

NOTES ON THE REPEATED SHEAR STRESS INDUCED  
PERMANENT STRAIN OF STEELS

T. Endo\*

## INTRODUCTION

It has been known since the early work of Bairstow that the unidirectional plastic strain is induced by repeated stress which is accompanied with mean stress. The phenomenon is known as fatigue deformation or as cyclic-stress induced creep. It was investigated by Bairstow [1], Kawamoto [2], Coffin [3], Taira [4], [5], [6]. Koterazawa [7], Shiratori [8], Udoguchi [9], Yamanouchi [10], Wood [11] and others. Benham [12] and Benham and Ford found that the cyclic creep curves are similar in shape to those observed for ordinary creep. The actual effects on the deformation are well illustrated by the work of Feltner and Sinclair [13], who subjected fcc metals copper and aluminum and hcp metal cadmium to repetitions of tensile stress at both 25°C and -196°C. Sinclair and Morrow [14] studied the related problem, the relaxation of mean stress in SAE4340 steel subjected to cyclic strain. Feltner [15] studied the dislocation arrangements in aluminum by the action of repeated tensile stress by using transmission electron microscopy. Gain investigated the effect of temperature on cyclic dependent deformation.

Although the above mentioned investigations have demonstrated the cyclic stress induced creep phenomena, the effects of mean stress and stress amplitude were not generally emphasized except in the work investigated by the present author [16]. This paper is to follow the previous ICF3 report where the angular plastic deformation of a tubular specimen of mild steel was measured under alternating torsion with static torsion. The analysis of the results on mild steel showed the existence of a differential equation of a master curve which expressed the relationship between the number of cycles and the cyclic stress induced permanent (monotonic) deformation under a constant stress amplitude. The master curve was similar to the logarithmic creep law under a static load.

The author intends to clarify the following in this investigation.

- a) some basic phenomenological relationship of fatigue deformation of steels under a simple shear stress system. Note that the stress system used here is so specific that the normal tensile stress is zero on the plane of maximum static and alternating shear stress.
- b) several fundamental materials which may be useful for understanding the mechanism of fatigue deformation.

The cyclic stress induced creep is frequently observed in the helical and leaf springs. On the other hand the research is also expected to give a fundamental idea for predicting fatigue life and fatigue deformation under complex loading. It is because the plastic deformation under service load is accompanied by a complex stress which is decomposed by a rain flow counting Algorithm into sets of repeated stresses and mean stresses [17].

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\*The Kyushu Institute of Technology, Tobata, Kitakyushu, Japan.

In this paper the following are reported:

- a) a phenomenological investigation of fatigue deformation of 0.15% and 0.46% carbon steels and a discussion of the above.
- b) optical microscopic observations of the fatigue deformation of a 0.15% carbon steel.
- c) the cyclic stress induced creep curve for a mild steel when the mean stress was switched to a different level.

#### SPECIMENS AND TESTING MACHINE

For this experiment a mild steel (0.15%) and a semi-hard steel (0.46%C) were used, which are approximately equivalent to SAE1015 and SAE1045. The mechanical properties of the mild steel reported in reference [17] are listed in the footnote. The semi-hard steel was annealed at 850°C, 30 min., and then machined to make the fatigue specimen. Tensile strength, upper yield point, elongation and contraction of area of semi-hard steel obtained by using solid specimens are  $\sigma_B = 553$  MPa,  $\sigma_S = 306$  MPa, 28%, 52% respectively. The chemical composition of the semi-hard steel was 0.46%C, 0.27%Si, 0.74%Mn, 0.15%P, 0.20%S, 0.02%Cu, 0.01%Ni, 0.10%Cr. The torsional yield point was 199 MPa.

Fatigue specimens were annealed in vacuum at 600°C, 30 min. prior to the static or fatigue torsion tests. The shear stress of tubular specimen is expressed by the average over the section.

