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A NUMERICAL APPROACH FOR STABLE CRACK-GROWTH AND FRACTURE CRITERIA

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I. INTRODUCTION

The behaviour of a cracked body in large-scale yielding conditions has been intensively studied in the past years. The well-known crack-tip parameters like J-integral or C.O.D. are generally computed and the influence of plasticity studied. However the computations are made for a stationary crack and give no information about stable crack-growth and corresponding fracture criteria.

Stable crack-growth has been studied by Andersson [1] by performing successive relaxation of crack-tip nodal forces in a finite-element programme. In this paper we attempt to refine this approach by introducing on the extension of the crack-line special finite-elements modelling the behaviour of the end-region and allowing the elimination of stress and strain singularities. Stable and unstable crack-growth will be connected to the fracture properties of the material submitted to complex loading.

II. FRACTURE CRITERION FOR AN ELASTIC BODY

The usual boundary conditions on the crack-line, mode I: $u_2 = 0$, $\sigma_{12} = 0$, ($\sigma_2 = \sigma_{12} = 0$ on the crack faces), lead to infinite stresses and strains at the crack-tip. Finite stresses and strains are obtained with the boundary condition $\sigma_2 = f(u_2)$ instead of $u_2 = 0$, as in the Barenblatt's model [2] and also in the Dugdale's model [3]. The main difficulty lies in the interpretation of the normal displacement u_2 in a continuum model. In this paper u_2 is interpreted from the strain $\varepsilon_2 = u_2/h$ of a strip with height 2 h located on the extension of the crack line, in a way similar to the rigid-plastic strip model introduced by Rice [4]. Dugdale's model is based on the Tresca criterion, which gives $\sigma_2 = f(u_2) = \sigma_y$ for an elastic-perfectly plastic material in plane stress conditions. In this paper a state of plane deformation is assumed; the relation $\sigma_2 = f(u_2) = g(\varepsilon_2)$ represents the local stress-strain curve in the strip and is related to the flow rule of the material.

The geometry of the crack-tip is modified by the insertion of the strip. But such a model is perhaps more realistic in this highly strained region than the usual reference to the initial geometry of a cut with zero crack-tip radius.

In this way, for an infinite linear-elastic medium (plane strain), the problem is reduced to the one-dimensional integral equation:

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$$u_2(x) = [2(1-\nu^2)/\pi E] \int_{-\infty}^{+\infty} \sigma_2(t) \ln|t-x| \, dt . \quad (1)$$

Andersson and Bergkvist [5] have resolved numerically this equation with a law $\sigma_2 = f(u_2)$ linearly increasing than decreasing; in this paper we consider a more general law. The non-linear part of $f(u_2)$ will be used to define the length of a "plastic zone" limited to the strip.

The model gives an interesting possibility to connect the global criterion of fracture with a local criterion at the crack-tip, defined by $\sigma_2 = \sigma_f$ or $\varepsilon_2 = \varepsilon_f$. We show numerically that in small-scale yielding the local criterion defined above yields the global criterion $K_I = K_{IC}$ with a good accuracy. We obtain $K_{IC}^2 = k\, S_f$, where S_f is the area under the curve $\sigma = g(\varepsilon)$ up to the limiting strain ε_f for which the stress vanishes.

The J-integral is found to be path-independent outside the "plastic zone". For a remote path, at the onset of fracture, its value is $J_c = [(1-\nu^2)/E]\, K_{IC}^2$, while for a path along the boundary of the "plastic zone" $J_c = 2h\, S_f$. This latter result is the same as given by Rice [6] but is based on a different model. So the constant k should be equal to $2hE/(1-\nu^2)$. This is verified numerically with a good accuracy.

III. ELASTIC-PLASTIC BODY. STABLE CRACK-GROWTH

A finite-element approach is convenient in the case of an elastic plastic body. The incremental plastic deformation, in plane strain conditions, is taken according to the Prandtl-Reuss flow-rule along with the von Mises criterion. An "implicit" algorithm recently given by Nguyen, Q. S. [7] is used. The implicit algorithm eliminates all the numerical and systematic errors usually found in the "explicit" method. Furthermore loading, unloading and reloading can be easily done.

The elastic-plastic constants are: $E = 200{,}000$ MPa, $\nu = 0.3$, $\sigma_y = 700$ MPa, linear hardening with a 1,000 MPa modulus. The law $\sigma = g(\varepsilon)$ in the strip is not the conventional curve obtained in the tension test, but a deduced curve corresponding to uniaxial strain. It is chosen to represent the complex stress state at the crack-tip. The fracture of each element occurs at a critical stress $\sigma_2 = \sigma_f = g(\varepsilon_f)$; for $\varepsilon > \varepsilon_f$ the curve $g(\varepsilon)$ drops to zero. During crack growth, the crack-tip nodal force is relaxed in five equal steps.

