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ASSESSMENT OF FAILURES BEYOND THE LINEAR ELASTIC REGIME

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INTRODUCTION

The failure of a flawed structure has been shown to be bounded by two limits, the linear elastic one and the plastic collapse one [1]. Both of these regimes are now well understood for most geometries. The most difficult regime to assess is the intervening regime, where fast brittle fracture follows plasticity. This situation is met where sections are relatively thin and the material is relatively ductile, or where stress are elevated locally beyond yield due to geometric constraint. Here the adoption of either of the two limits can lead to an overestimate of the defect tolerance of a structure so that resort has to be made to some form of "post-yield" assessment. The following is an attempt to define and validate one of these post yield routes, and to show that even after appreciable plasticity, brittle fracture can still be described by the linear elastic failure parameter, K_{1C}.

THE MODEL

The model chosen is based upon the Bilby Cottrell Swinden [2] theory of yielding ahead of a crack. This is a crack opening displacement approach to fracture which can be reinterpreted in terms of K_{1C} following the suggestions of Heald Spink and Worthington [3]. Because of its theoretical basis this approach can be treated analytically [4] making it very versatile. In this way it has distinct advantages over empirical crack opening displacement approaches which are not universally applicable.

The basic equation can be generalised in terms of the failure stress, σ_f

$$\sigma_f = \frac{2}{\pi}\sigma_1 \cos^{-1} \exp - \frac{\pi^2 K_{1C}^{\ 2}}{8Y^2 a\sigma_1^{\ 2}} \tag{1}$$

where Y is the linear elastic compliance of the cracked body, and takes crack shape into account, and σ_1 is the collapse stress of the cracked body. The value of σ_1 must take into account the geometric constraint local to the crack. It can be obtained from conventional limit analysis, from slip line field theory, from finite element analysis or from the testing of scale models, whichever is the most appropriate. This has advantages over the J integral approach in that small scale tests can be used to confirm the predictions and avoid over-reliance on finite element techniques.

Following Harrison Loosemore and Milne [6] equation (1) can be plotted as a universal function in terms of the ratios $K_r = K_1/K_{1C}$ and $S_r = \sigma_f/\sigma_1$, Figure 1, where K_1 is calculated elastically at σ_f. It should be noted

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that the stress ratio, S_r could equally be written in terms of loads or pressures, and need not be converted to stresses.

VALIDATION

For the model to be generally applicable it must be capable of describing failure in any geometry. Over the previous 10 years a whole fund of test data has become available on structural geometries, such as cylinders and spheres. Much of this data suffers from the lack of relevant materials data, particularly fracture toughness and so a direct comparison with the assessment line in Figure 1 is not possible. However, using this assessment line it is possible to obtain the ratio K_r and hence predict a K_{IC}. This should be constant with varying geometry and compare favourably with the expected values of K_{IC}.

Cylinder tests of Nichols Irvine Quirk and Bevitt [7]

These tests were performed on a variety of steel cylinders of varying diameters. Collapse can be expected in these geometries when $M\sigma = \bar{\sigma}$, where $\bar{\sigma}$ is a flow stress and M is the stress magnification factor due to bulging [8]. There is some speculation as to the actual value of the flow stress; the value adopted here is $\frac{1}{2}(\sigma_y+\sigma_u)$.

Table 1 groups the results obtained from these tests on .36 C steel, in order of temperature. It is apparent that failure occurred mainly at stresses well below the collapse limit. The predicted values for K_{IC} are reasonably constant at a given temperature and increase with temperature.

HSST 3 inch vessels of Derby [9]

Data on these tests are again not very complete. Failure was after considerable plastic bulging at the higher temperatures; thus the pressure at these temperatures was taken as the limit pressure. Consequently three of the tests failed with $S_r < 1$, as indicated in Table 2. (σ_u was assumed independent of temperature over this temperature range). Values for K_I were difficult to predict, since there are no standard solutions for external part penetrating longitudinal cracks in pipe geometries, but they were obtained by applying a bulging factor to the solutions of Merkle et al [10]. The predicted values for K_{IC} are within the limits expected.

HSST large test vessels

For these vessels σ_1 was taken from the solutions of Duffy et al [8] for part penetrating defects, i.e.

$$\sigma_1 = \bar{\sigma}\left[\frac{t/a - 1}{t/a - 1/M}\right],$$

where t is the thickness of the vessel.

