Fracture 1977, Volume 2, ICF4, Waterloo, Canada, June 19 - 24, 1977

ON THE THRESHOLD FOR FATIGUE CRACK GROWTH

A. J. McEvily and J. Groeger*

INTRODUCTION

In the assessment of the significance of flaws in a cyclically loaded structure a number of material parameters are of interest. If, for example, the rate of fatigue crack growth can be written as

$$\frac{\Delta a}{\Delta N} = \frac{A_{(e)}}{\sigma_y E}\left(\Delta K - \Delta K_{Th_{(R,e)}}\right)^2 \left(1 + \frac{\Delta K}{K_{Ic} - K_{max}}\right) \tag{1}$$

where a is the crack length
N is the number of cycles
$A_{(e)}$ is a material constant, a function of the environment, e.
σ_y is the yield strength
E is Young's modulus
ΔK is the stress intensity factor range
$\Delta K_{Th(R,e)}$ is the stress intensity range at the threshold level, a function of R and environment, where $R = \sigma_{min}/\sigma_{max}$
K_{Ic} is the plane strain fracture toughness
K_{max} is the maximum stress intensity in the loading cycle and can be expressed as $K_{max} = \frac{\Delta K}{1-R}$

then there are five material constants involved, namely $A_{(e)}$, σ_y, E, $\Delta K_{Th(R,e)}$ and K_{Ic}. In recent years the magnitude and nature of one of these constants, $\Delta K_{Th(R,e)}$, has been a topic of increasing interest. Indeed the question has been raised as to whether or not $\Delta K_{Th(R,e)}$ is evidence [2] suggests that it is.

In considering the factors responsible for a threshold the residual compressive stress of the crack tip appears to be of importance. Elber's concept of crack closure [3,4] has also been given attention by some investigators. In terms of crack closure the simplest explanation of the existence of a threshold would be if ΔK_{eff} equalled to $\Delta K - \Delta K_{cl}$, where ΔK_{eff} denotes the effective stress intensity range, and ΔK_{cl} indicates the stress intensity range over which the crack is closed. In this case ΔK_{Th} would correspond to ΔK_{cl}. However, Schmidt and Paris [5] as well as Kikukawa [6] et al., have determined experimentally that ΔK_{Th} is larger than ΔK_{cl}. Nevertheless ΔK_{cl} figures importantly in Schmidt and Paris' treatment of the dependence of ΔK_{Th} on R. In view of the attention given to the role of crack closure on ΔK_{Th} it seems worthwhile to summarize a few of the pertinent experimental findings.

---

*Metallurgy Department, University of Connecticut, Storrs, Connecticut 06268, U.S.A.

Elber's concept of crack closure is that residual tensile strains normal to the crack surface lead to contact between the two surfaces of a crack before the minimum load in a fatigue cycle is reached. This contact is macroscopic in nature and there is ample experimental evidence as indicated by strain gauge readings to show that in the plane stress region at the specimen surface such closure does take place. What is less clear is whether or not such closure occurs in the plane strain region along the crack front at mid-thickness of the specimen. On this point, Lindley and Richards [7] have shown by sectioning that closure contact is made primarily in the plane stress region along the surface of a plate specimen where thinning and extension is more pronounced than in the interior. A similar observation has been made in the case of PMMA [8]. Closure has also been studied by ultrasonic surface waves, [1,9,10] with a starting semi-elliptical surface flaw of 6 mm in length and 1.5 mm in depth. It was observed that as the crack deepened, the closure level was reduced, which might be taken to indicate that this trend was due to a greater proportion of plane strain conditions along the crack front with increase in crack depth. In any event it is of interest that the closure range in air is of the order of only 10% or less of the total range ($\Delta K = 6$ MPa$\sqrt{m}$) in the case of Ti-6Al-4V, for example. With respect to the influence of the environment on the closure level, it was found [10] that the closure level is higher in dry nitrogen than in air of relative humidity 15-80%. It is also of interest that fatigue striations were not observed on the fracture surfaces of specimens tested in dry nitrogen (one wonders if the increase in closure level at the surface interferes with the process of striation formation).

Closure has also been studied by the potential drop technique. Shih and Wei [11] found that closure in through-cracked plate specimens (5 mm thick) was not observed in Ti-6Al-4V for R values greater than 0.3. Their results indicate that any closure near the threshold level would be minimal. Irving et al., [12] found no crack closure in air, but did observe closure in vacuum (10^{-3} Pa). However, Bachman and Munz [13] found that closure level was the same in vacuum (5×10^{-4} Pa) as in air in testing 12 mm thick CT specimens and that the level of K_{cl} was independent of K_{max}. Evidently there are some conflicting conclusions with respect to crack closure which are in need of clarification.

