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# NOTCH FATIGUE LIFE PREDICTIONS USING SMOOTH BAR FATIGUE DATA

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## INTRODUCTION

For a number of years, the Materials and Process Technology Laboratories of the Aircraft Engine Group, General Electric Co., has been obtaining smooth bar low and high cycle fatigue data for a number of materials used in jet engines. In most applications notches are present in the form of cooling holes, slots, etc., and numerous component simulation tests are performed before an engine is put into service. However, for purposes of preliminary design it is not economically feasible to carry out extensive test programmes and it is useful to develop procedures for estimating allowable stresses in the fatigue life regime of interest. For these reasons a survey of available analytical techniques for estimating notch behaviour from smooth bar fatigue and tensile properties was carried out. The most useful approaches are discussed below.

## CALCULATION OF NOTCH ROOT STRESSES AND STRAINS

The crucial step in estimating notch life fatigue behaviour is to calculate the stress and strain states at the notch root. Once this is done, the life can be estimated by comparison to smooth bar fatigue curves. This of course tacitly assumes that multiaxial effects will not significantly change the failure mode and that comparisons can be made on the basis of equivalent shear. While this can lead to problems [1], especially at elevated temperatures, it is customary to adopt this approach [2].

The basic equation was derived by Neuber [3] and relates the theoretical stress concentration factor $K_t$ to the product of the plastic stress and strain concentration factors, $K_t$ and $K_\varepsilon$ respectively:

$$K_t^2 = K_\sigma \cdot K_\varepsilon \tag{1}$$

$$K_\sigma = \frac{\sigma}{S} \tag{2}$$

$$K_\varepsilon = \frac{\varepsilon}{e} \tag{3}$$

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where $\sigma$ = notch root stress, S = nominal net section stress, $\varepsilon$ = notch root strain and e = nominal net section strain.

Applying equations (1) - (3) to the maximum stresses gives the following:

$$K_t^2 = \frac{\sigma_{max}}{S_{max}} \cdot \frac{\varepsilon_{max}}{e_{max}} \quad (4)$$

The plastic portion of the cyclic stress strain curve generally follows an equation of the form:

$$\sigma = A(\varepsilon_p)^{n'} \quad (5)$$

where $\sigma$ = stress, $\varepsilon_p$ = plastic strain, A = cyclic strength coefficient and n' = cyclic strain hardening exponent. The elastic portion is given by:

$$\varepsilon_{e\ell} = \sigma/E \quad (6)$$

Equations (5) and (6) apply equally for maximum stresses or stress amplitudes. Considering the maximum stress, one obtains:

$$\varepsilon_{max} = \varepsilon_{e\ell}^{max} + \varepsilon_p^{max} = \frac{\sigma_{max}}{E} + \left(\frac{\sigma_{max}}{A}\right)^{\frac{1}{n'}} \quad (7)$$

A similar expression is obtained for the net section strain in terms of the applied stress in:

$$e_{max} = e_{e\ell}^{max} + e_p^{max} = \frac{S_{max}}{E} + \left(\frac{S_{max}}{A}\right)^{\frac{1}{n'}} \quad (8)$$

Substitution of (7) and (8) into (4) after rearrangement gives:

$$\left(K_t S_{max}\right)^2 \left[1 + \frac{E}{S_{max}}\left(\frac{S_{max}}{A}\right)^{\frac{1}{n'}}\right] = \sigma_{max}^2 \left[1 + \frac{E}{\sigma_{max}}\left(\frac{\sigma_{max}}{A}\right)^{\frac{1}{n'}}\right] \quad (9)$$

Similarly, an equation may be written for the stress amplitudes:

$$\left(K_t S_a\right)^2 \left[1 + \frac{E}{S_a}\left(\frac{S_a}{A}\right)^{\frac{1}{n'}}\right] = \sigma_a^2 \left[1 + \frac{E}{\sigma_a}\left(\frac{\sigma_a}{A}\right)^{\frac{1}{n'}}\right] \quad (10)$$

where $S_a$ and $\sigma_a$ are the net section and local stress amplitudes respectively. We have chosen to use stress concentration factors ($K_t$) instead of fatigue strength reduction factors ($K_f$) in the Neuber relationship since in the vast majority of cases $K_t > K_f$. This introduces an element of conservatism into subsequent life calculations.





