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PROBABILITY OF FATIGUE FAILURE BASED ON RESIDUAL STRENGTH

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INTRODUCTION

The residual strength of a structure, given by the maximum load it can support before failure, has served as a useful parameter in many fatigue reliability studies. It is usually assumed that the initial static strength of a structure reduces with the number of cycles applied and failure occurs when it equals the maximum stress in the fatigue loading. It should, however, be recognized that, although the residual strength changes continuously with the number of cycles applied, it represents weakening of the material due to microscopic cracks in the crack initiation stage, while it stands for the stress at which the growth of a predominant crack becomes unstable in the crack propagation stage. It can therefore be expected that, in these two stages of the fatigue process, the probability distribution of the residual strength will have different properties.

In the following, it will be shown that, if the initial strength of a material follows the Weibull probability distribution, then the residual strength, after a given number of cycles, also follows this distribution. Furthermore, in each stage of the fatigue process, the shape parameter of the distribution remains unchanged, while the location and scale parameters decrease with the number of cycles applied. Test results for specimens of a steel alloy clearly indicate that the one-component distribution of the initial strength develops into a two-component distribution of the residual strength after fatigue cycling. The component with high residual strength corresponds to the crack initiation stage and that with low residual strength to the crack propagation stage.

RESIDUAL STRENGTH

Crack Initiation Stage

In the crack initiation stage, the weakening of strength occurs due to microcracks generated at strain incompatibility centres spread throughout the cross-section of a specimen. This weakening can be characterized by a damage parameter D with its value D=0 in the virgin state and D=1 at failure. Expressing D in terms of the residual strength R, we have

$$D = (R_o - R)/(R_o - S) \qquad (1)$$

where R_o is the initial strength and S is the maximum stress applied.

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Assuming further that the damage per cycle is stress dependent and is given by

$$dD/dN = n \{S/(1-D)\}^m \tag{2}$$

we have, from equations (1) and (2)

$$dR/dN = -n\ S^m\ (R_o-S)^{1+m}/(R-S)^m \tag{3}$$

Integrating equation (3) we get a relation between the initial and the instantaneous residual strength:

$$R_o = S + (R-S)/\{1-n(1+m)S^mN\}^{1/(1+m)} \tag{4}$$

If the probability distribution of the initial strenth is given by

$$P(R_o \leqq r) = F(r) \tag{5}$$

then from equations (4) and (5) we observe that, for a given N, the probability of failure P_f will be given by

$$P_f = P(R \leqq S) = P\left[R_o \leqq S + (R-S)/\{1-n(1+m)S^mN\}^{1/(1+m)}\right]$$

$$= F\left[S + (R-S)/\{1-n(1+m)S^mN\}^{1/(1+m)}\right] \tag{6}$$

Assuming the Weibull distribution for the initial strength, we have

$$F(r) = 1 - \exp\{-(\frac{r-a}{b})^c\} \tag{7}$$

where a, b and c are the location, scale and shape parameters, respectively.

From equation (6) we have

$$P(R \leqq S) = 1 - \exp\{-(\frac{R-a'}{b'})^c\} \tag{8}$$

where $$a' = S + (a-S)\{1-n(1+m)S^mN\}^{1/(1+m)} \tag{9}$$

and $$b' = b\{1-n(1+m)S^mN\}^{1/(1+m)} \tag{10}$$

From equation (8) we see that the shape parameter remains unchanged, while the location and scale parameters decrease with the number of cycles applied as shown by equations (9) and (10).

Crack Propagation Stage

In the crack propagation stage, the residual strength is given by

$$R(C)^{1/2} = K_c\ \alpha \tag{11}$$

where C is the crack length and α is a constant.