In the present investigation we assume that crack propagation is controlled by plane strain conditions along the crack front. It is also assumed that crack closure in the plane strain region is either non-existent or negligible. Further, equation (1) was derived on the assumption that the role of crack opening displacement. With respect to the threshold level the further assumption is made that the threshold level corresponds to a constant crack opening displacement range which is independent of mean stress, that is

$$K^2_{max} - K^2_{min} = K^2_{Th_{(o)}} \tag{2}$$

where $\Delta K_{Th_{(o)}}$ is the threshold stress intensity range corresponding to R equal to zero loading. Then, since

$$K_{max} = \frac{\Delta K}{1-R} \tag{3a}$$

and

$$K_{min} = \left(\frac{\Delta K}{1-R}\right) R \tag{3b}$$

it follows that

$$\Delta K_{Th_{(R)}} = \sqrt{\frac{1-R}{1+R}}\, \Delta K_{Th_{(o)}} \tag{4}$$

MATERIALS, SPECIMENS AND TESTS

To check on the validity of equation (4), a series of tests were carried out in air (50% relative humidity) and in vacuum (10^{-5} Pa) in order to determine the threshold level. The specimens were of the CT type (ASTM specification E-399) and were machined from a 94 mm thick Ti-6Al-4V pancake forging which had been β processes at 1310°K, water quenched and then heat treated at 980°K for two hours. The specimens were 6.35 mm thick, with an H dimension of 57 mm and a W dimension of 50 mm. To facilitate observation of the crack tip which was viewed with a 10X telescope, the specimen surface was polished through 1μ diamond paste. The specimens were tested in displacement control to obtain a decrease in K with increase in crack length. The cyclic frequency was 30 Hz and the initial crack growth rate was 2.5 nm/cycle and cracks extended of the order of 25 mm before the arrest condition was reached. It was considered that the arrest condition had been reached when no detectable crack growth was observable over at least 2×10^6 cycles.

RESULTS AND DISCUSSION

The results of the threshold determinations as a function of mean stress shown in Figure 1. A pronounced effect of environment is noted as well as a dependency on mean stress level. A comparison of the predictions of equation (4) with experiment indicates that reasonable agreement is obtained. However, it should be clear from the introduction that further work is needed to substantiate the assumptions made in deriving equation (4).

For R = 0.35, Irving and Beevers [14] obtained for Ti-6Al-4V a threshold of 8 MPa$\sqrt{m}$ in vacuum (1 mPa) as compared to 12 MPa$\sqrt{m}$ in the present case. Their threshold in air was less than 3 MPa·m$^{1/2}$ in contrast to the 4 MPa·m$^{1/2}$ observed in the present study. The reason for the difference in results is now known but may reflect a difference in microstructure as a result of different processing procedures.

Thus far we have considered the threshold for crack growth in terms of fracture mechanics considerations related to macro-cracks. There is also a threshold for propagation of micro-cracks, and this level has been considered to correspond to the endurance limit of steel, for example. An important question in dealing with very small flaws is whether or not the fracture mechanics approach is applicable. In a recent study by Kitagawa and Takahashi [2] for flaw sizes less than 0.5 mm in a particular steel the fracture mechanics approach was not valid and the threshold was determined by the stress level rather than the stress intensity level. This trend is schematically indicated in Figure 2. This type of behavior can be analyzed as follows. We assume that the condition for propagation is given by the following equation:

$$K_N\, \sigma_{net} = \sigma_{end} \tag{5}$$

where σ_{net} is the net section stress
σ_{end} is the endurance limit, a function of R
K_N is the Neuber stress concentration faction given as

$$K_N = 1 + \frac{K_{T-1}}{1 + \sqrt{\rho'/\rho}} \qquad (6)$$

where K_T is the theoretical stress concentration factor based on the net section
ρ' is a material constant
ρ is the tip radius

The stress intensity factor K is related to the stress concentration factor K_T by

$$K = \lim_{\rho \to o} K_T \, \sigma_{net} \sqrt{\frac{\pi\rho}{4}} \qquad (7)$$

If we substitute K_N for K_T and consider that tip radius, ρ, of a crack approaches ρ_e in the limit rather than zero, we obtain

$$K = K_N \, \sigma_{net} \sqrt{\frac{\pi\rho_e}{4}} \qquad (8)$$

If σ_{end} is substituted for $K_N \, \sigma_{net}$ (9)

Then since K is of the form $B\sigma\sqrt{\pi a}$

$$\sigma_{Th} = \frac{\sigma_{end}}{B} \sqrt{\frac{\rho_e}{4a}} \qquad (10)$$

This equation indicates that for values of $B\sqrt{4a}$ equal to or less than ρ_e that σ_{Th} would be given by the condition that $\sigma_{Th}\sqrt{a}$ be equal to a constant, namely

$$\frac{\sigma_{end}}{2B} \sqrt{\rho_e} \, ,$$

in which case the slope of a log σ_{Th} - log a plot would have a slope of minus two as indicated in Figure 2. In this approach the key parameter is the constant ρ_e. The physical meaning of this parameter in terms of closure, residual stress fields, etc., is a subject for further consideration.

