

## SOME EXPERIMENTS RELEVANT TO MINER'S RULE IN FATIGUE

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## INTRODUCTION

Miner rule ( $\sum n/N = 1$ ) is sometimes used for the estimation of fatigue lives of specimens subjected to repeated loads of varying amplitudes. However, there are many evidences which demonstrate that the rule has no general application [1 - 5]. Clearly, the fatigue damage increases with initiation of cracks and their growth, but strengthening of material occurs as a result of the development of dislocation structures or strain-aging. Therefore, the cumulative fatigue damage is not considered as a linear function of stress cycles, as be assumed in the Miner rule. In this paper, some experiments were made to examine these two effects on the values of  $\sum n/N$ . On the basis of the results obtained, data are tabulated, by which whether lives are longer or shorter than those predicted by the Miner rule can be estimated.

## EXPERIMENTAL

Specimens were machined in the form of waisted cylinder from normalized iron bars and then annealed at 923 K in vacuo. Although the stress concentration factor of such specimens was 1.01, they are assumed to be a plain specimen. Tests were made under push-pull load conditions using a Shimadzu Type UF 500 testing machine.

First, the cumulative damage was examined by two-level fatigue tests: That is, various cycle ratios ( $n_1/N_1$ ) at the first stress level were applied, and then the tests were continued to failure at the second stress level. ( $n_1$  and  $N_1$  are the number of cycles applied and the fatigue life at the first stress level, respectively.)

Next, tests with electropolishing and annealing were carried out. These were as follows: After cycling at the first stress level, the specimens were electropolished or annealed and then further cycled at the second stress level as above.

All these tests will be called L-H, H-Polish-L, L-Ann-H tests etc., respectively, where H and L mean the high stress level 169 N/mm<sup>2</sup> (the life =  $7.0 \times 10^4$  cycles) and the low stress level 127 N/mm<sup>2</sup> (the life =  $2.2 \times 10^6$  cycles); Ann and Polish denote annealing done at 973 K in vacuo and electropolishing performed to such an extent that microcracks were polished away, respectively.

Observations were also made for surface- and internal structures in specimens fatigued up to various stages at both the low and the high stress levels. The aim in these observations is to study the effects of the surface damage such as microcracks and of strengthening due to stress-

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induced dislocation substructures.

#### DAMAGE TESTS

*L-H Tests:* The specimens were first cycled at the low stress level for 5, 10, 20, 40 and 80% of the expected life ( $2.2 \times 10^6$  cycles) respectively, and then cycled to failure at the high stress level. The results are shown by the marks  $\bigcirc$  in Figure 1, where the damage ratio ( $1 - n_2/N_2$ ) is plotted against the first cycle ratio ( $n_1/N_1$ ). ( $n_2$  and  $N_2$  are the number of cycles applied to failure at the second stress level and the life at this level, respectively.) The Miner relation is expressed by a straight line of unit slope through the origin. The other marks  $\bullet$  and  $\ominus$  are the experimental plots from the tests with electropolishing (L-Polish-H tests) and the tests with annealing (L-Ann-H tests), respectively. Every plotted point is the average from three results.

As seen in this figure, the values of  $\Sigma n/N$  are larger than those predicted by the Miner rule when  $n_1/N_1 > 10\%$ , although  $\Sigma n/N$  is nearly equal unity when  $n_1/N_1 = 5\%$ . Also, the values of  $\Sigma n/N$  in L-H tests are somewhat smaller than those in L-Polish-H tests, but considerably larger than those in L-Ann-H tests.

*H-L Tests:* Figure 2 shows the results of the tests starting with the high stress amplitude, in which the same marks as shown in Figure 1 are used. In these tests, the values of  $\Sigma n/N$  are very large when  $n_1/N_1 < 10\%$ , but become suddenly smaller than unity when  $n_1/N_1 > 20\%$ . Such a behaviour of  $\Sigma n/N$  in H-L tests is in marked contrast with that in L-H tests.

Effects of electropolishing and annealing on  $\Sigma n/N$  in H-L tests are in the same direction as those in L-H tests, respectively. That is, the former enlarges the cumulative cycle ratios ( $\Sigma n/N$ ) and the latter reduces them.

#### SURFACE- AND INTERNAL STRUCTURES, AND DISCUSSION

To understand the results obtained in the above sections, surface- and internal structures were examined for specimens cycled up to various stages at both the low and the high stress levels.

The surface observations revealed that fine cracks were not developed even at 80% the life at the low stress level, but were initiated as early as 15% the life at the high stress level. Some of the results are given in Figures 3 and 4. In these figures, whether or not cracks has been formed in slip bands was checked by electropolishing the surface layer by few microns.