In general, the notch geometry ($K_t$), net section stress amplitude ($S_a$) and max stress ($S_{max}$) are known so that the left hand sides of equations (9) and (10) are also known. On the right hand side, only the local stresses are unknown and they can easily be determined by an iterative procedure which is easily done by computer. The stresses can be put in terms of strains by substitution into (7) and (8). Of course the max strain and strain amplitude define the A ratio:

$$A_\varepsilon = \frac{\text{alternating strain}}{\text{mean strain}} = \frac{\varepsilon_a}{\varepsilon_{max} - \varepsilon_a} \quad (11)$$

The A ratio thus defines the mean strain and one procedure for obtaining the life is to enter the smooth bar life versus strain range curve at the mean strain of interest. In the case of plane-strain, Young's modulus E is divided by $1-\nu^2$ where $\nu$ is Poisson's ratio. This is not rigorous* but appears to be reasonable from comparison to plane-strain elastic problems.

## FAILURE CRITERIA AND COMPARISON WITH EXPERIMENTAL DATA

Having calculated the appropriate notch root strains, a failure criterion must be adopted. The ideal situation would be to have experimental life versus strain range curves for a wide range of mean strains. However, such data is not usually available and other approaches must be adopted. Strain controlled smooth bar fatigue curves ($A_\varepsilon$ = 1) have been obtained for a number of materials including Inconel 718 Astroloy and Ti-6Al-4V. To a very high degree of accuracy, these curves can be represented by 3 straight line segments on a log-log basis. Both mechanical property and appropriate fatigue data are shown in Table 1. In addition, notch fatigue experiments have been carried out in the load control mode ($A_\sigma$ = 1). It is assumed that for sufficiently sharp notches, the material at the notch root is strain cycled. This assumption, which is physically reasonable, has been made in other investigations of crack propagation [4, 5, 6]. This comparison of smooth bar strain cycling data to notched bar load cycling data appears to be appropriate. Figures 1, 2 and 3 show representative results of the notch fatigue experiments along with predicted lives.

The curves labelled "Neublife" are based on the 3 line segments approximating the smooth bar data and in general they approximate the experimental data quite well.

As indicated previously, mean strain effects are taken into account only in as much as $A_\varepsilon$ = 1 smooth bar data was used in the predicative scheme. A more conservative approach can be adopted based on a representation of the fatigue curve that has a form similar to the Universal Slopes equation:

$$\Delta\varepsilon = \frac{3.5 f_1 \sigma_u}{E} N^{-\alpha} + 0.75 f_2 \varepsilon_f N^{-\beta} \quad (12)$$

* It is worth pointing out that Neuber's analysis is applicable only to anti-plane strain shear loading and extension to tensile notches, while apparently reasonable and widely used, has not been rigorously demonstrated to be correct.

where $\Delta\varepsilon$ = total strain range, $\sigma_u$ = ultimate tensile strength, $\varepsilon_f$ = true fracture strain, $\alpha, \beta$ = exponents which represent the slopes of the elastic and plastic portions of the logarithmic smooth bar fatigue curve, $f_1$, $f_2$ = factors to account for deviations of experimental results from the constants in the Universal Slopes equation.

