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FATIGUE LIFE PREDICTION BY AMPLIDUTE TRANSFORMATION

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INTRODUCTION

In order to achieve a high level of structural reliability of fatigue loaded components, fatigue life predictions have to be made in several stages of the design and development process. For that purpose appropriate procedures of fatigue life prediction must be available.

Miner's rule is the best known and the simplest of those procedures. In its original version or in some modified versions it is applied to stress-time histories and to fatigue data presented in terms of nominal stress (nominal stress approach). There are no difficulties in a formal application of Miner's rule to any type of stress-time history, although this attribute does not give rise to the expectation of a particular reliability of the result. Based on a comprehensive literature survey Schijve [1] gave a detailed analysis of the associated problems. Last not least, present experience has been gathered mainly from studies and design applications based on the nominal stress approach. Also most of the available S-N data are given in terms of nominal stress.

Considering more recent procedures the fatigue life periods of crack initiation and of crack propagation are treated separately. Fracture mechanics methods are applied for fatigue life prediction in the crack propagation period. In the crack initiation period fatigue life is considered to be primarily a function of those stresses and strains that occur locally at the potential points of crack initiation (local stress-strain approach). Due to local plasticity a non-linear relation exists between the history of the nominal stress and that of the local stress which is mainly dependent on the magnitude and on the sequence of the peak stresses. Among numerous publications on that topic a survey by Conle, Nowack and Hanschmann [2], and the pilot studies of Crews and Hardrath [3], of Martin, Topper and Sinclair [4], and of Wetzel [5] shall be mentioned here. It was shown that the incorporation of cyclic stress-strain concepts can lead to major improvements in accuracy, particularly when dealing with complex stress-time histories. On the other hand, however, the applicability of present concepts seems to be restricted more or less to structural elements having a well defined and simple notch geometry, as geometrical effects are dealt with in a rather simplified manner by using Neuber's rule [2].

With the idea that present procedures allow some further development, and in order to extend their applicability with special reference to the background of existing experience an investigation at LBF was aimed at combining the particular advantages of the nominal stress approach with the improvements attainable by the local stress-strain approach, including further on some aspects of the fracture mechanics procedures. The result of that development is the procedure of amplitude transformation. The following

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is a brief outline of its underlying concept. A more detailed description is given in reference [6].

BASIC ELEMENTS OF KNOWN PROCEDURES

In following the nominal stress approach, Figure 1d, the stress-time history S(t) is analysed by a cycle counting procedure CC in order to specify individual cycles and to derive the corresponding amplitude frequency distribution AFD. When dealing with complex stress-time histories it may be necessary to distinguish stress amplitudes S_a of different mean stress S_m by establishing a joint frequency distribution. Accordingly a set of S-N curves $N_f(S_a\ ;\ S_m)$ for different mean stress conditions will be required for calculating and summing up the damage increments $1/N_f$ of each cycle identified.

Also when following the local stress strain approach, Figure 1a, the input of the stress-time history S(t) is provided in terms of nominal stress. The corresponding sequence of the local stress and strain σ, ε is obtained from a computer simulation of the elastic-plastic material behaviour by means of a hysteresis model and by applying Neuber's rule. For that purpose an additional input of the cyclic stress-strain curve of the material and of the notch factor K_t of the structure is required. Observing the hysteresis of the local stress-strain function the completion of an individual cycle CYC is identified each time a hysteresis loop closes. For each cycle a damage parameter P is derived from the hysteresis loop, considering either the elastic and/or plastic strain ranges or a combination of strain range and stress or stress range, Figure 2. From input data describing the constant amplitude endurance $N_f(P)$ of the material as a function of the particular type of damage parameter P selected, the damage increment $1/N_f$ of each cycle is calculated and linearly summed up.

THE PROCEDURE OF AMPLITUDE TRANSFORMATION

The procedure of amplitude transformation may be understood as a modification of the local stress-strain approach by including specific elements of the nominal stress approach. Its particular feature is that a conventional S-N curve $N_f(S_a)$, describing in terms of nominal stress the structural fatigue strength of the component under consideration, will be used for calculating the damage increments along with the local stress-strain approach. In the following this S-N curve will be called the "reference S-N curve".