Figures 5 and 6 show typical dislocation structures observed in fatigued specimens, together with those in specimens annealed after stress cycling. It was supposed that the development of dislocation structures depended on the stress amplitude more than on the number of stress cycles. Most of the stress-induced dislocations seemed to be annealed out during the annealing, but twist boundaries still remained.

In the L-H tests (Figure 1), it was found that electropolishing had not a great effect on  $\Sigma n/N$ . This suggests that the surface layer was not so much damaged. In fact, no crack was observed when cycled up to 80% the life at the low stress level. On the other hand, annealing led  $\Sigma n/N$  to

considerably small values. This suggests that strengthening effect, which came from the development of dislocation structures or strain-ageing, was diminished to an appreciable extent during the annealing. However, this effect seemed not to be fully faded away by annealing at this temperature, because the values of  $\Sigma n/N$  were still larger than unity even in the tests with annealing (L-Ann-H tests). This is consistent with the observation which showed that dislocation structures such as twist boundaries still remained after annealing (Figures 5 and 6). (Compressive residual stress may have the same effect, but in the present test it can be discriminated from the strengthening effect mentioned here.)

In the H-L tests (Figure 2), (1)  $\Sigma n/N > 1$  when  $n_1/N_1 > 15\%$ , and (2)  $\Sigma n/N < 1$  when  $n_1/N_1 > 20\%$ . The former result seems to be due to the fact that no crack initiated when  $n_1/N_1 < 15\%$  and the material would be much more strengthened than when cycled at the low stress level. On the other hand, the cracks appearing in cycling beyond 20% the life are considered to be responsible for the latter result. Therefore, it is supposed that, once a crack initiates in plain specimens, the damage due to the crack becomes so dominant that the strengthening effect is masked [6].

If the Miner rule is obeyed in two-level tests, the damage or strengthening occurring in a specimen cycled up to  $A_1$  at the first stress level must be equal to that in the specimen which would be cycled up to  $A_2$  at the second stress level. (For  $A_1$  and  $A_2$ , refer to Figure 7). Therefore, on the basis of the above discussion, data are tabulated, by which whether the fatigue life is shorter or longer than that predicted by the Miner Rule can be estimated (Table 1).

This table, however, will not be applicable to sharply notched specimens, because the stress concentration factor of a crack initiated at the root of the notch is relatively low, so that the damage due to the crack is not so serious as in the case of plain specimens [6].

Nonferrous metals such as Cu show somewhat different manner in  $\Sigma n/N$  from that mentioned above [7]. In these metals, their yield stresses are relatively low, and fine cracks are easy to be initiated in cycling at a low stress level. Also, their work-hardening rate are fairly high. Such metallurgical features may explain the behaviour of  $\Sigma n/N$  in these metals.

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Table 1 Estimation of  $\Sigma n/N$

A <sub>1</sub>	A <sub>2</sub>	$\Sigma n/N$
Crack	No Crack	$\Sigma n/N < 1$
No Crack	Crack	$\Sigma n/N > 1$
Crack	Crack	$\Sigma n/N \begin{cases} \geq 1 & \text{if Crack } A_1 \geq \text{Crack } A_2, \\ < 1 & \text{respectively.} \end{cases}$
No Crack	No Crack	$\Sigma n/N > 1$ in H-L Tests* $\Sigma n/N \neq 1$ in L-H Tests**

\* Strengthening when cycled initially to A<sub>1</sub> at high stress level will be more pronounced than when cycled to A<sub>2</sub> at low stress level.

\*\* Strengthening when cycled initially to A<sub>1</sub> at low stress level will not be so pronounced as when cycled to A<sub>2</sub> at high stress level. However, it will be elevated after small numbers of cycles at the subsequent high stress level.