Equation (12) is a 4 parameter fit to the smooth bar fatigue curve. Note that when $\alpha = 0.12$, $\beta = 0.60$, $f_1 = f_2 = 1$, the Universal Slopes equation results. It has been shown [7] for low cycle fatigue, that when mean plastic strains are present (i.e., $A_\varepsilon\ \infty$), the true fracture strain is reduced by the mean plastic strain $\varepsilon_m^p$. By analogy, the elastic strain should also be reduced if non-zero mean stresses are present. However as much fatigue data shows, the mean stress effect is small for low stresses so the elastic mean stress effect should not be as pronounced as in the plastic portion. A convenient empirical function that fits these observations is:

$$\sigma_e = \sigma_u \sqrt{1 - \frac{\sigma_m}{\sigma_u}} \qquad (13)$$

where $\sigma_e$ = "effective" stress replacing $\sigma_u$ in (12)

$\sigma_m$ = mean stress .

Equation (13) is a modification of the well-known Gerber parabola for accounting for mean stress. The result of incorporating these considerations into equation (12) is:

$$\Delta\varepsilon = \frac{3.5 f_1 \sigma_u}{E} \cdot \sqrt{1 - \frac{\sigma_m}{\sigma_u}} \cdot N^{-\alpha} + 0.75 f_2 (\varepsilon_f - \varepsilon_m) N^{-\beta} . \qquad (14)$$

Since $\Delta\varepsilon$, $\varepsilon_m$ and $\sigma_m$ are known from the Neuber notch analysis, all quantities other than N are available from smooth bar fatigue and mechanical property testing, equation (14) can be solved for N using an easily computerized iterative procedure. The results of this procedure are also shown on Figures 1, 2, and 3 by the curves marked "Neubman". Since fatigue data is not always available in the true LCF regime (i.e., plastically dominated) $\beta$ has been chosen to be 0.6 in agreement with the Universal Slopes equation. The parameter $f_2$ is then fixed by drawing a line of slope - 0.6 (on a log-log basis) through the lowest life data point and extrapolating to N = 1. Subtracting the extrapolated value of the elastic strain yields the plastic strain. Equating the plastic strain at N = 1 to 0.75 $f_2\varepsilon_f$ fixes $f_2$. The "elastic" parameters $\alpha$ and $f_1$ are determined in a similar fashion except that a log linear approximation to the high cycle regime is used. The appropriate fatigue parameters are listed in Table 2. Note that for the elastic portion of the curve (which is most important for life regimes of engineering interest) $\alpha$ is occasionally greater than 0.12, the value used in the Universal Slopes equation, which implies nonconservatism if 0.12 is used.

As mentioned previously, the main goal of this work was to calculate allowable stresses for life ranges of interest. The degree of predictability





is conveniently represented on plots of predicted versus observed stress at lives of interest. A typical graph is shown in Figure 4. In all cases, the predictions tend to be usefully conservative estimates to the actual data. The Neubman procedure was used as the predictive instrument in constructing these plots.

## SUMMARY AND CONCLUSIONS

1. Neuber notch analysis has been used to calculate both the maximum stress and stress amplitude at the notch root of load cycled fatigue bars. The stresses can be converted to strain range and mean strain by means of the cyclic stress-strain curve.

2. Cyclic lives were estimated using two techniques. In the first, a 3 segment approximation of smooth bar fatigue data was used. The predicted stresses at lives of interest were in good agreement with the results of fatigue tests on notched bars. A slightly more conservative procedure consisted of approximating the smooth bar data by the sum of two log linear segments to maintain the form of the Universal Slopes equation. The effects of mean stress and strain were taken into account and the results were a usefully conservative estimate to the experimental data.