There are two rows of the flow chart indicating two separate computer runs. The first row, Figure 1b, shows how the constant amplitude stress cycles of the reference S-N curve are treated in order to fit with the local stress-strain approach. Constant amplitude cycles $S_m \pm S_a$ are specified with a mean stress S_m equivalent to that applied when establishing the reference S-N curve. The damage parameter P corresponding to these constant amplitude cyclies is calculated in rather the same way as for cycles of the actual stress-time history. By repeating the calculation for several stress levels of the reference S-N curve a relationship between the nominal stress amplitude S_a and the damage parameter P will be obtained. In a simplified verison, indicated by dashed lines, the relationship $(S_a;P)$ may be used for scaling the ordinate of the reference S-N curve in order to derive constant amplitude endurance data $N_f(P)$ suitable for use with the local stress-strain approach.

The final version of the procedure of amplitude transformation differs only slightly from the simplified version: The stress-time history is treated accordingly to the local stress-strain approach in order to calculate the damage parameter P of each cycle as defined from the hysteresis loop, Figure 1c. However, instead of determining N_f as a function of P the corresponding nominal stress amplitude $S_a(P)$ is calculated for each cycle of the actual stress-time history as a function of the damage parameter P by making use of the pre-derived relationship between S_a and P. $S_a(P)$ is the transformed stress amplitude, its corresponding mean stress is that of the reference S-N curve. The transformed stress amplitudes may be classified in order to get an amplitude frequency distribution AFD, and they may be used to find the number of cycles $N_f(S_a)$ from the reference S-N curve for calculating the damage increments $1/N_f$.

DETAILS OF REALIZATION

The hysteresis model described by Wetzel [5] was used for computing the local stress and strain values. In addition, it was provided that a stabilized cyclic stress-strain curve will hold for the major period of the fatigue life. In order to have a stable envelope of the hysteresis loops a preload by the peak to peak cycle of the stress-time history maybe applied to the memory of the hysteresis model.

Referring to experimental data by W. Schütz [7] the mean stress sensitivity M of materials has to be considered within two extreme cases, Figure 3: In the case M = 0 (very low strength, ductile materials) the endurance N_f is a function of the stress amplitude S_a only. In the case M = 1 (very high strength or fully elastic materials) the endurance N_f is a function of the maximum stress or $(S_a + S_m)$ respectively. The Smith parameter [8], Figure 2a, has been shown to give figures of M only between M = 0 and M = 0.4 [6]. Therefore the damage parameter has been re-defined on the basis of a simple fracture mechanics model, assuming a crack-like defect at the notch root to be undamaging if the local stress is compressive, Figure 2b. By observing the small defect being situated in plastically strained material the definition of Figure 2c was derived, where the reference level BB is shifted downwards if the instantaneous compressive stress is lower than it has been before [6]. Accordingly the damage increments of all succeeding cycles will be increases.

Furthermore a modified version of Miner's rule [9] was adopted in order to take account of the damaging effect of transformed stress amplitudes ranging below the fatigue limit. By Miner's rule in its original version those cycles will not contribute any damage as the S-N curve states $N = \infty$. From a fracture mechanics point of view the stress intensity range ΔK will have to exceed a threshold value ΔK_{th} for crack growth to occur. Due to the cumulative increase of the crack length, however, the stress range $\Delta\sigma_e$ corresponding to ΔK_{th} will decrease. Thus small cycles may be non-damaging when occurring at the beginning of the fatigue life, but cycles of the same stress range may become damaging after some crack growth has developed. It may be shown [9] that the reference S-N curve of slope k has to be extended below the fatigue limit by a line of shallower slope of (2k - 1) in order to take account of that effect, Figure 4. Practically this modification proves as a means to extend the validity of the derived prediction into the period of crack propagation fatigue life.