For plain strain crack growth, the crack growth rate has been found to be proportional to the crack length for most of the crack propagation stage [1]. We therefore have

$$dC/dN = \beta C \tag{12}$$

where β is a constant given by

$$\beta = \gamma S^p \tag{13}$$

Following the procedure outlined for the crack initiation stage, we get

$$P(R \leqq S) = 1 - \exp\{-(\frac{R-a''}{b''})^c\} \tag{14}$$

where $$a'' = a\ \exp(-\frac{1}{2}\ \gamma S^pN) \tag{15}$$

and $$b'' = b\ \exp(-\frac{1}{2}\ \gamma S^pN) \tag{16}$$

TEST PROGRAM

Specimens were prepared from 22mm dia. bars of a Cr-Mo-V steel, which were machined to give a smooth reduction to 10mm x 15mm cross-section. On one 15mm side a small surface flaw was introduced by electro-discharge machining. The flaw was 0.15mm on the surface and 0.05mm in depth. The processes of machining and introducing flaws were carried out in randomized order.

The test program consisted of 6 test series. In each series, a preselected number of specimens were drawn randomly from the lot containing all specimens. The specimens in Series No. 1 were pulled in tension to estimate the initial strength. The specimens in Series Nos. 2, 3, 4 and 5 were subjected to pre-specified number of cycles at the maximum stress of 380 MPa and the minimum stress of -235 MPa. The unfailed specimens were then pulled in tension to estimate the residual strength. The specimens in Series No. 6 were subjected to fatigue cycling until failure.

Table 1 shows the residual strength data for the unfailed specimens and the fatigue life in cycles for the failed specimens.

STATISTICAL ANALYSIS OF TEST DATA

Methods of Analysis

The test data is analyzed by means of two plots: the (x_i, Ez_i) plot and the $(S_k, k^{-1/c})$ plot.

The (x_i, Ez_i) Plot

The standardized variable z is defined by

$$z = (x-a)/b \tag{17}$$

where x is the random variable in the Weibull distribution.

The expected value of the order statistic z_i, denoted by Ez_i, is given by (2)

$$Ez_i = i\, C_i^M\, (1/c)! \sum_{r=0}^{i-1} (-1)^{i-1-r}\, C_r^{i-1}\, (M-r)^{-(1+1/c)} \tag{18}$$

As can be seen in this equation, Ez_i is, for a given size of the sample M, a function of the shape parameter c only.

If now the observations are denoted by x_i, the parameters a and b can be estimated by use of the relation

$$x_i = a + b \,.\, Ez_i \tag{19}$$

Thus, if x_i is plotted against Ez_i, the data points (x_i, Ez_i) will, with due regard to "the sampling scatter", fall on a straight line. The parameter c is estimated by fitting a regression line to the data points by means of the least-square principle for different values of c and accepting that value of c which provides the least sum of the squared deviations of the fitted line.

The $(S_k, k^{-1/c})$ Plot

By grouping independent observations randomly into a number of subsamples, a new statistic, denoted by S_k is obtained. The relation of this statistic to the order statistic x_i, derived in a work to be published, is given by

$$S_k = \sum_{i=1}^{M+1-k} r_{ki} \,.\, x_i \tag{20}$$

where

$$r_{ki} = C_{k-1}^{M-i} \,/\, C_k^M \tag{21}$$

If the observed values of S_k, i.e. the linear function of the observations x_i given by equation (20), is equated to its expected value, we arrive at the relation

$$S_k = a + b(1/c)!\,k^{-1/c} \tag{22}$$

which provides another graphical method for estimating the distribution parameters.

Results of Analysis

Fatigue Life

The (x_i, Ez_i) plot of the fatigue life data of 9 specimens in Series No. 6 showed that this sample was homogenous. The estimated values of the distribution parameters are: a = 51146 cycles, b = 69162 cycles and c = 1.5.

By pooling fatigue life data of Series Nos. 5 and 6, a truncated sample of size M=31 is obtained. Only the 15 smallest order statistics of this sample are known. By use of equation (20), they were transformed into 15 S_k values. The $(S_k, k^{-1/c})$ plot of this data gave the estimated parameter values as: a = 48937 cycles, b = 56863 cycles and c = 2.3 which are close to the estimated parameter values listed above.