We study a three-point bend specimen (width W, span S = 4 W, thickness B, initial crack-length a_o, $b = W-a_o$). In order to avoid the effect of element size the elements adjacent to the uncracked ligament near the crack-tip have the same dimension, and remain unchanged for specimens of different sizes. The side s of these triangular elements is taken equal to 0.1 mm for a width W from 5 to 200 mm. The strip height h is no longer the characteristic length of the process of fracture as it was the case for an elastic body. In fact h may be related to the crack-tip radius, and the results are independent of h if it is sufficiently small (here for $h < 0.01$ mm). It is s which is the characteristic length: fracture occurs when σ_2 (mean) $> \sigma_f$ over a distance s from the crack-tip.

The load-deflection curves P (d) for $\sigma_f = 2{,}300$ MPa are given in Figure 1 along with the crack-growth curves P (Δa) for W = 20 and 5 mm. The points of crack-growth initiation and unstable crack-growth depend on the size of the specimen. Moreover the second one depends on the loading conditions. With load-control the instability occurs after a few steps of crack-growth. With displacement-control the crack-growth is stable; it goes on under a quasi-constant, than decreasing load (curve 5a); the maximum load can be somewhat greater than that obtained with load-control (curve 3a).

IV. FRACTURE CRITERIA

Figure 2 shows different critical values of the stress-intensity factor K_I as function of specimen size. These values are deduced from Figure 1 as follows: K_{max} is the value of K_I at maximum load; K_{max}^d at the load obtained by extrapolation of the linear part of the P - d curve up to the displacement at maximum load; K_Q at the load defined by the intersection with "5%-secant"; K_B is computed according to the equivalent energy concept introduced by Witt [8]; K_{Ji} is deduced from the J-integral at the onset of stable crack-growth. K_Q and K_{max} decrease for $b < 1$ to $1.5\,(K_{IC}/\sigma_y)^2$ as it is verified with medium-strength steels if the thickness is sufficient. K_{max}^d and K_B are more constant and bracket the value of K_{IC}; these two values give a good estimate of the fracture toughness K_{IC} with "medium-size" specimens [between 0.25 and $1.5\,(K_{IC}/\sigma_y)^2$]. With smaller specimens K_{max}^d and K_B are no longer well-defined for the computed value of the displacement at maximum load is not accurate.

The J-integral is computed for two cases: $\sigma_f = 2{,}300$ MPa and 3,200 MPa. $V = -(1/B)[dU/da]_d$ and $V^* = -(2/B)(U^*/b)$ are also computed[1]. The obtained values are converted into K_J, K_V and K_V^* by the usual plane strain formula $J = [(1-\nu^2)/E]K^2$. They fit well together, at least until full plasticity, which justifies the experimental determination of J (see Figure 3).

K_J at the initiation of crack-growth, i.e., K_{Ji}, is practically independent of the specimen size and depends only of the material considered: $K_{Ji} \simeq 54$ MPa$\cdot$m$^{1/2}$ for $\sigma_f = 2{,}300$ MPa (see Figure 2), $K_{Ji} \simeq 100$ MPa$\cdot$m$^{1/2}$ for $\sigma_f = 3{,}200$ MPa (for W = 20 mm, $a_o/W = 0.5$, 0.6, 0.7 - for W = 50, 100 and 200 mm, $a_o/W = 0.5$). The J-integral gives a good criterion for the initiation of stable crack-growth. However, since K_J deviates very little from the linear curve, K_I^d (Figure 3) is a more simple, but approximate, criterion for the initiation.

For the two cases investigated K_{Ji} is notably smaller than K_{IC}, about 30% for the weaker material ($\sigma_f = 2{,}300$ MPa) and 70% for the tougher one ($\sigma_f = 3{,}200$ MPa: in that case it was not possible to reach the point of instability, because of computer limitations; K_{IC} is greater than 300 MPa$\cdot$m$^{1/2}$; this shows the high dependence of K_{IC} with σ_f). It does not seem that the value of the J-integral at the onset of stable crack-growth allows the direct determination of fracture toughness on small specimens.

V. FURTHER DEVELOPMENTS

The magnitude orders of the computed values K_{IC} and K_{Ji} are quite good. However the model will be refined in the following ways.

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[1] These two values are given by the well-known relations used for the experimental determination of J, the former with a few specimens, the latter with a single deep-cracked specimen ($a/W \geq 0.6$) [9].

First, the uniaxial strain hypothesis will no longer be imposed to the special crack-elements. The algorithm for the finite-elements in plasticity will be used also for the special crack-elements, that is to say the increments of stresses will be given as functions of actual stresses, hardening parameter and increments of strains $\Delta\varepsilon_1 = \Delta u_1/\Delta x_1$, $\Delta\varepsilon_2 = \Delta u_2/h$, $\Delta\varepsilon_{12} = 0$.