FIRST RESULTS

The significance of the procedure of amplitude transformation can be demonstrated best by presenting the results obtained for simplified flight by flight sequences, Figure 5, that were investigated by D. Schütz [10]. The experiments, using axially loaded notched specimens of $K_t = 2.5$ in two types of material, Table 1, were aimed at clearing up the question, whether adjusting the mean stress of the S-N data according to the residual stresses from local plasticity will improve the fatigue life prediction by Miner's rule. For comparison three predictions by amplitude transformation are presented here, considering the two extreme cases of M = 0 and M = 1, and the specific material data of the cyclic stress-strain curve according to

$$\varepsilon = \sigma/E + 0.002 \cdot (\sigma/\sigma_{zs})^{1/n} \qquad (1)$$

where E = 73500 (71000) N/mm², σ_{zs} = 418 (408) N/mm², and 1/n = 9.46 (9,90) for the Al Cu Mg 2 or (AZ 74/72) aluminum alloy respectively. For either material the reference S-N curve is that of Figure 4.

The results are summarized in Table 1: For the sequences containing ground-air-ground cycles (GAG cycles) the fatigue life predictions are unconservative (predicted life greater than the experimental life) in all cases, except for the prediction by amplitude transformation using the specific material data. For the sequences without ground-air-ground cycles the predictions are conservative in all cases, except for the predictions derived by Schütz when using an adjusted mean stress. Predictions by the local stress-strain approach may be expected to be similar to those obtained by the adjusted mean stress. Identical predictions as in the case of M = 0 will be obtained by the nominal stress approach when using the range-pair or the rain-flow cycle counting method and neglecting any mean stress effect.

From the results it may be concluded (a) that there is a detrimental effect of the ground-air-ground cycle due to plasticication of the notch root in local compression, and only the procedure of amplitude transformation using the specific material data and the damage parameter according to Figure 2c can take account of that interaction effect, and (b) that there is a favourable effect from the occurrence of the tensile peak stresses, whereas the smaller amplitudes are the damaging ones; up to now, none of the available procedures can take account of this kind of interaction effect. The procedure of amplitude transformation, however, allows some further improvement in this respect by deriving an appropriate definition of the damage parameter. As a whole the results are characterized by the average and by the range of variation of the indicated life ratios. The procedure of amplitude transformation using the specific material data proves to be superior to all others, as the predictions are conservative throughout, and the range of variation of 2.2 is the smallest.

SUMMARY

The procedure of amplitude transformation is based on the hypothesis of linear damage accumulation and it is aimed at improving the fatigue life prediction of structural components. A new concept enables changes of the local mean stress and interactions resulting from a complex loading sequence to be considered more effectively than by current methods. The basic idea is to decrease (or to increase) the nominal stress amplitude of each cycle analysed in such a way that favourable (or detrimental) effects of mean stress and of interactions will be covered. In a computer programme the degree of amplitude transformation is derived from the hysteresis loop of the local stress and strain at the fatigue critical point of stress concentration by means of a properly defined damage parameter. A conventional S-N curve, describing in terms of nominal stress the structural fatigue strength of the component under consideration, is used for calculating the damage increments of the identified cycles.

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Table 1 Fatigue Life Predictions in Terms of the Number of Flights to Failure Expressed by the Ratio of Predicted Life to Experimental Life, Probability of Survival P_s = 0.5

			PREDICTED / EXPERIMENTAL LIFE					
MATERIAL	STRESS-TIME HISTORY	EXPERIMENTAL No. of flights	ratio	NOMINAL STRESS	ADJUSTED MEAN STRESS	AMPLITUDE TRANSFORMATION M = 0	SPECIFIC	M = 1
AlCuMg 2 (2024 T 3)	NORMAL SEQUENCE	2 600	1.00	2.63	2.86	1.35	0.54	2.46
	" , WITHOUT GAG CYCLE	12 200	1.00	0.86	1.31	0.65	0.42	0.64
	NORMAL SEQUENCE MODIFIED	2,050	1.00	2.56	3.57	1.58	0.68	2.68
	" , WITHOUT GAG CYCLE	8 700	1.00	0.82	1.89	0.75	0.49	0.74
AZ 74/72 (≈ 7075 extr.)	NORMAL SEQUENCE	2 250	1.00	1.67	1.92	1.57	0.56	2.85
	" , WITHOUT GAG CYCLE	14 500	1.00	0.45	1.25	0.54	0.31	0.54
AVERAGE RATIO			1.0	1.5	2.1	1.1	0.5	1.7
RANGE (MAX/MIN)			1.0	5.8	2.9	2.9	2.2	5.3