CONCLUSIONS

In this study we have been concerned with factors influencing the threshold level for fatigue crack propagation. We have found that

A. The dependency of the threshold level on mean stress can be rationalized in terms of COD concepts.
B. The transition from stress control to stress intensity control at the threshold level can be related to the effective tip radius of a fatigue crack, ρ_e.

ACKNOWLEDGEMENT

The authors are pleased to acknowledge the support of the U.S. Air Force Office of Scientific Research under Grant No. 74-2703A. The capable assistance of Mr. Richard Shover in carrying out the test program is also gratefully appreciated.

REFERENCES

1. MARCUS, H. L., MORRIS, W. L., BUCK, O., in Prospects of Fracture Mechanics, (ed.) Sih, Van Elst, and Broek, Noordhoff, Leyden, 1974, 179.
2. KITAGAWA, H. and TAKAHASHI, S., to be presented at the Second Int. Conf. on Mechanical Behavior of Materials, Boston, 1976.
3. ELBER, W., Eng. Fracture Mechanics, 2, 1970, 37.
4. ELBER, W., Damage Tolerance in Aircraft Structures, ASTM STP 486, 1971, 230.
5. SCHMIDT, R. A. and PARIS, P. C., Progress in Flaw Growth and Fracture Toughness Testing, ASTM STP 536, 1973, 79.
6. KIKUKAWA, M., JONO, M. and TANAKA, K., to be presented at the Second Int. Conf. on Mechanical Behavior of Materials, Boston, 1976.
7. LINDLEY, T. C. and RICHARDS, C. E., CERL Report, 1973.
8. PITONIAK, F. J., GRANDT, A. F., MONTULLI, L. T. and PACKMAN, P. F., Eng. Fracture Mechanics, 6, 1974, 663.
9. HO, C. L., BUCK, O. and MARCUS, H. L., Progress in Flaw Growth and Fracture Toughness Testing, ASTM STP 536, 1973, 5.
10. BUCK, O., FRANDSEN, J. D. and MARCUS, H. L., Eng. Fracture Mechanics, 7, 1975, 167.
11. SHIH, T. T. and WEI, R. P., Eng. Fracture Mechanics, 6, 1974, 19.
12. IRVING, P. E., ROBINSON, J. L. and BEEVERS, C. J., Eng. Fracture Mechanics, 7, 1975, 619.
13. BACHMAN, V. and MUNZ, D., Int. J. of Fracture, 11, 1975, 713.
14. IRVING, P. E. and BEEVERS, C. J., Research Report, Univ. of Birmingham, Dept. of Physical Metallurgy and Science of Materials, 1973.

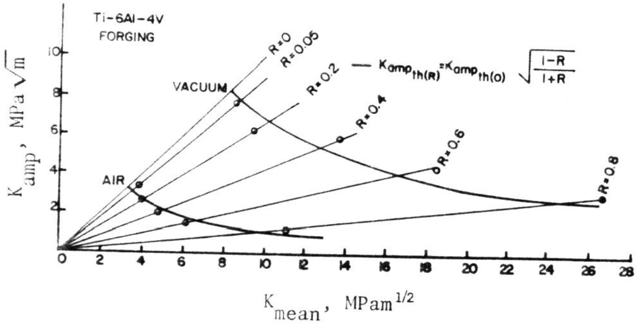

Figure 1 The dependency of K_{amp} at threshold on K_{mean} for Ti-6Al-4V alloy in air (50% relative humidity) and in vacuum (10^{-5} Pa). Predicted trends are also indicated.

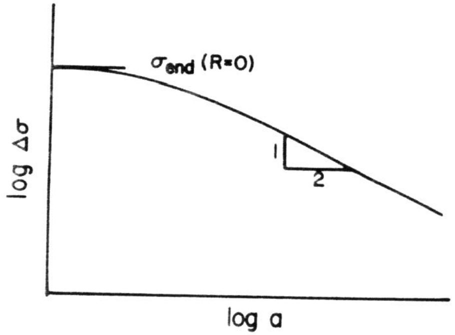

Figure 2 The dependency of the threshold stress range for crack propagation on flaw size for R=0 test conditions.