The testing machine is a constant moment type i.e. the mean moment and repeated moment are automatically kept constant irrespectively of the fatigue deformation of specimen. Overloading at the start of the test was carefully avoided. Mean stress had been applied before alternating stress was applied. The shear stress was determined by the relative angle of twist between the chucks of specimen, and by the gauge length of 40 mm. The inner and outer diameter of specimens were 10 mm and 12 mm respectively. The fatigue tests and microscopic examination were carried out after the removal of polished layer by electro-polishing. The fatigue limit of the mild steel and the semi-hard steel were 113 MPa and 127 MPa, respectively.

#### EXPERIMENTAL RESULTS AND DISCUSSION

The typical phenomenological relation between the cyclic-shear-stress-induced creep and the number of cycles  $N$  is shown in Figures 1 and 2. The parameters in the figures indicate the mean stress in MPa units. These typical figures show the following:

- a) The form of curves is expressed by the following equation:

$$\gamma = C \cdot \log N + C' \quad (1)$$

$$\text{or} \quad \gamma = C \cdot \log(\gamma N) \quad (1')$$

Heat treatment of mild steel:

900°C, 30 min. prior to machining, 600°C, 30 min. in vacuum prior to static or fatigue test.

Mechanical properties of 0.15%C mild steel:

Tensile strength  $\sigma_B = 415$  MPa, Upper yield point  $\sigma_{SU} = 266$  MPa, Lower yield point  $\sigma_{S1} = 253$  MPa, Upper yield point under torsion  $\tau_{SU} = 185$  MPa, Lower yield point under torsion  $\tau_{S1} = 153$  MPa.

Namely the rate of shear strain  $d\gamma/dN$  is inversely proportional to the number of cycle  $N$ .

$$d\gamma/dN = C/N, \quad (2)$$

where the value  $C$  is independent of the mean stress in the case of mild steel for a constant stress amplitude. Equation (2) expressed the form of the master curve in differential equation form for mild steel which was reported in the previous paper by the present author.

In the case of super-purity polycrystalline aluminum a similar relationship exists when the axial strain and the number of cycles are plotted on log scales. Figure 3, drawn from data by Ronay [18], shows the axial strain accumulation of aluminum subjected to a static axial stress and cyclic reserved torsion.

In the case of the semi-hard steel, however, only a weak dependence on mean stress is shown. The above mentioned dependence of mean stress makes it difficult to express the fatigue deformation in a simple differential equation shape. In this case the constant  $C$  depends not only on stress amplitude but also on mean stress.

- b) The constant  $C$  of mild steel is independent of mean stress and is a function of stress amplitude. The relation is shown in Figure 4, which shows two relationships of the form  $C = B(\tau_0)^n$ , where  $B$  and  $n$  are empirical constants. The branch point approximately corresponds to the torsional fatigue limit under alternating stresses.
- c) Fatigue deformation curves can be approximately expressed by the constant  $C$ , which shows the inclination of the fatigue deformation curve, and by the initial shear strain  $\gamma_0$ , which is defined as the permanent strain when  $N$  is one hundred (for engineering convenience).

Several typical examples of structural change observed by an optical microscope are now reported. The surface of the electro-polished mild steel specimens were observed at room temperature using chromium shaded plastic replicas. For comparison a static torsion test and an alternating torsion test were made. The photograph taken on the statically twisted specimen is shown in Figure 7, which shows the surface at ( $\tau = 194$  MPa,  $\gamma = 19.0\%$ ). When the mild steel was subjected to an alternating stress of a magnitude near the fatigue limit ( $\tau_w = 114$  MPa), typical fatigue slip traces appeared in a few crystallites that were favourably oriented with respect to the direction of the axis of the specimen. An example is shown in Figure 8. This specimen number 17 was subjected to an alternating stress of  $\tau_a = 117$  MPa, where  $\tau_a$  stands for stress amplitude. Figure 8 shows one of the most slipped grains at  $N = 2.7 \times 10^6$ , where  $N$  stands for the number of cycles. When the shear stress was larger than 124 MPa, the very clear slip bands appeared over most grains. When the stress was smaller than the fatigue limit by ten percent, it was difficult to find slip even after  $10^6$  cycles. This agrees with the work by Hempel [19], Nisitani and Murakami [20] and others who observed electro-polished specimens. An example of a micrographic examination of cyclic-stress-induced creep is shown in Figures 9 and 10. Figure 9 shows the surface of specimen number 12 with  $\tau_a = 117$  MPa,  $\tau_m = 88$  MPa,  $N = 8 \times 10^2$ ,  $\gamma = 9.54\%$ , where  $\tau_m$  stands for mean stress. Figure 10 corresponds to the surface appearance when  $\tau_a = 117$  MPa,  $\tau_m = 88$  MPa,  $N = 5 \times 10^5$ ,  $\gamma = 19\%$ . The surface at the start of the test was almost the same as Figure 9, and the surface at  $N = 1.2 \times 10^6$ ,  $\gamma = 21\%$  was quite similar to Figure 10. Specimen number 13 was subjected to the following stress:  $\tau_a = 117$  MPa,  $\tau_m = 30$  MPa, and