Residual Strength

The residual strength data of Series Nos. 1 and 2 showed no significant difference and were therefore pooled together to obtain a sample of size M=18. The (x_i, Ez_i) plot of Series Nos. 1+2, 3, 4 and 5 is shown in Figure 1 for c=10. At approximately this value of c the regression fit gave the minimum sum of the squared deviations. As seen in the figure, the initial strength data appears as a homogenous sample of a single component. At N=65620 cycles, two components appear in the distribution, the lower one corresponding to the cases where failure occurred from a fatigue crack grown from the machined-in flaw. At higher number of cycles, the residual strength has a truncated distribution with an additional component, shown by a dotted line in Figure 1. This component shows an abrupt decrease in the residual strength and corresponds to the later stage of crack propagation. The slope and intercept of the (x_i, Ez_i) plot show a decreasing tendency with increasing number of cycles for each component, supporting the predicted behaviour. The estimated parameter values from the (x_i, Ez_i) plot agree well with those obtained by using the S_k statistic.

REFERENCES

1. WAREING, J., Met. Trans. A, 6A, 1975, 1367.
2. LIEBLEIN, J., Ann. Math. Statist., 26, 1955, 330.

Table 1   Test Data. Residual Strength x_i in MPa and Fatigue Life N_{fi} in number of cycles

Series	1	2	3	4		5		6
N	0	32810	65620	91550		115300		
i	x_i	x_i	x_i	x_i	N_{fi}	x_i	N_{fi}	N_{fi}
1	1827.3	1824.0	1556.0	-	75900	-	60230	65620
2	1833.8	1843.7	1667.1	-	82820	-	68680	76430
3	1882.9	1896.0	1726.0	751.8		-	69750	85220
4	1909.0	1909.0	1794.6	840.1		-	80650	91550
5	1915.6	1912.3	1811.0	1225.8		-	84290	115300
6	1922.1	1915.6	1863.3	1542.9		-	87840	123400
7	1931.9	1918.8	1866.5	1588.7		-	89950	128700
8	1951.5	1928.6	1869.8	1608.3		-	95680	137800
9	1954.8	1961.3	1896.0	1778.3		-	110500	198400
10			1896.0	1811.0		-	112630	
11			1902.5	1837.1		601.5		
12			1905.8	1850.2		647.2		
13			1909.0	1873.1		679.9		
14			1915.6	1886.1		791.1		
15			1928.6	1892.7		1591.9		
16			1931.9	1896.0		1712.9		
17			1931.9	1896.0		1794.6		
18			1935.2	1931.9		1814.2		
19						1817.5		
20						1889.4		
21						1905.8		
22						1922.1		

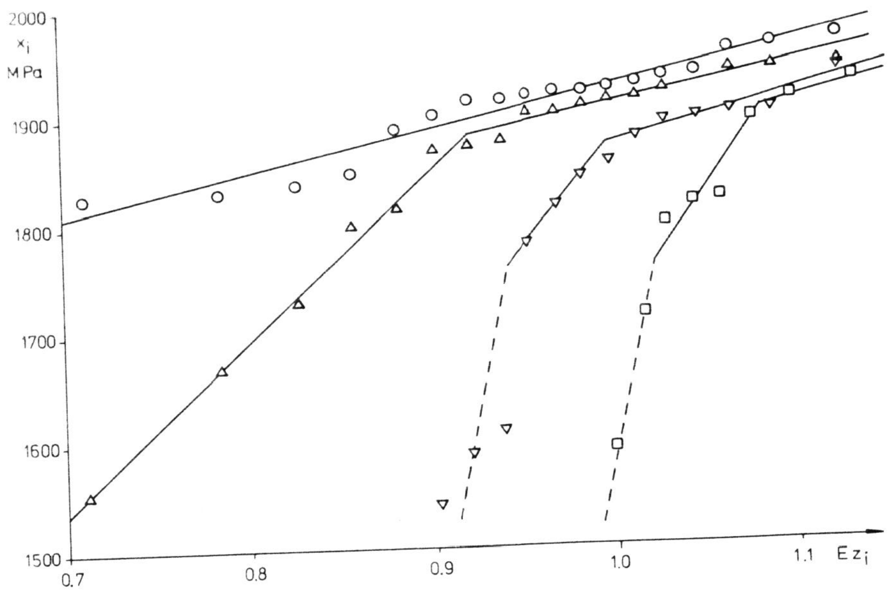

Figure 1   The (x_i, Ez_i) Plot. Circles: Series Nos. 1+2, triangles: Series No. 3, inverted triangles: Series No. 4, squares: Series No. 5.