean stress is compressive, as in the case of the disc or sphere tests.

No size correction has been made in this study. While the effective specimen size for the spheres and discs were about the same, the effective specimen size for the ring specimens were much larger. According to most authors this should result in the ring test values of this study being too low relative to the disc and sphere test values.

TORSION RESULTS ON MARBLE

Sato and Nagai [9] have recently presented fracture data for Italian Ondagata light marble tested in torsion both with and without fluid pressure applied at the outside diameter of specimens whose surfaces were previously sealed by a thin polymer coating. In all cases the foliation markings in the marble were oriented parallel to the specimen axes. Figure 3 shows the specific twisting moment M/d_o^3 versus torsional strain γ for coated solid cylinders tested at different values of hydrostatic pressure p. Curve I is the elastic curve and under atmospheric pressure, fracture occurs at point A (p = 0) on this curve, as we might expect for a perfectly brittle material. However, when fluid pressure (b) is applied, initial fracture occurs at a point A on curve I but the specimens continued to deform to large strains before completely coming apart at points marked by a cross (X). In all cases fracture surfaces were along helices inclined 45 degrees to the specimen axis, suggesting brittle fracture. Points A on curve I mark the conditions of initial fracture for different values of hydrostatic pressure p. Values of maximum shear stress τ_m at the surface for initial fracture were computed as follows:

$$\tau_m = \frac{16M_A}{\pi d_o^3} \tag{18}$$

The principal stresses lie on planes inclined 45 degrees to the specimen axes and are

$$\sigma_1 = \tau_m - p \tag{19}$$

$$\sigma_2 = -\tau_m - p \tag{20}$$

$$\sigma_3 = -p \tag{21}$$

Therefore, from equations (4) and (7)

$$\sigma_{ef} = (1 + \nu)\ \tau_m - p(1 - 2\nu) \tag{22}$$

and

$$\sigma_h = -p \tag{23}$$

Assuming Poisson's ratio to be 0.25 at fracture equation (22) becomes:

$$\sigma_{ef} = 1.25\ \tau_m - 0.5p \tag{24}$$

Median values of σ_{ef} versus σ_H are shown plotted in Figure 4. All points lie relatively close to the straight line despite the fact that specimen diameters varied from 8 to 15 mm. This suggests there is little or no size effect for this variation in specimen size.

Discs measuring 12.5 mm diameter by 3.17 mm thick were machined from the same marble used for the torsion tests, with foliation markings again running parallel to the specimen axes. Parallel flats having a chordal length of 2.54 mm were ground on these spcimens oriented either parallel to the foliation markings or normal to these markings. A soft brass shim of 0.25 mm thickness was placed between each flat and the loading anvils and the specimens were loaded to fracture (fracture load = P, lb).

Since no systematic difference was found for the two foliation orientations, all points were analyzed together.

The median value of σ_{ef} for the disc tests was found to be 5.10 kg/mm^2. From equations (5) and (13) for the disc test (ν = 0.25).

$$\sigma_H = -\left(\frac{2}{3}\right)\frac{\sigma_{ef}}{1.75} = -0.381\sigma_{ef} \tag{25}$$

and hence the median value of σ_H for the disc tests was -(0.381)(5.10) = -1.94 kg/mm^2.

From Figure 4 the value of σ_{ef} for the torsion tests for a value of σ_H of -1.94 kg/mm^2 is seen to be 4.00 kg/mm^2 (point A in Figure 4).

DISCUSSION

The fact that the values of σ_{ef} from the disc and torsion tests (5.10 kg/mm^2 and 4.00 kg/mm^2 respectively) are not the same might be due to three causes: (a) the magnitude of the assumed value for Poisson's ratio at fracture (0.25), (b) the presence of points of higher stress concentration at the surface of a torsion specimen than at the centre of a disc (surface defect effect), (c) a difference in the size of the critically loaded volumes for the torsion and disc tests (size effect).

The assumed value of Poisson's ratio at fracture appears reasonable for a material that fractures before plastic flow occurs. The fact that other choices of Poisson's ratio fail to yield closer coincidence further suggests that cause (a) is not important.

Since fracture occurs at the surface of a torsion specimen, the strength of such a specimen will be influenced by the presence of surface defects and surface roughness. On the other hand, fracture begins at the centre of a transversely loaded disc and surface imperfections will not have an influence in such a case.

The critical volume in which fracture begins is very small for the disc tests relative to that for the torsion tests. The probability of finding a flaw of a given magnitude in the critical volume in a disc test is therefore small relative to that for a torsion test. As a result we should expect σ_{ef} for the disc test to be greater than that for the torsion test at the same value of σ_H.

The fact that σ_{ef} for the disc test is greater than that for the torsion test at the same value of σ_H is consistent with the presence of both a surface defect effect and a size effect. However, to determine the relative magnitudes of these two effects would require considerable further work with these materials.

In reference [10] it is shown that the lack of influence of specimen diameter in the range of torsion specimen size of 8 to 15 mm diameter is consistent with the presence of a very weak size effect but that a change in effective specimen volume of 50 to 1 which is the value estimated for the disc and torsion tests could give rise to a significant size effect.