is shown in Figures 11 and 12. The photographs were taken at ( $N = 8.82 \times 10^3$ ,  $\gamma = 3\%$ ) and ( $N = 10^5$ ,  $\gamma = 7.4\%$ ).

If Figures 10, 11 and 12 are compared with Figure 8 it may be said that the effect of mean stress is not evident. After inspecting the other photograph, the author concludes that the effect of mean stress on the surface appearance is generally small, and the maximum stress cannot be considered important, except when the stress amplitude is small. Figures 8 and 9 and the related photograph show that the slip bands increase in number initially and continue to show a steady increase in extent during fatigue deformation. The behaviour was similar to that of the slip bands appearing under alternating stress. These results coincide with the results on brass [21]. It may be worth noting that the slip bands of a specimen which is subjected to a smaller mean stress (Specimen No. 13, Figure 12) are nearly the same as the slip bands of a specimen which is subjected to a larger mean stress (Specimen No. 12, Figure 9), although the amount of plastic strain is different.

#### Prolongation of Fatigue Life Under Mean Stress

Figure 1 shows that the fatigue life of annealed steels under a constant stress amplitude with a superimposed larger mean stress is long compared with the life which is combined with a lower mean stress. The results are consistent with the best results on a 0.45% carbon steel under axial stress obtained by the present author [22]. The test results are shown in Figure 5. The abscissa shows the number of cycles of zero-compression  $N_1$  which was applied at the first step of the fatigue test. Each specimen was then subjected to zero-tension fatigue loading, until it broke. The ordinate shows the number of cycles  $N_2$  to failure under zero-tension, and the total number of cycles  $N_t = N_1 + N_2$ . The absolute value of the pulsating tension which followed the pulsating compression was equal to that of the pulsating compression and was 667 MPa for the test shown in Figure 5. The fatigue limit under pulsating tension was 588 MPa ( $\sigma_m = 294$ ,  $\sigma_a = 294$  MPa). Tensile strength  $\sigma_B = 817$  MPa, yield point  $\sigma_Y = 683$  MPa. If crack initiation is approximately independent of mean stress when the cyclic hardening is not great, the total number of cycles to failure  $N_t$  is expected to be constant, provided that the number of cycles  $N_1$  is smaller than the number of cycles  $N_n$  which corresponds to the 'before crack propagation period'. In this case, a fraction of  $N_2$  is used for crack initiation and the remainder for crack propagation. If number of cycles  $N_1$ , which is larger than  $N_n$ , is applied first, the survival number of cycles  $N_2$  under the pulsating tension is expected to be constant, because  $N_2$ , in this case, corresponds to crack propagation under a tensile mean stress. The total number of cycles  $N_t = N_1 + N_2$  is the sum of the 'before crack propagation period'  $N_n$  and the crack propagation period. The  $N_m$  in Figure 5 is the mean of  $N_2$ . The experimental results shown in Figure 5 meet the foregoing expectation. A similar investigation was made for brass by Nisitani and Yamasita [21]. Annealed metals generally show cyclic hardening. This results in a larger life to failure under mean stress, except that excess damage is given by the maximum stress. It may be said that the overall effect of a mean stress on the fatigue crack initiation and the deformation is small.