$\bar{\sigma}$ was again taken as $\frac{1}{2}(\sigma_y+\sigma_u)$, and K_I was obtained using the ASME XI procedures. The predicted values of K_{IC} are within the range expected, Table 3.

Spherical vessel tests of Lebey and Roche [11]

Here different sized vessels made of different thicknesses of AMMO steel were tested with varying crack lengths. σ_1 was taken as $0.8\ \sigma_u$, since this is approximately the limit stress for the shorter cracks. Table 4 lists the predictions for these tests using the initiation of crack growth as the failure point. For a given sphere the variation in the predicted values of K_{IC} are within the range normally experienced in steels of this nature.

3-point bend specimens of Lubahn and Yukawa [12]

Here the specimen size was varied, so that the collapse stress could be taken as the failure stress in the smallest specimen. The predicted values for K_{IC} were very constant and of the value expected of a Ni-Mo-V steel, Table 5.

CKS specimen tests of Begley and Landes [13]

In this case σ_1 was obtained by the curve fitting technique of Chell and Milne [14]. This is important at high values of S_r (for the smaller series of tests $S_r \to 1$) in order to obtain precision in the region where the curve becomes asymptotic to $S_r = 1$, and the predictions were obtained using equation (1) rather than Figure 1. Nevertheless these predicted values of K_{IC} are in full agreement with those of Begley and Landes and the data obtained from large scale tests (Table 6).

DISCUSSION AND CONCLUSIONS

In the simplified form of Figure 1 equation (1) has been shown to predict consistent values for K_{IC} in both the linear elastic and the large scale yielding regimes. Hence it is proposed as a means for assessing the integrity of a structure regardless of the operational stress level. It is not evident from the foregoing, however, that an infinite value for K_{IC} is predicted at $S_r = 1$. This is consistent with the known behaviour that plastic collapse is independent of K_{IC}. Thus an increasing uncertainty occurs in the predicted K_{IC} as the Assessment Line in Figure 1 becomes asymptotic to $S_r = 1$. In this region σ_1 needs to be known very accurately for adequate predictions of K_{IC}, as was required for the CKS specimen tests. This is no disadvantage however, especially for assessment purposes where it is prudent to ensure pessimism by using upper bound criteria for S_r and K_r.

The advantages of the above approach using the assessment line of Figure 1 are manifold:

1) Any appropriate analytical technique can be used to obtain the parameters K_I and σ_1. Thus the approach is as versatile and sophisticated as our knowledge of stress analysis.
2) Where an analysis is suspect, or inadequately defined, resort can be made to model testing for the evaluation of σ_1.
3) The effect of secondary stresses can be easily studied; e.g. if it can be demonstrated that a secondary stress influences only the linear elastic regime it is easy to allow for this without being unduly pessimistic.
4) The influence of various factors of safety on the final assessment can be readily explored; e.g. the effect of using factors in σ, or mat-

erials data, or in defect size etc.

5) The most likely regime of failure is immediately apparent.

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Table 1

Test	Temp. (°C)	2a (mm)	σ_f	K_1	S_r	K_r	K_{1C} Predicted
V7T1	1	628.6	66.4	166.3	.47	.96	173
V13T1	10	304.8	95.75	115.5	.43	.97	119
V1T2	12	152.4	190	105.6	.62	.92	115
V8T2	13	304.8	123.5	128.7	.52	.95	135
V14T2	17	304.8	139	124.3	.46	.97	128
V2T1	29	304.8	130	134.2	.55	.94	143
V5XT1	45	152.4	222	124.3	.707	.88	141
V12T3	50	304.8	120.4	146.3	.55	.94	156
V4T4	51	304.8	145	150.7	.62	.92	164
V4T1	62	152.4	227	126.5	.73	.86	147
V3T1	62	609.6	88	215.6	.57	.94	229
V6T1	77	304.8	177.6	182.6	.77	.84	217
V12T1	79	304.8	161.4	195.8	.73	.86	227
V14T1	80	304.8	187	166.1	.62	.92	180
V5T1	84	304.8	183.8	190.3	.77	.84	226
V3XT1	88	609.6	105	108.9	.407	.97	112

Table 2

Temp. (°C)	P_f	K_1	S_r	K_r	K_{1C} Predicted
-45	20.7	40.7	.885	.75	54
-18	20.7	40.7	.885	.75	54
-3	21.4	42.1	.914	.72	58.5
+16	23.4		1.0		
+54	23.4		1.0		