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Table 1 Data Used for Notch Fatigue Life Predictions

| MATERIAL | TEMP °C | STRENGTH COEFFICIENT MPa | STRAIN HARDENING EXPONENT N | REDUCTION IN AREA % | YOUNG'S MODULUS E ($10^3$MPa) | POISSON'S RATIO ν | TENSILE STRENGTH MPa | STRESS CONCENTRATION $K_T$ | A-RATIO $A_σ$ |
|---|---|---|---|---|---|---|---|---|---|
| ASTROLOY | 566 | 1896 | .095 | 20.6 | 180 | .32 | 1324 | 2<br>3 | 1.0<br>1.0 |
| ASTROLOY | 649 | 1448 | .075 | 23.2 | 174 | .33 | 1293 | 2<br>3 | 1.0<br>1.0 |
| INCONEL 718 | RT | 2069 | .097 | 40.2 | 200 | .31 | 1365 | 2<br>3 | 1.0<br>1.0 |
| INCONEL 718 | 510 | 2000 | 0.14 | 43.8 | 172 | .32 | 1116 | 2<br>3 | 1.0<br>1.0 |
| INCONEL 718 | 538 | 2413 | .163 | 40.6 | 170 | .33 | 1110 | 2<br>3 | 1.0<br>1.0 |
| INCONEL 718 | 649 | 1724 | .15 | 26.5 | 162 | .33 | 1083 | 1.5<br>2<br>3 | 1.0<br>1.0<br>1.0 |
| Ti-6Al-4V | RT | 1875 | .112 | 36.8 | 115 | .32 | 983 | 3.0<br>1.8<br>3.0<br>3.0 | 0.5<br>1.0<br>1.0<br>∞ |
| Ti-6Al-4V | 316 | 1214 | .112 | 56.3 | 101 | .34 | 693 | 3.0<br>1.8<br>3.0<br>3.0 | 0.5<br>1.0<br>1.0<br>∞ |

Table 2 Fatigue Parameters

| Material | Temperature (°C) | α | $f_1$ | β | $f_2$ |
|---|---|---|---|---|---|
| Astroloy | 566 | 0.131 | 1.11 | 0.60 | 0.879 |
| Astroloy | 649 | 0.089 | 0.70 | 0.60 | 1.27 |
| Inconel 718 | RT | 0.055 | 0.64 | 0.60 | 1.69 |
| Inconel 718 | 510 | 0.065 | 0.68 | 0.60 | 1.35 |
| Inconel 718 | 538 | 0.065 | 0.68 | 0.60 | 1.59 |
| Inconel 718 | 649 | 0.070 | 0.58 | 0.60 | 1.95 |
| Ti-6Al-4V | RT | 0.161 | 1.71 | 0.60 | 0.58 |
| Ti-6Al-4V | 316 | 0.111 | 1.36 | 0.60 | 0.43 |





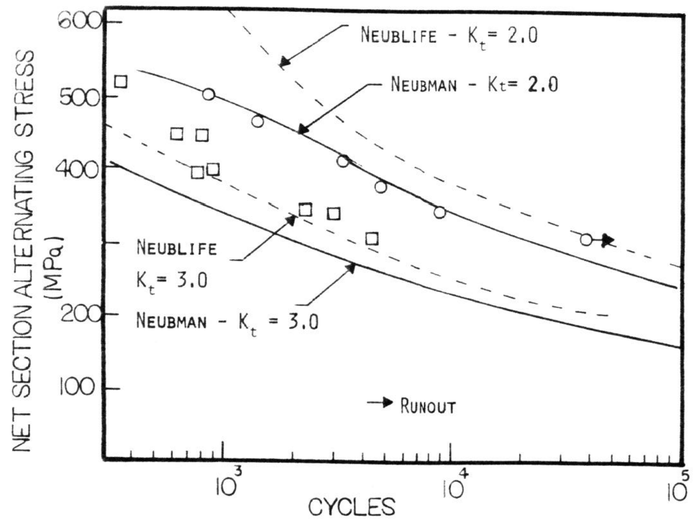


Figure 1 Effect of $K_t$ on Notched Fatigue Life Predictions for Astroloy at 649° C