#### Effect of Change of Mean Stress

An example of the effects followed by the changes of mean stress during cyclic-stress-induced-creep of mild steel is shown in Figure 6. The curves AB and KLM are the usual fatigue deformation curves, which are obtained

under  $\tau_a = 117$  MPa,  $\tau_m = 29$  MPa and  $\tau_a = 117$  MPa,  $\tau_m = 88$  MPa respectively. The curve given by the open circles shows the effect of change of mean stress. Namely, the mean stress was switched from 29 MPa to 88 MPa, when the number of cycles was  $4 \times 10^4$ . The stress amplitude was kept constant. The principal feature of the change is that the fatigue deformation curve after the switching is quite similar in shape to the curve which was obtained on the virgin material by the switched stress. Consequently 'the factor of deformation under repeated shear stress'  $C$  of the equation (1) after switching is almost the same as the value for a virgin material. Even when the mean stress level is switched several times, the constants are almost unchanged in the case of mild steel. The effect of previous stressing at the different mean stress level can be expressed by the other constant  $C'$  of equation (1). The value  $C$  or  $C'$  is physically equivalent to  $\gamma_0$ .

#### CONCLUSION

Observations of cyclic-stress-induced-creep shows the relative importance of stress amplitude on the strain, on the life and on the microscopic structure. Principal conclusions obtained by macroscopic observations and by optical microscopic observations are as follows:

- a) Cyclic shear stress induced strain of steels is expressed by the following relationship, if regions where the stress is very small and where the cyclic-stress-induced strain stops increasing are excluded.

$$\gamma = C \cdot \log N + C', \quad d\gamma/dN = C/N,$$

where  $\gamma$  is cyclic stress-induced shear strain or fatigue strain, and  $N$  stands for the number of cycles at the time when  $\gamma$  is observed.

- b) The value of  $C$  is expressed by the following relationship:

$$C = F \left( \tau_m \right) \times \left( \tau_a \right)^n$$

where  $F$  is a function of mean stress in general, and is nearly constant for mild steel, and is weakly dependent on mean stress in the case of a semi-hard steel.

- c) The dependence of mean stress on the number of cycles to failure was discussed. The concept that the stress amplitude is primarily important for the fatigue deformation and the crack nucleation under repeated stress, and that the tensile mean stress principally works during the crack propagation period, was applied to the cyclic-stress-induced-creep problem. The additional related experimental background was shown.
- d) Microscopic structural changes observed by an optical microscope showed the importance of stress amplitude to the formation of slip bands of mild steel under cyclic-stress-induced creep conditions.

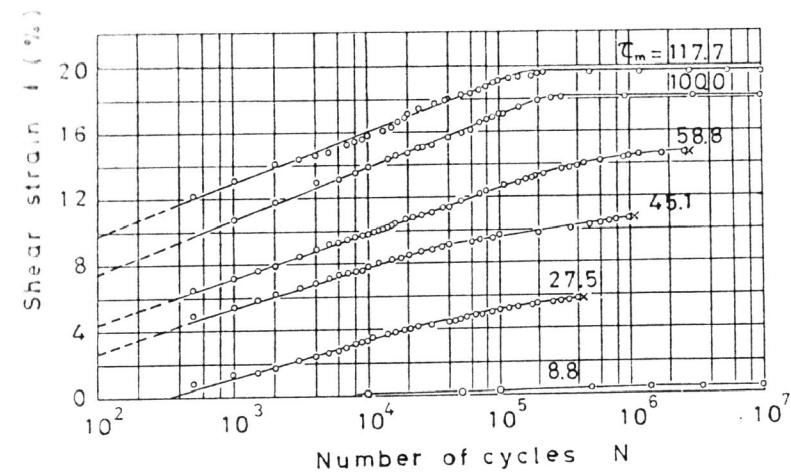
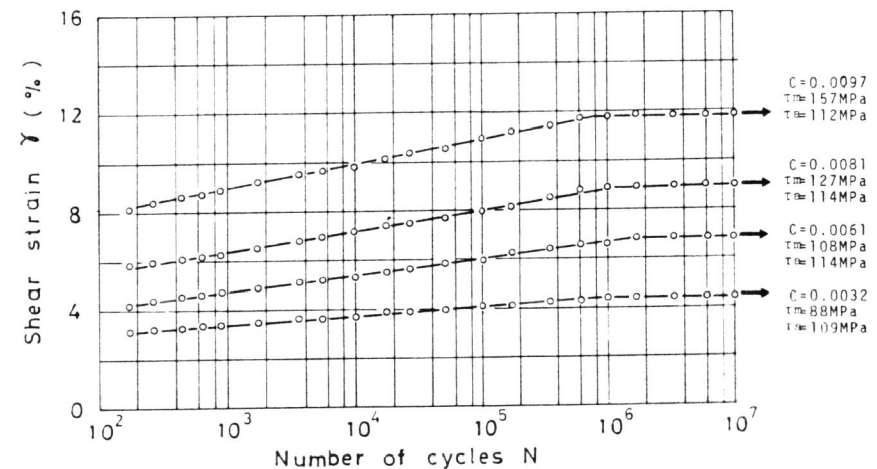
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\*Papers written in Japanese.