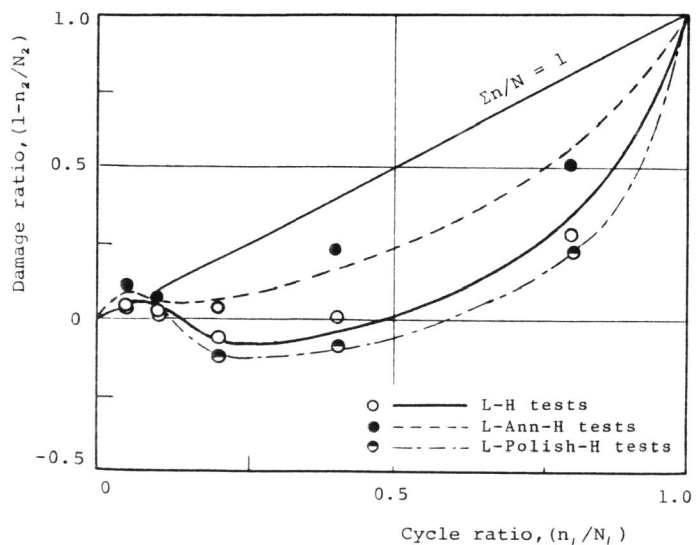


Figure 1 Damage Curves in Tests Starting with Low Stress Level

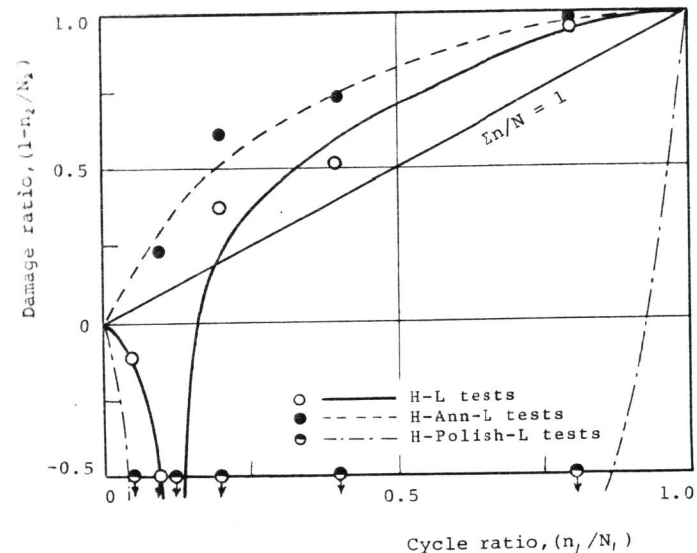


Figure 2 Damage Curves in Tests Starting with High Stress Level

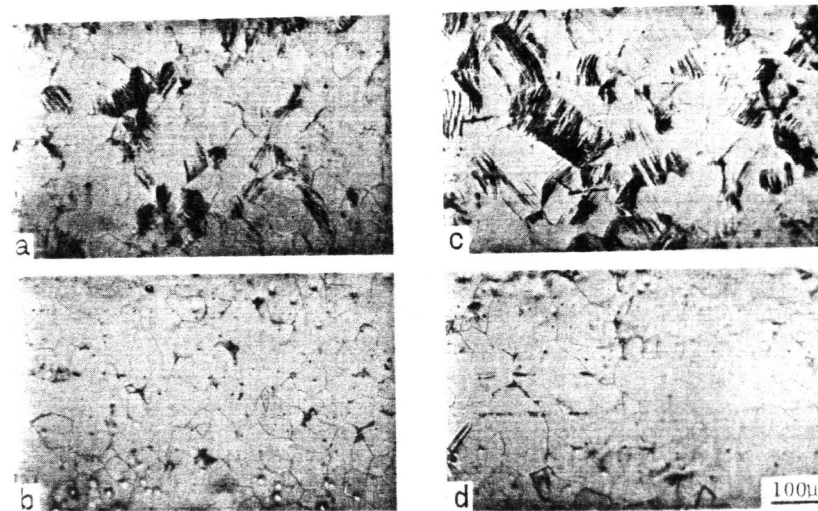


Figure 3 Surface Aspects: (a) and (c), After 40% and 80% the Life at the Low Stress Level, respectively; (b) and (d), After Electropolishing the Same Area as (a) and (c), respectively