Table 3

Vessel	Temp. (°C)	a (mm)	2C (mm)	σ_f	K_1	S_r	K_r	K_{1C} Predicted
1	54	65	209.5	365.4	194.7	.686	.89	219
2	0	64.25	210.8	355.8	171.6	.61	.92	186
3	54	53.6	215.9	397.1	183.7	.74	.86	213
4	24	76.2	209.5	337.8	176.0	.706	.88	200
6	88	47.5	133.4	406.8	215.6	.74	.86	279

Table 4

Sphere No.	2a (mm)	σ_f	K_1	S_r	K_r	K_{1C} predicted
9	35	330	116	.77	.84	138
	70	205	177	.48	.95	186
	105	132	203.6	.31	.98	207
10	40	318	127.5	.74	.86	151
	59	200	101	.46	.95	106
13	35	336	126	.78	.82	153
	50	254	139	.59	.92	151
	65	217	161.5	.5	.94	171
	72	180	153	.42	.96	159
	80	166	163.5	.39	.97	168
	95	132	173	.31	.98	177
	102	125	197	.29	.984	199
	110	115	186	.27	.99	188
	125	100	190.4	.23	.99	192
15	15	368	63.25	.86	.77	82
	25	368	94.5	.86	.77	123
10	56	203	152	.47	.95	160
	61	204	132.5	.47	.95	139
	75	185	165	.43	.96	172
	99	113	142	.26	.99	143

Table 5

Specimen size (mm)	σ_f	K_1	S_r	K_r	K_{1C} Predicted
5	1379	74.8	.93	.69	108.
10	1206	93.5	.814	.81	115.5
18	1103	112.2	.744	.855	127.6
40	724	112.2	.49	.96	117.7
100	414	101.2	.28	.99	102.3
240	276	103.4	.19	.995	103.4

Table 6

Specimen size (mm)	$a/_w$	σ_f	K_1	S_r	K_r	K_{1C} predicted	K_{1C} predicted from Begley & Landes (13)
25.4	.604	31.7	98.5	.99	.54	182	194
25.4	.573	37.2	102.5	.995	.49	208	
25.4	.547	41.4	104	.995	.49	212	
50.8	.576	41.7	140.8	.91	.72	195	202
50.8	.552	39.8	144.1	.9	.73	198	
50.8	.526	44.8	148.7	.9	.73	204	
50.8	.5	49.1	150.5	.86	.77	196	

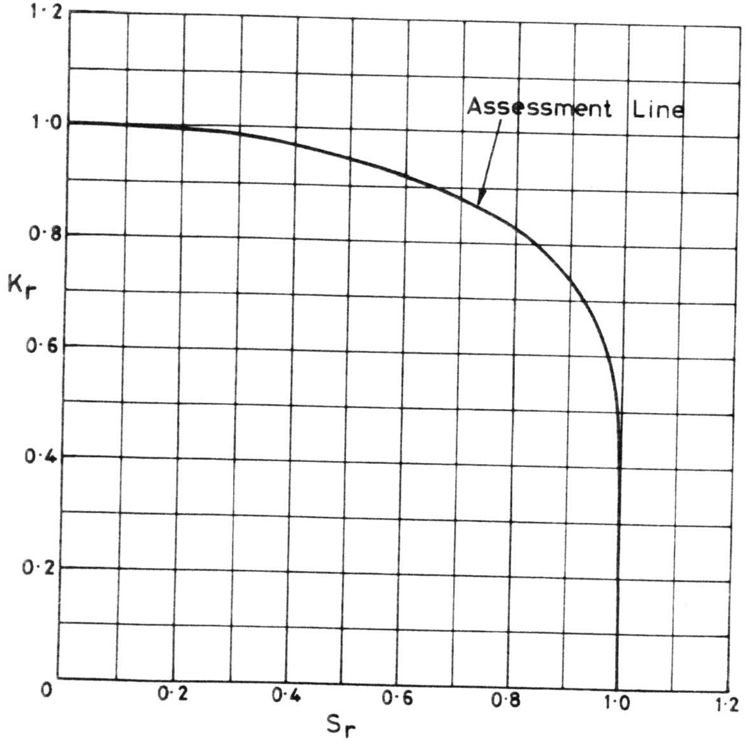

Figure 1