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A CRITERION FOR STRENGTH OF STRUCTURAL MATERIALS IN COMPLEX STRESS STATE AT LOW TEMPERATURES

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INTRODUCTION

Equivalence conditions (or strength conditions) are used for estimating the strength of materials in any arbitrary stess system. These are based on particular assumptions, often of hypothetic nature [1].

However, it is difficult to select among the criteria proposed so far the most substantiated ones. The reliability of each criterion is restricted by both the type of materials and the range of principal stress ratio. As to selecting the criteria for ultimate state of materials at low temperatures, no recommendations are available in the literature.

INITIAL PHYSICAL ASSUMPTIONS

Since the metal ability to deform plastically changes as the temperature is varied, one can take, as basis for the theory of strength, Prandtl's conception of two types of fracture: brittle (by break-away) and tough (by shear) [2]. Prandtl's scheme is known to have found a wide development in works of the Soviet school of mechanics [3,4,5] and it does not contradict to the latest advances in the physics of solids. Accordingly, the plastic strain due to shear stresses loosens the material and prepares it for rupture while a disintegration of the material solidity occurs under normal tensile stresses.

Thus, the occurrence of the ultimate state is stipulated by the material capacity to resist to both shear and tensile stresses. Consequently, the ultimate state is determined by two criteria: a criterion of crack appearance, some function of the shear stress τ_{kl} and a criterion of crack propagation, the maximum tensile stress σ_{max} as the highest of the three stresses. The condition that defines the ultimate state of material may be given by

$$f(\tau_{kl}, \sigma_{max}, C_i) = 0 \qquad (1)$$

where C_i is a constant of the material.

THE CRITERION STRUCTURE

The closer is the material state to a perfectly brittle one, the less becomes the part played by shear stress in the fracture process. Con-

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versely, for a material in the plastic state, functions of shear stress only may be adopted as equivalent stress since the yielding itself, without fracture, may prove to be unsafe. For structurally-uniform materials, these obvious conditions are sufficiently met by the criterion (1) written in the form [6,7];

$$\chi\sigma_i + (1 - \chi)\sigma_1 = \sigma^+ \tag{2}$$

Equation (2) contains as a function of shear stress, the stress intensity $\sigma_i = \sqrt{2}\sqrt{\tau_1^2+\tau_2^2+\tau_3^2}$.

The parameter $\chi = \sigma^+/\sigma^-$ (σ^+,σ^- are limiting stresses in tension and compression) characterizes the degree of responsibility for macroscopic fracture by shear strain that provides for conditions favourable for "loosening" the material.

In the stress space, the condition (2) is interpreted into a surface in which is inscribed a hexahedral pyramid corresponding to the Coulomb-Mohr criterion. If $\chi \to 1$, then the limiting surface is reduced to Mises' cylinder. In the case where $\chi \to 0$ the surface disintegrates into three pairs of planes (the theory of maximum normal stress).

The increase in nonuniformity of the stress field and "defectiveness" of material due to lowering the temperature suggests that the resistance to fracture needs to be considered with allowance made for statistical regularities. Bearing in mind the structure of equation (2), the ultimate state criterion may be given for this case by

$$\eta = \chi\sigma_i + (1 - \chi)\sigma_1\varphi, \tag{3}$$

where φ is an influence function that reflects the statistical regularities of deformation and fracture.

Thus, the first summand in the criterion (3) reflects regularities of shear processes while the second one expresses dynamic and statistical regularities in crack propagation and formation of fracture surfaces. A further investigation of the criterion is reduced to a reasonable selection of the function φ.

We assume that the degree of softening of a solid body associated with the presence of defects is directly proportional to the probability of accumulating a critical number of propagating cracks at a given stress level. Then the magnitude φ in the criterion (3) can be evaluated as a ratio of some tolerance for the probability W_0 of solidity disintegration at weak sites Z' in the major type of testing (for example, in uniaxial tension) to the corresponding probability W in any arbitrary stress state.

$$\varphi = \frac{W_0}{W} \tag{4}$$

The critical number Z of propagating cracks, upon reaching of which the fracture occurs, depends generally on the material nature and structure. An essential part is played also by the mode of stress state.

One can assume in the first approximation that

$$Zcr = \alpha - \beta S, \tag{5}$$

where α, β are constants reflecting the material properties associated with the presence of weak sites (defects) in it; $S = \sigma_1+\sigma_2+\sigma_3/\sigma_i$ is a parameter of the stress state, which has, by anology with σ_{max}/τ_{max}, the meaning of loading severity.

Using the theorem of multiplication of probabilities of independent event, we may write the probability of development of Zcr defects out of all defects present in the material as

$$W = q^{Zcr}(1 - q)^{Z-Zcr} \tag{6}$$

where q is the probability of evolving each crack.

Using (4), (5) and (6) and taking into account that for uniaxial tension S = 1 we obtain

$$\varphi = \left(\frac{1 - q}{q}\right)^{\beta(1-S)},$$

or

$$\varphi = C^{1-S}, \tag{7}$$

where $C = (1-q/q)^{\beta}$ is a constant of material.

Thus, the expression for equivalent stress according to the criterion (3) can be written in the form

$$\eta = \chi\sigma_i + (1 - \chi)\sigma_1 C^{1-S}. \tag{8}$$

A statistical nature of the function φ is confirmed by the correlation of the constant V with a homogeneity factor in the brittle-strength theory of Weibull. Indeed, if a material is in a perfectly brittle state, then $\chi = 0$. In this case, as follows from (8) the constant C will be equal to the ratio of the limiting stress in uniaxial tension to the limiting stress in pure shear

$$C = \frac{\sigma_t}{\sigma_s} \tag{9}$$

According to Weibull theory, the ratio of the limiting stress in pure bending of a beam of rectangular cross-section to the limiting stress in torsion of a round bar is defined, for the same probability of the specimen fracture by the expression

$$\frac{\sigma_{ben}}{\sigma_{tor}} = \left(\frac{V + 2}{4V + 4}\right)^{1/V} \tag{10}$$

Neglecting the effects of the stress gradient we obtain from (8) and (10)

$$C = \left(\frac{V + 2}{4V + 4}\right)^{1/V} \tag{11}$$

The correlation (11) is confirmed experimentally in [8] and can be used for estimating the constant C with use of the factor V which, being a quite indicative characteristic of the structural non-uniformity of the material, can be readily assessed from the results of quite simple experiments. A special study has shown that neglecting the effects of structural defects can lead to noticeable errors with $V < 10$.

Involving the statistical aspects of strength by introduction of the auxiliary function (7) leads to a good agreement of the analysis with experimental results obtained in tests on various structurally-nonuniform materials (graphite, brittle thermoreactive plastics, grey iron and others) including those at low temperatures [9,10,11].

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