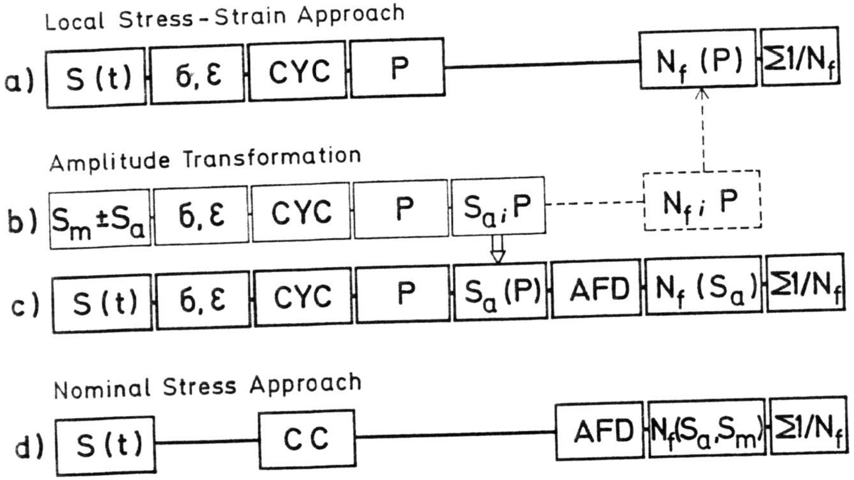

Figure 1 Procedures of Fatigue Life Prediction (Explanation Given in Text)

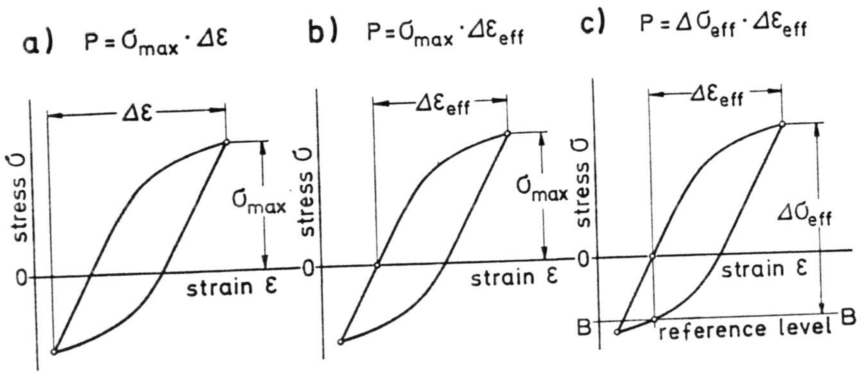

Figure 2 Definition of Damage Parameters

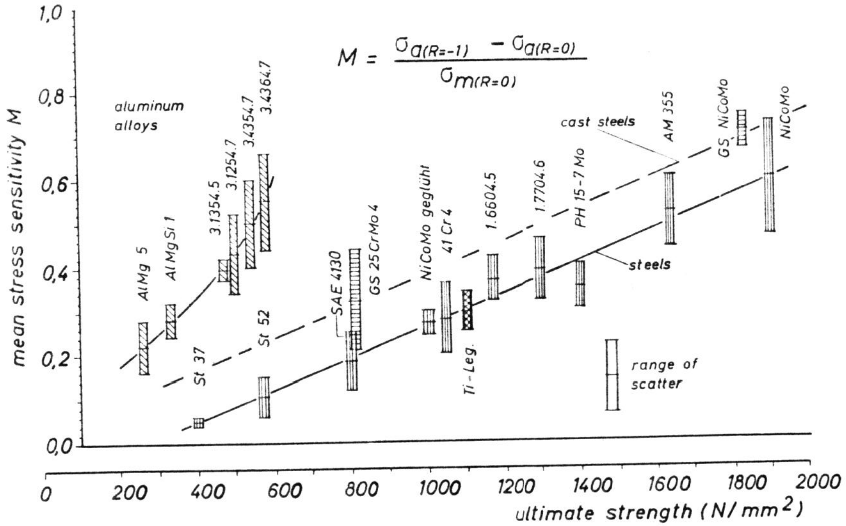

Figure 3 Mean Stress Sensitivity of Various Materials According to W. Schütz; Axially Loaded Specimens, K_t = 1.0 to 5.0, N = 10^4 to 10^6