Figure 1  $\gamma$  versus N (0.15% C.S.)Figure 2  $\gamma$  versus N (0.46% C.S.)

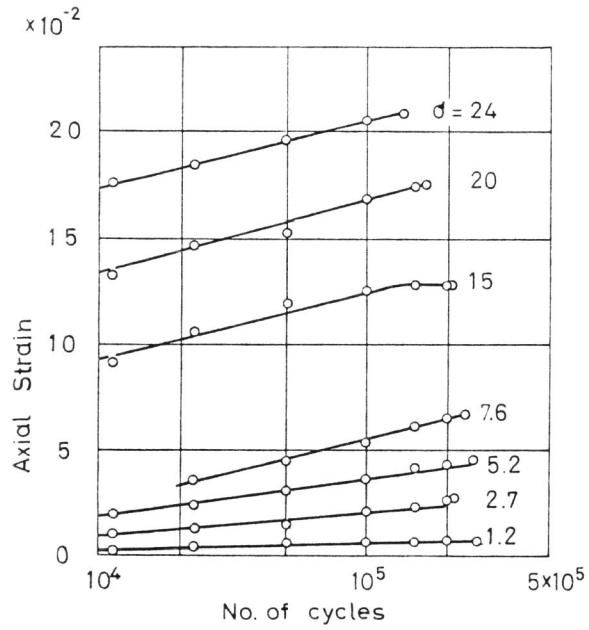


Figure 3  $\gamma$  versus N (Pure Al., after Ronay)

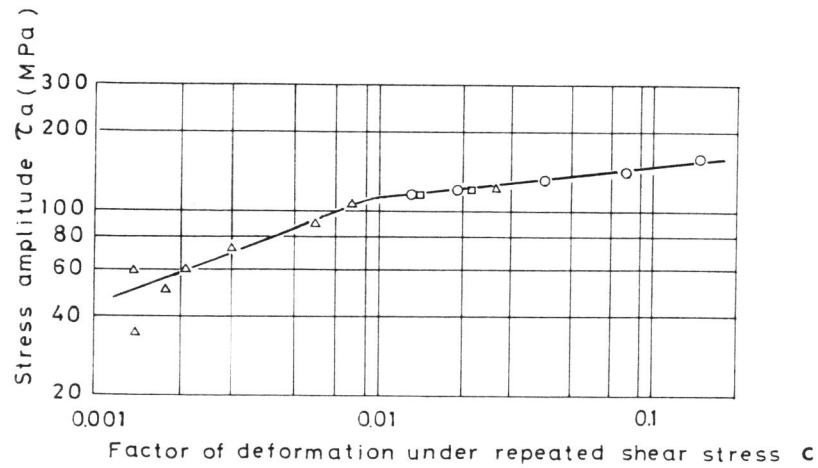


Figure 4  $\tau_a$  versus C (0.15% C.S.)

