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A FINITE ELEMENT METHOD ANALYSIS OF AN ELASTIC-PLASTIC SOLID CONTAINING HOLES IN THE VICINITY OF A CRACK TIP

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INTRODUCTION

Inclusions can sometimes reduce fracture toughness strongly i.e., in alloys where fracture is accompanied by extremely limited plastic flow. This feature is characteristic of modern high strength steels with, often unintentionally, a high inclusion content of sulphides and oxides. The reduction of fracture toughness due to inclusions must be a result of their influence upon the stress and strain distributions in a body, especially in the vicinity of the crack tip. The most accurate description of these distributions in an elastic-plastic body is today obtained by means of the finite element method.

The path independent J-integral is a function of stress and strain along a path enclosing the crack tip which characterizes the state at the crack tip and which can be used as a fracture criterion for both elastic and elastic-plastic conditions, as suggested by Rice [1]. In this work an attempt is made to estimate the reduction of fracture toughness due to inclusions by calculating the J-integral for equivalent paths around domains with and without inclusions.

According to a discussion in a previous paper [2] inclusions such as oxides and sulphides shall be considered as holes already from zero load. The results of this work is compared to experimental results given in the above previous paper.

THE FINITE ELEMENT PROGRAMME

The finite element programme used in this work was originally developed by Härkegård and Larsson [3]. They adopted a principle according to Yamada et al [4], based on the von Mises yield criterion and the Prandtl-Reuss equations, permitting an incremental treatment of elastic-plastic problems. The programme has subsequently been extended by Markström to include routines for calculations of path-independent integrals. Markström and Carlsson [5] have later studied, among other things, the effect of element size on the value of the J-integral and also the effect of different orientations of the integration path. One important result is that if the integration path intersects plasticized domains, then path independence is preserved only if the calculation includes the total elastic-plastic work. With their results in mind, it has been possible to choose an optimal element size and permissible integration paths.

The present problem is attacked by using the 'boundary layer' approach, that is, by assuming that the boundary stresses σ_{ij} of an elastic-plastic

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domain around the crack tip are given by the singular term in the elastic stress solution:

$$\sigma_{ij} = \frac{K}{\sqrt{2\pi R}} f_{ij}(\theta) , \qquad (1)$$

where K is the stress intensity factor, R the radius of the domain considered and $f_{ij}(\theta)$ are given by the elastic solution.

The domain around the crack tip is divided into approximately 300 elements. Triangular constant strain elements are used. The element size at the crack tip is approximately 0.03 times the crack length. The load is applied at the boundary nodal points and the load distribution is determined according to equation (1) and to the principle of virtual work. For each load increment at most one element is plasticized.

Three geometries containing holes are studied. The first one comprises one hole, symmetrically situated in the elastic domain between the two branches of the plane strain plastic zone, and the second one also one hole asymmetrically situated with its centre on a line through the crack tip and the point of maximum extent of the plastic zone. The third one is a combination of the former two geometries, and is shown in Figure 1. As a reference a geometry without holes is used. The proportions of the geometry at the crack tip are chosen according to observations of the fracture surfaces of a real material.

COMPUTATION OF THE J-INTEGRAL

A necessary condition for path independence of the J-integral is that the domain inside the integration path is simply connected. In order to check if the presence of holes there implies any restriction of the path independence, the J-intregral was computed for paths either including or excluding the hole(s) in all geometries. All such integration paths coincide except in a neighbourhood of the hole(s), where for a given geometry one path is traced so as to include the hole(s) and the other so as to exclude it (them). For any pair of integration paths the corresponding difference in J is smaller than 1% for any value of J. Thus the presence of holes in the domain inside an integration path does not imply any restriction of the path-independence of the J-integral.

RESULTS AND DISCUSSION

The values of the J-integral of the geometries containing holes are given as functions of that of a massive material, in Figure 2 for 'small loads', that is, up to loads for which the crack tip plastic zone encloses all holes, and in Figure 3 for the whole load range investigated, where i = 1 refers to the two-hole model, i = 2 to one hole assymmetrically situated and i = 3 to one hole symmetrically situated.

In the whole load range the presence of holes implies increased stress and strain in every element of the hole-geometries. It may be concluded that the larger J-value of the hole-geometries correspond not only to a larger volume of plasticized material, but also, especially at the crack tip, to a more intense stress and strain distribution. Therefore, considering equally tough matrix materials, the larger value of the J-integral of the hole-geometries compared to that of a massive material is likely to correspond to a lower apparent fracture toughness of an inclusion material.

If this assumption is correct the results from all geometries indicate that the reduction of fracture toughness due to inclusions increases as the fracture toughness of the matrix material increases.

In Figure 4 the reduction of fracture toughness according to the model is compared to experimental results. In spite of its simplicity the model yields reliable results. Under plane strain conditions the model predicts accurately the reduction of fracture toughness and yields conservative, safe, values when extrapolated to mixed plane stress-plane strain conditions.