Second, instead of a critical stress σ_f, a local criterion $F(\sigma_1, \sigma_2, \sigma_3) = 0$ will be used. It will be related to tests on notched - but not cracked- specimens of a given material.

With these improvements an agreement is hoped between numerical and experimental values of K_{IC} for the given material. Moreover theoretical and experimental results in large-scale yielding conditions will be compared.

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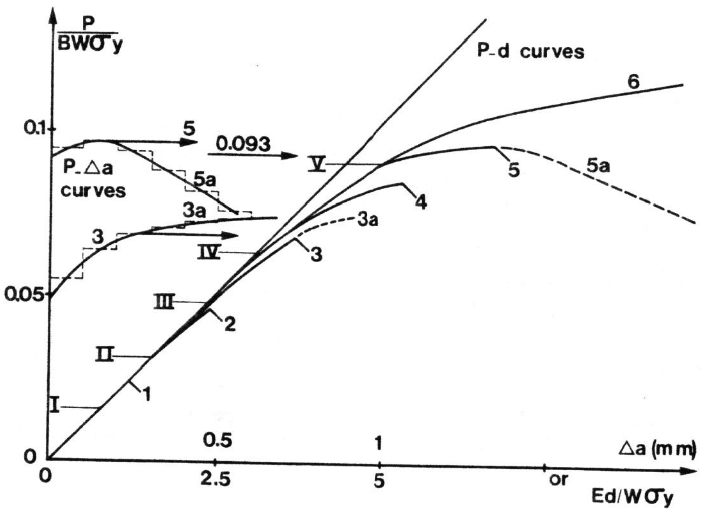

Figure 1 Computed Dimensionless P versus d Curves with σ_f = 2,300 MPa for Different Specimen Sizes (1 : W = 200 mm, 2 : 50 mm, 3 : 20 mm, 4 : 10 mm, 5 : 5 mm; a_o/W = 0.5) with Load Control (Curves 1 to 5) or Displacement Control (Curves 3a and 5a). Points of Crack-Growth Initiation are Shown on Each Curve by Roman Numerals (I to V). Curve 6 is for a Stationary Crack. Load P versus Crack-Growth Δa Curves are Shown for W = 20 mm and 5 mm. $P/BW\,\sigma_y$ = 0.093 is the Limit Load from the Slip-Line Field Theory

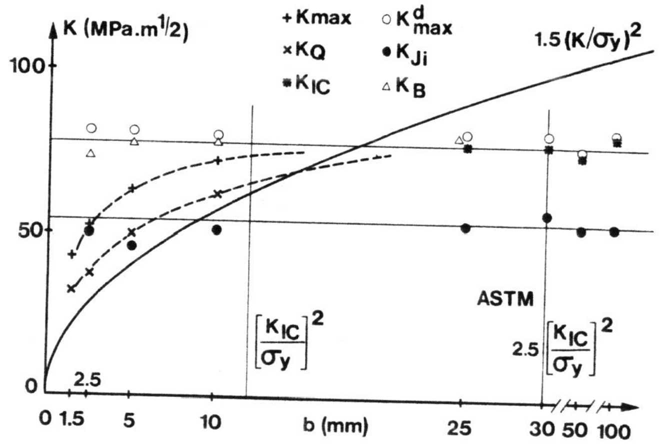

Figure 2 Computed Critical Values of K_I and K_J for σ_f = 2,300 MPa as Function of Specimen Size (W = 200, 100, 50, 20, 10 and 5 mm; a_o/W = 0.5 and 0.7). Symbols are Defined in the Main Text

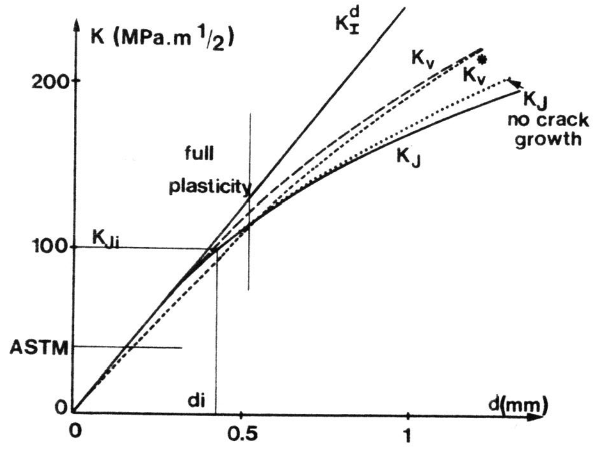

Figure 3 Computed K versus d Curves with σ_f = 3,200 MPa for a W = 20 mm, a_o/W = 0.6 Specimen (——— K_J, ... K_J for a Stationary Crack, —— —— —— K_V, ----- K_V*). Symbols are Defined in the Main Text