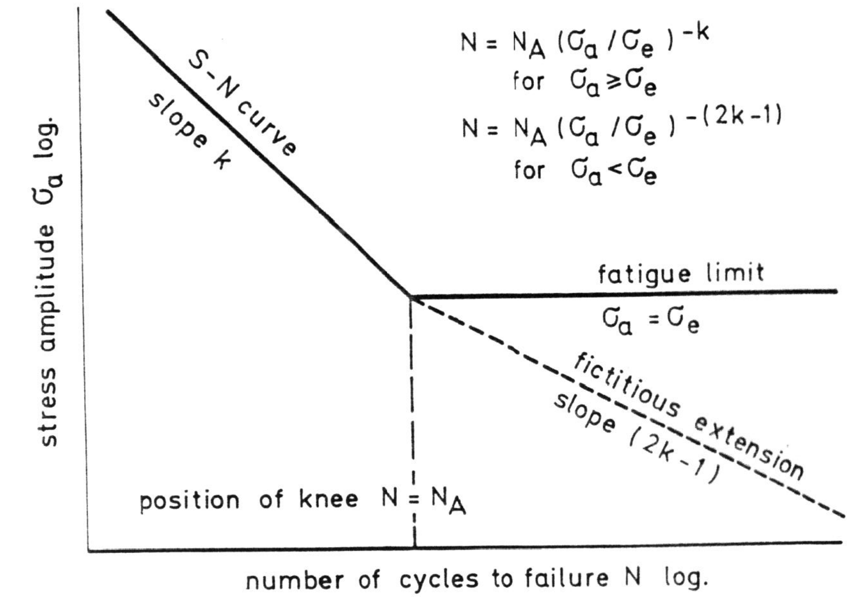

Figure 4 Reference S-N Curve and Its Fictitious Extension Below the Fatigue Limit. Data from D. Schütz: Al Cu Mg 2 or AZ 74/72, Notched K_t = 2.5, Mean Stress 140 N/mm², Fatigue Limit σ_e = 53 N/mm², knee N_A = 6000 000, Slope k = 4.05

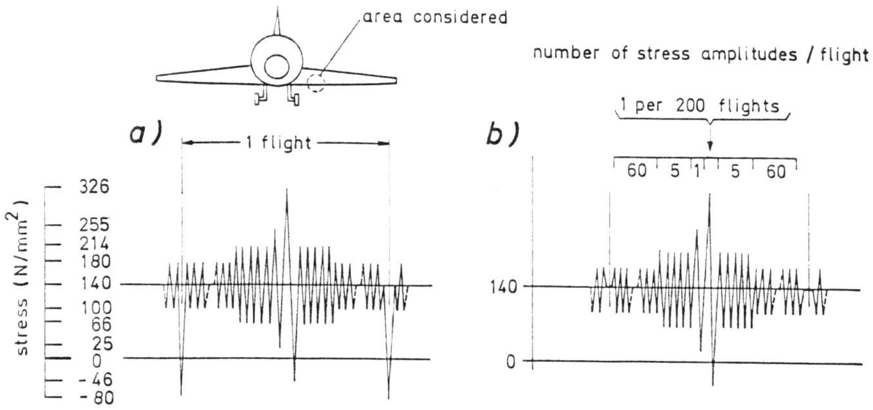

Figure 5 Stress-Time Histories (Flight by Flight Sequences) Investigated by D. Schütz; (a) Normal Sequence, (b) Sequence Without Ground-Air-Ground Cycle