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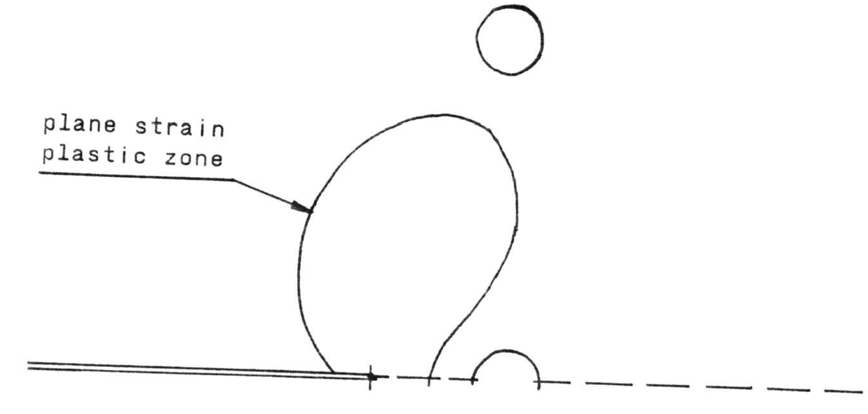

Figure 1 The 'Two Hole Model'

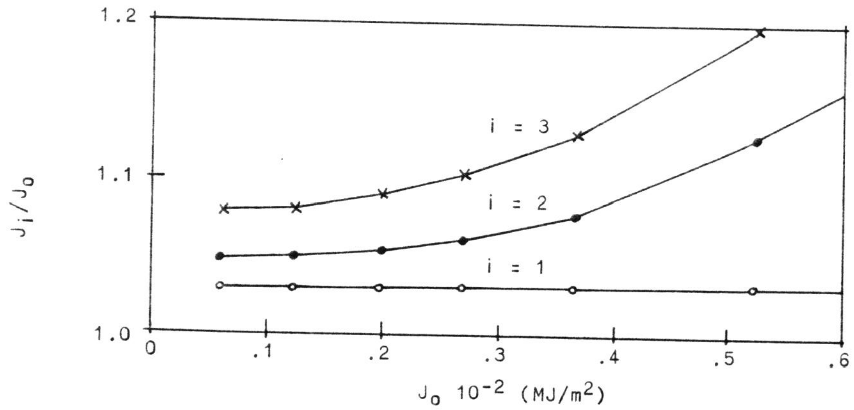

Figure 2 Effect of Holes Upon the J-Integral, 'Small Load' Range

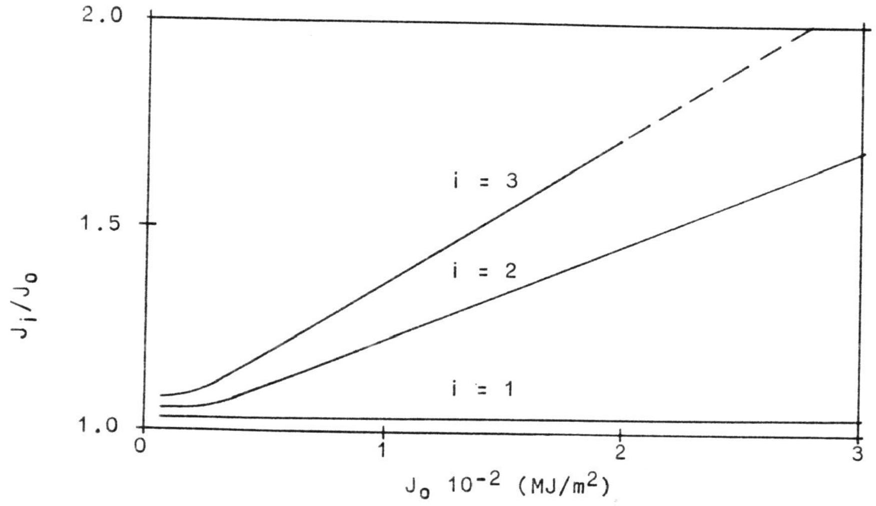

Figure 3 Effect of Holes Upon the J-Integral

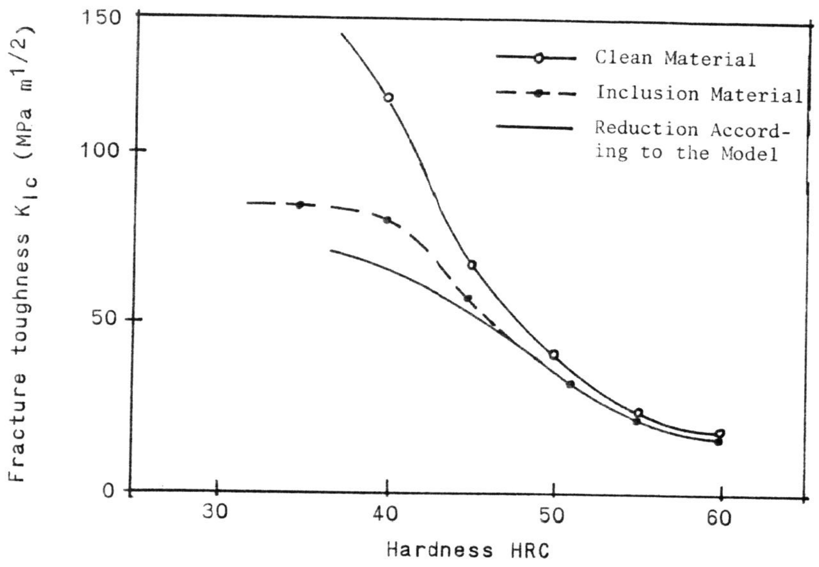

Figure 4 Comparison of Experimental and Theoretical Results