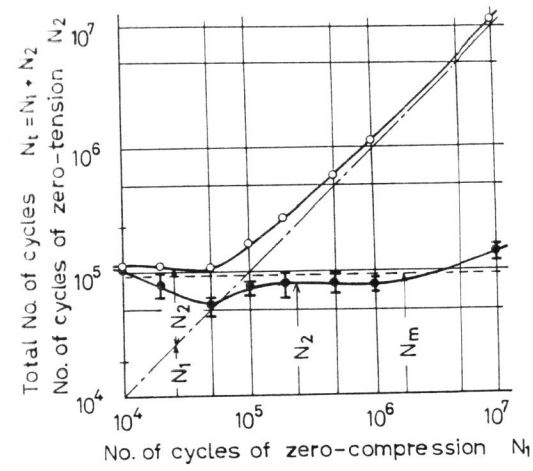


Figure 5  $N_t, N_2$  versus  $N_1$  (0.46% C.S.)

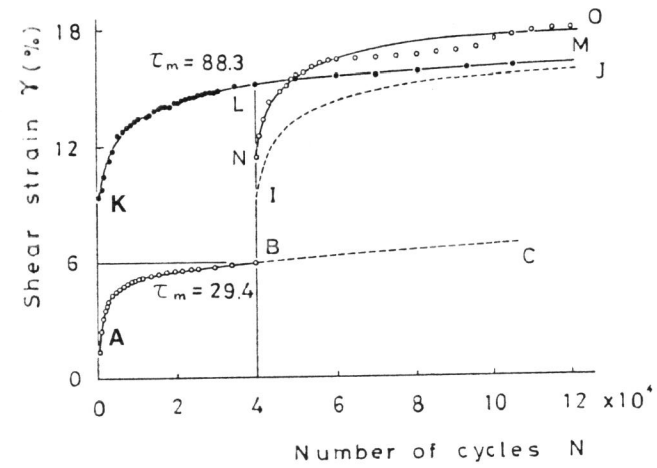


Figure 6  $\gamma$  versus N (0.45% C.S.)



Figure 7 Static Torsion (No. 3)

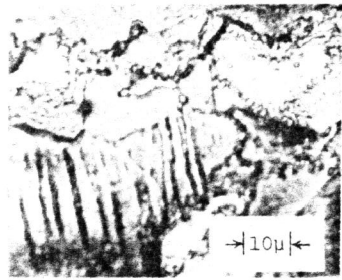


Figure 8 Alternating Torsion (No. 3)

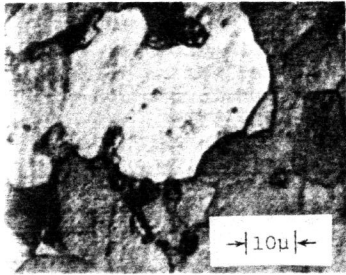


Figure 9 Cyclic Stress Induced Creep (No. 12)

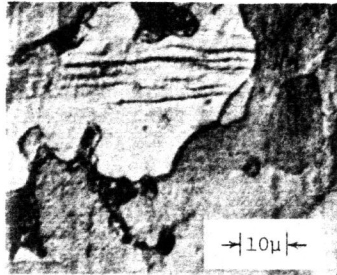


Figure 10 Cyclic Stress Induced Creep (No. 12)

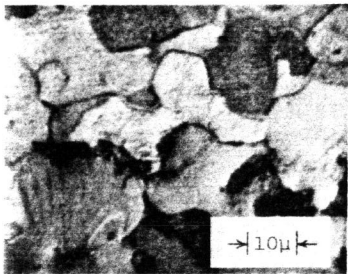


Figure 11 Cyclic Stress Induced Creep (No. 13)

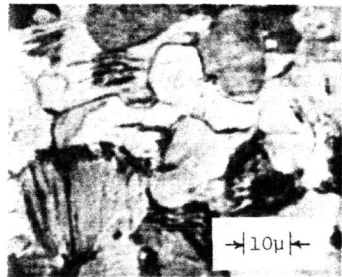


Figure 12 Cyclic Stress Induced Creep (No. 13)