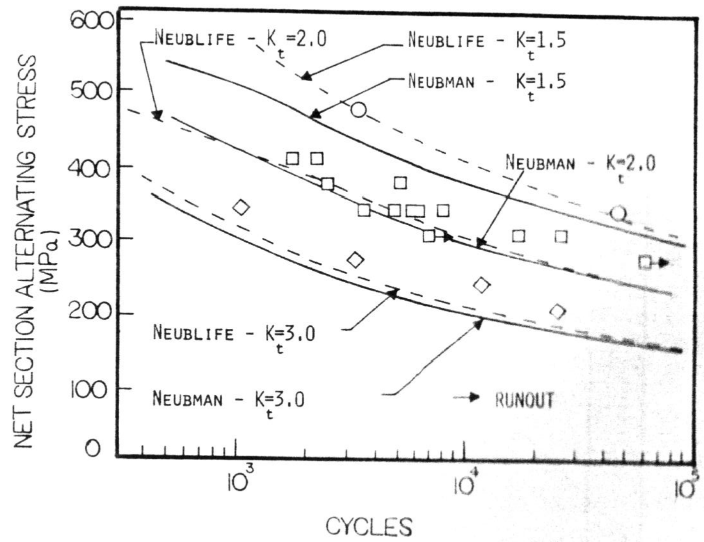


Figure 2 Effect of $K_t$ on Notched Fatigue Life Predictions for Inconel 718 at 649° C

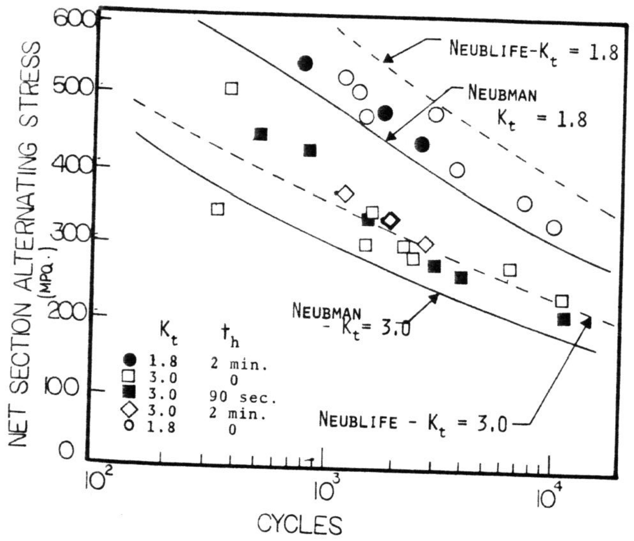


Figure 3 Effect of $K_t$ on Notched Fatigue Life Predictions for Ti-6Al-4V at RT. The Filled and Partially Filled Points Represent Hold Times at Peak Load

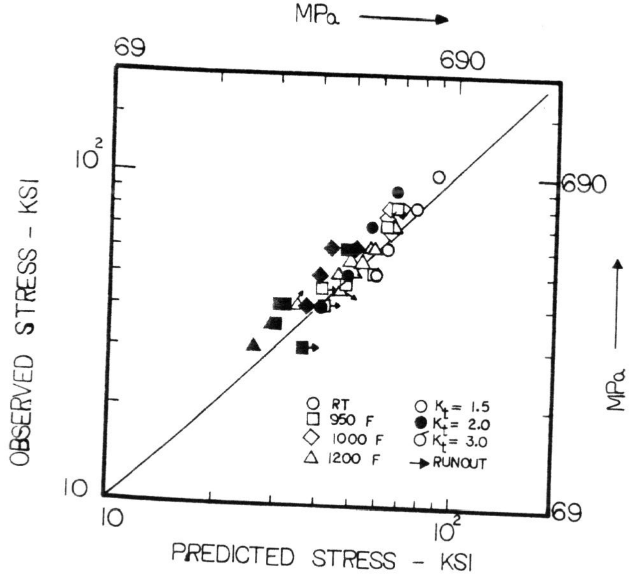


Figure 4 Comparison of Predicted and Observed Stress Amplitudes for LCF Lifetimes $10^3$ - $10^5$ Cycles, Inconel 718