Figure 4 may be used to predict the uniaxial tensile strength for the Ondagata marble studied by Sato and Nagai. For uniaxial tension ($\sigma_2 = \sigma_3 = 0$)

$$\sigma_{ef} = \sigma_1 \tag{26}$$

$$\sigma_H = \frac{\sigma_1}{3} \tag{27}$$

The dashed curve in Figure 4 corresponds to these two equations and the intersection of the dashed and solid lines (point B) is the tensile strength (1.30 kg/mm^2). The tensile stress at fracture in torsion may be obtained by dividing the value of σ_{ef} at $\sigma_H = p = 0$ in Figure 4 by $1 + \nu = 1.25$. Thus, the median value of τ_m for $p = 0$ is 1.80/1.25 = 1.44 kg/mm^2. The median tensile stress at fracture in torsion (1.44 kg/mm^2) is thus predicted to be greater than the median value of tensile stress at fracture in a uniaxial tensile test (1.30 kg/mm^2), a result found experimentally by several investigators of brittle fracture in the past. The theory discussed here provides two reasons for this difference:

1. Fracture occurs at a critical value of tensile strain rather than at a critical value of tensile stress.
2. The critical tensile strain for fracture decreases with an increase in positive hydrostatic stress.

CONCLUSIONS

1. Brittle fracture occurs under a complex state of stress when the principal tensile strain reaches a critical value.
2. The critical strain at fracture increases as the hydrostatic stress (mean principal stress) becomes more compressive.
3. The size of the critically loaded volume plays a much smaller role in determining tensile strength than previously supposed. For example, a few hundred percent increase in critical volume will have a negligible influence.
4. The uniaxial tensile stress at fracture is lower than the maximum tensile stress at fracture in torsion.

ACKNOWLEDGEMENTS

The authors wish to acknowledge a grant from the RANN Programme of the National Science Foundation to the Processing Research Institute of Carnegie-Mellon University used to support this investigation. The marble discs employed in this study were kindly provided by Professor Y. Sato of the Tokyo Institute of Technology.

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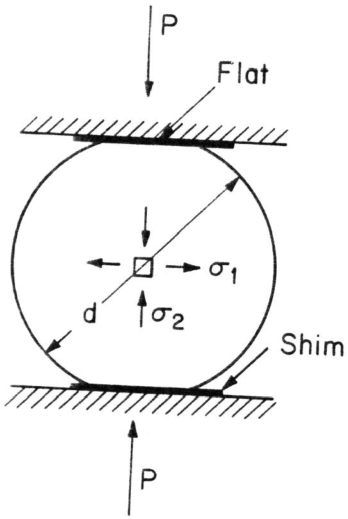

Figure 1 The Disc Test

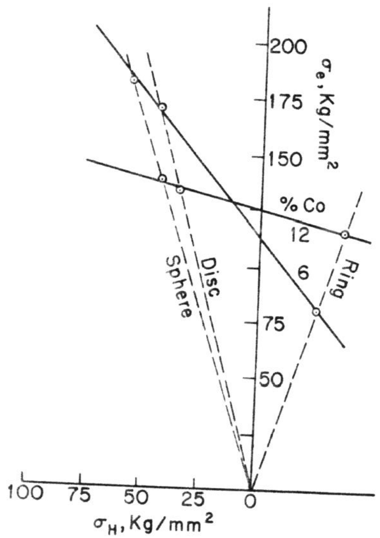

Figure 2 Variation of Median Effective Fracture Stress σ_{ef} with Hydrostatic Stress σ_H for Ring, Disc, and Spherical Tungsten Carbide Specimens Containing 6 and 12 Percent Cobalt

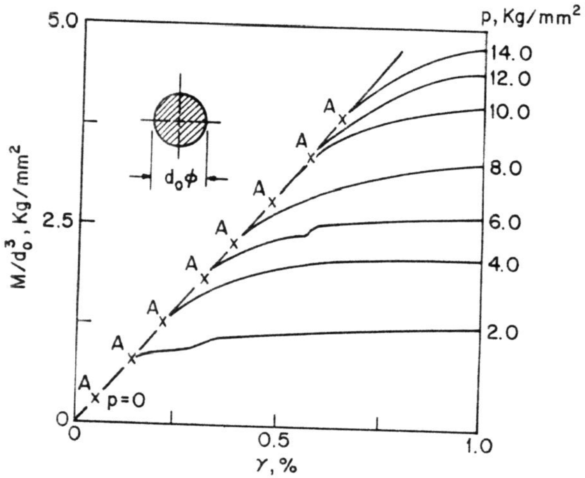

Figure 3 Variation of M/d_0 with Strain γ for Coated Marble Solid Cylinders Subjected to Different Values of Hydrostatic Pressure (p, kg/mm^2)

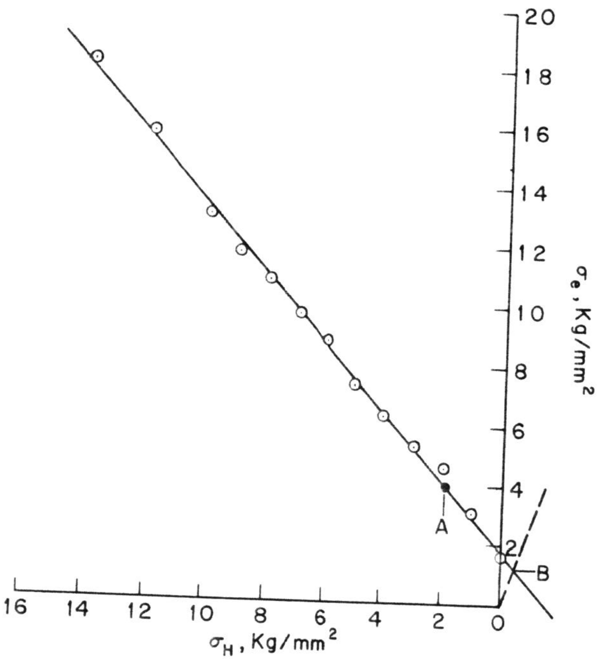

Figure 4 Variation of σ_{ef} with σ_H for Torsion Tests of Ondagata Marble at Fracture