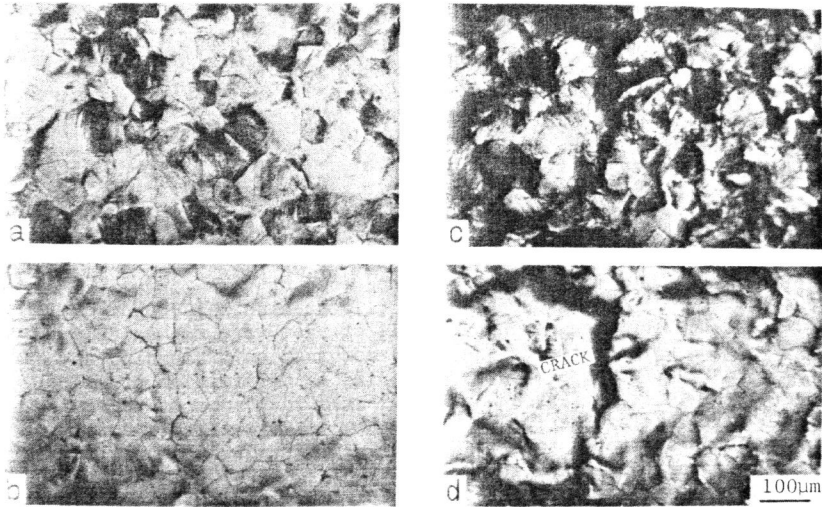


Figure 4 Surface Aspects: (a) and (c), After 10% and 20% the Life at the High Stress Level, respectively; (b) and (d) After Electropolishing the Same Area as (a) and (c), respectively

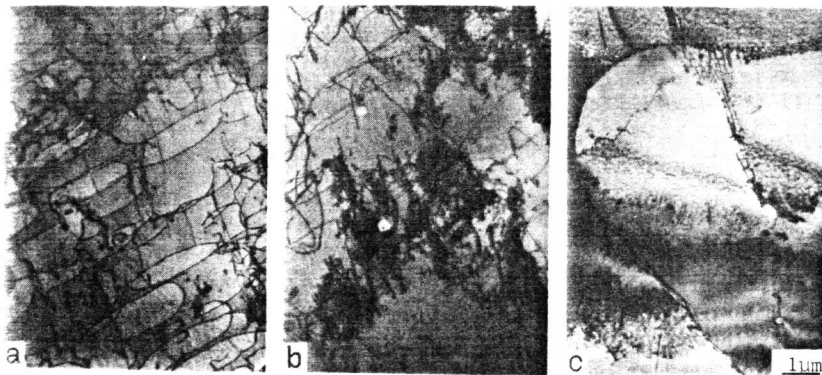


Figure 5 Dislocation Structures: (a) and (b), After Cycling 40% and 80% the Life at the Low Stress Level, respectively; (c), Annealed After Cycling 80% the Life

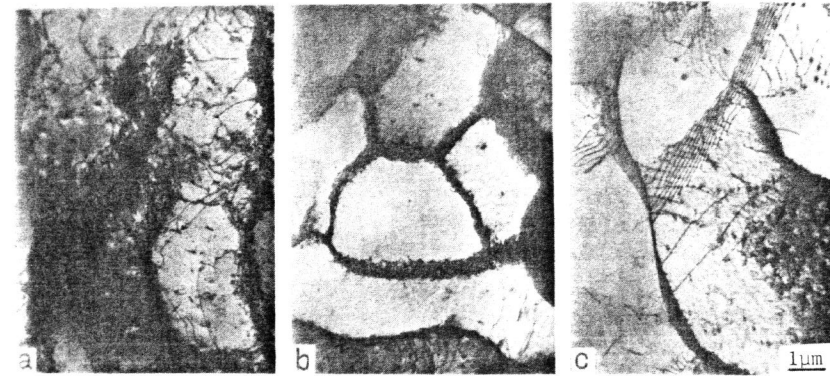


Figure 6 Dislocation Structures: (a) and (b), After Cycling 5% and 40% the Life at the High Stress Level, respectively; (c), Annealed After Cycling 40% the Life

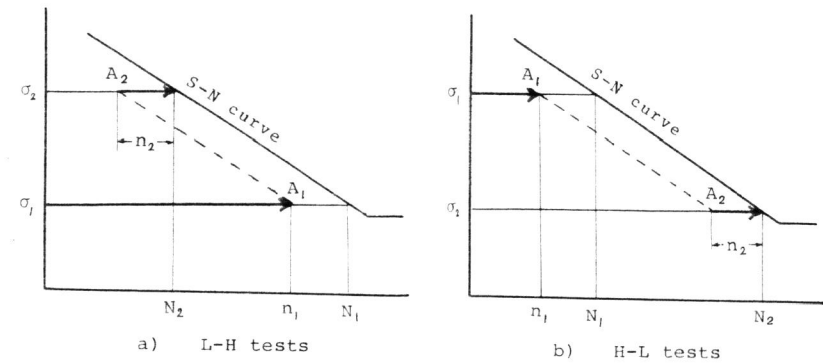


Figure 7 Miner Rule in Two-Level Tests. [ $A_1$  is a Point ( $n_1, \sigma_1$ ) and  $A_2$ , the Intersection of a Line ( $\sigma = \sigma_2$ ) and a Line Parallel to the S-N Curve Through the Point  $A_1$ . The Abscissa  $N$  is Plotted to a Log Scale.]